

Estimating the Implied Required Return on Equity with a Declining Growth Rate Model

by

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Abstract

We illustrate the calculation of the implied required return on equity using a new declining growth model as compared to a single constant growth model, a two-stage constant growth model, the H Model, and the Ohlson-Juettner model. The declining growth model allows the flexibility to more realistically match the cash flow patterns according to the life cycle theory of the firm. Using Value Line data, we demonstrate the differences in estimates from each of the valuation models. The declining growth model provides a versatile new method for calculating the implied required return on equity.

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Although there are various methods currently available for estimating the required return on equity for a company, we propose a new method based on a new declining growth rate valuation model, which is simple to apply and versatile in its application.

The required rate of return is an important calculation as part of the process for determining the Weighted Average Cost of Capital (WACC). Corporations need to calculate the WACC for project analysis in capital budgeting and for valuation purposes in mergers and acquisitions. Utilities need to estimate WACC to support rate increases at regulatory rate hearings. From the corporate perspective, Brotherton, Eades, Harris, and Higgins (2013) report from a recent survey that 90% of respondents from corporations, consultants, and textbooks estimate the required return on equity using CAPM, with beta being the measure of risk. The remaining respondents used variations of the CAPM or a Dividend Discount Model (DDM) to estimate the required return on equity. From the regulatory perspective, various sources present multiple methods for calculating the cost of capital for utilities that are recognized by regulatory agencies. For example, the Brattle Group (2013) and Witmer and Zorn (2007) document two main classes of models used to estimate the required return on equity. The realized returns class includes the Sharpe-Lintner CAPM, the Fama French model, the Consumption CAPM, and the Arbitrage Pricing Theory (APT) model. This class of models may be less reliable when interest rates on government bonds are unusually low. A second class of models utilize a forward-looking approach to calculate an implied return on equity assuming the current stock price reflects an efficient market. This class is primarily based on the constant Dividend Discount

Model (DDM) or variations such as the Residual Income Model and Abnormal Earnings models. In each of these cases, an implied return on equity is calculated as the discount rate that results in the current stock price with the particular model that is chosen. The most common patterns for the present value of future cash flows include four main approaches: (1) A single stage constant growth model according to Gordon (1962), (2) A two-stage constant growth model such as applied by Claus and Thomas (2001), (3) An H-model estimation using the Fuller and Hsia (1984) equation, and (4) Variations of the Ohlson-Juettner (2005) abnormal earnings model. We offer an additional option in these forward-looking approaches – a cash flow pattern with a declining growth rate according to Holland (2018). In this model, an initial high short-term rate, g_s , declines asymptotically to a lower long-term mature rate of g_L . This new valuation model provides an additional flexible method for matching actual firm performance.

This paper is divided into seven sections. The first section sets the stage with a basic introduction and background for calculating the implied required return on equity using a simple constant dividend growth model. The second section focuses on the use of multi-stage valuation models to calculate an implied return on equity. The next two sections apply the H-Model and the Ohlson-Juettner (2005) abnormal earnings model to estimate an implied return on equity. The fifth section focuses on estimating the implied return on equity from the Holland (2018) valuation model with declining growth rates, which is the main contribution of this paper. The sixth section compares the results from the declining growth rate model to the existing valuation approaches using Value Line data for the Dow 30 stocks. Finally, the last section is a summary of the paper.

Background: The Constant Growth Single Stage Model

At the most fundamental level, the cash flows to the holder of a stock are the future dividends. Thus, the value of a stock would be the present value of all expected future dividends, or

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + R_E)^t} \quad (1)$$

where V_0 = the value at time zero,

D_t = the dividend at time t , and

R_E = the required rate of return for equity cash flows.

As a practical matter, estimating future dividends over an extended period of time can be difficult. Therefore, models have been developed that simplify the present value of future dividends. The most widely recognized of these models is the constant dividend growth model, mentioned by Williams (1938) and then popularized by Gordon and Shapiro (1956) and Gordon (1962).

$$V_0 = \frac{D_1}{R_E - g_L} \quad (2)$$

where g_L = a constant long-term growth rate.

There are two conditions in the use of this model that restrict its usefulness: (1) The constant growth must be less than the required rate of return, and (2) the constant growth must be small enough in the long run to reflect only growth in the economy (or growth in population). These restrictions limit the direct application of this model because the initial growth rate is frequently larger than the estimated long-term growth in the economy, and often is larger than

the required return on equity in the short-term. Thus, a direct application of the constant growth model for valuation is generally restricted to a very few mature companies with stable cash flows and low growth. However, a nice feature of this model is that a simple closed form equation is available to calculate a required return on equity. Assuming that the current stock price P_0 is equal to the calculated fair value V_0 , the constant dividend growth model can be re-arranged to reflect an implied required return on equity. Solving for R_E yields

$$R_E = \frac{D_1}{P_0} + g_L \quad (3)$$

However, as noted above, this model applies mainly to stable, mature companies. Thus, the accuracy of this formula is uncertain for companies that are not in a mature, low constant growth situation.

Multi-Stage Models

To overcome the restrictions of the single-stage constant dividend growth model with one long-term growth rate, analysts often apply multi-stage models. The simplest form is a two-stage model. In this case, dividends are estimated for a finite number of years (T) using a larger short-term growth rate. Then a terminal value (V_T) estimates the remaining dividends with the constant dividend growth model using a low long-term growth rate (g_L) suitable for a mature company, as follows:

$$V_0 = \sum_{t=1}^T \frac{D_t}{(1 + R_E)^t} + \frac{V_T}{(1 + R_E)^T} \quad (4)$$

where

$$V_T = \sum_{t=1}^{\infty} \frac{D_{T+t}}{(1 + R_E)^t} = \frac{D_{T+1}}{(R_E - g_L)} \quad (5)$$

This two-stage model could also be used to estimate an implied return on equity by assuming the current stock price P_0 is equal to the calculated fair value V_0 and solving for the required return on equity. However, this requires an iterative trial and error solution because there is no direct closed form equation to solve for the required return on equity. As an example of this solution method, Claus and Thomas (2001) use a Residual Income Model (RIM) and estimate 5 years of cash flows followed by a constant growth terminal value. They then employ an iterative (trial and error) procedure to calculate an estimate of the implied return on equity.

A slight variation of this simple two-stage model is a multi-stage model in which several constant growth segments are assumed for a fixed number of years (i.e, fixed term annuities) and applied in a step function manner. Finally, a constant growth perpetuity (such as Equation 2) is often used as a terminal value to address the remaining cash flows after the last segment. A two-stage model with a short-term constant growth of g_S for S years followed by a constant long-term growth of g_L thereafter is as follows:

$$V_0 = \frac{D_1}{R_E - g_S} \left[1 - \left(\frac{1 + g_S}{1 + R_E} \right)^S \right] + \frac{D_{S+1}}{(R_E - g_L)(1 + R_E)^S} \quad (6)$$

The approach of this constant growth two-stage model can be extended into a three-stage model by adding one more mid-term constant growth annuity from year S to year M growing at the rate of g_M . This would again be followed by a constant growth perpetuity growing at the long-term rate of g_L beginning at year L as a terminal value. Such a three-stage approach is as follows:

$$\begin{aligned}
V_0 = & \frac{D_1}{R_E - g_S} \left[1 - \left(\frac{1 + g_S}{1 + R_E} \right)^S \right] \\
& + \frac{D_{M+1}}{(R_E - g_M)(1 + R_E)^M} \left[1 - \left(\frac{1 + g_M}{1 + R_E} \right)^{M-S} \right] \\
& + \frac{D_{L+1}}{(R_E - g_L)(1 + R_E)^L}
\end{aligned} \tag{7}$$

Although a bit tedious, assuming again that the current stock price P_0 is equal to V_0 , Equations 6 or 7 could also be used to solve for an implied required return on equity through a trial and error procedure.

The H-Model

The H-Model of Fuller and Hsia (1984) is a somewhat more intuitive approach than the multi-stage approaches of Equations 6 and 7. The H-Model uses an initial growth of g_S for a half-life of H years, plus a second stage growth of $(g_S + g_L)/2$ for H years after that as an approximation of a declining growth rate. This is followed by a constant growth perpetuity with a growth rate of g_L as a terminal value. In this case, H is equal to $\frac{1}{2} L$ years. The H-Model is

$$V_0 = \frac{D_0 (1 + g_L)}{R_E - g_L} + \frac{D_0 H (g_S - g_L)}{R_E - g_L} \tag{4}$$

A nice feature of the H-Model is that it can be used also to calculate an implied required return on equity in a direct closed form solution. Assuming the current stock price P_0 is equal to V_0 and solving for R_E yields

$$R_E = \frac{D_0}{P_0} [(1 + g_L) + H (g_S - g_L)] + g_L \tag{5}$$

Note that if the second term in the brackets were zero, this formula would simplify to the simple constant growth formula in Equation 3. This means that the second term in the brackets accounts for the additional risk associated with a company that currently has a higher growth rate than a mature, constant low growth rate.

The Ohlson-Juettner (2005) Model

The Ohlson-Juettner (OJ) Model is an earnings capitalization valuation model which includes abnormal earnings and a built-in function to compensate for the effect on growth from the payout of dividends. It also contains a feature that includes a declining growth rate cash flow stream, although this feature is not clearly explicit. The OJ Model is

$$V_0 = \frac{E_1}{R} + \frac{Z_1}{R - g_L} \quad (10)$$

$$Z_t = \frac{1}{R} [E_{t+1} + R D_t - (1 + R) E_t] \quad (11)$$

$$Z_{t+1} = (1 + g_L) Z_t \quad (12)$$

where Z_t = the capitalized abnormal earnings factor for any year t ,
 E_t = the forward earnings for year t , and
 D_t = the dividend to be paid in year t .

Substituting Equation 11 evaluated at $t=1$ into Equation 10 yields

$$V_0 = \frac{E_1}{R} + \frac{E_2 + R D_1 - (1 + R) E_1}{R (R - g_L)} \quad (13)$$

Recognizing that $E_2 = E_1 (1+g_2)$ and solving for R yields

$$R_E = \frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) + \sqrt{\left(\frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) \right)^2 + \frac{E_1}{P_0} (g_2 - g_L)} \quad (14)$$

Note that if the second term in the square root function were zero (i.e., $g_2 = g_L$), this equation would simplify to the constant growth formula in Equation 3, which applies to mature, low constant growth companies. This means that this term accounts for the additional risk associated with companies in which the near-term growth rate from year 1 to year 2, or g_2 , is higher than the long-term growth rate, g_L .

The Holland (2018) Declining Growth Model

In the valuation approaches of Equations 2 through 9, there are step functions with a constant growth rate – this is true, even for the H-Model approximation of a declining rate. However, holding a growth rate constant in fixed term segments or even as a long-term mature terminal growth usually does not precisely reflect the way a growth rate normally declines in a cash flow stream. Holland (2018) illustrates a valuation model that simulates a gradually declining growth rate in cash flows over time. This model is based on the difference between a

high cash flow stream growing at the long-term growth rate and a low cash flow stream growing at a slower rate than the long-term growth rate. This simulates a normalized cash flow stream with a cash flow of 1 at time zero and which has an initial growth rate of g_S from time zero to time 1 that over time declines asymptotically to the long-term growth rate of g_L . The simplest version of this model is to assume a zero-growth rate on cash flow stream L. This yields

$$H_0 = \frac{g_S}{g_L} \quad (15)$$

$$L_0 = \frac{g_S}{g_L} - 1 \quad (16)$$

$$C_t = C_0 [H_t - L_t] = C_0 \left[\frac{g_S}{g_L} (1 + g_L)^t - \left(\frac{g_S}{g_L} - 1 \right) \right] \quad (17)$$

$$V_0 = C_0 \left(\frac{H_0(1 + g_L)}{R_E - g_L} - \frac{L_0}{R_E} \right) = C_0 \left(\frac{\frac{g_S}{g_L} (1 + g_L)}{R_E - g_L} - \frac{\frac{g_S}{g_L} - 1}{R_E} \right) \quad (18)$$

where H_t = High cash flow at time t ,

L_t = Low cash flow at time t ,

C_t = a declining cash flow at time t with a declining growth,

g_S = the initial short-term growth rate,

g_L = the mature long-term growth rate, and

t = the time period.

This declining growth valuation model can also be used to estimate an implied required rate of return on equity by assuming the calculated fair value V_0 is equal to the current price P_0 .

Assuming the cash flow stream is a dividend, and solving for R_E yields a closed form equation of

$$R_E = \frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) + \sqrt{\left(\frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) \right)^2 + \frac{D_0}{P_0} (g_S - g_L)} \quad (19)$$

Notice that if the second term in the square root function in Equation 19 were equal to zero (i.e., $g_S = g_L$), the equation would simplify to the simple constant growth formula in Equation 3, which applies directly to mature, low growth companies. This means that the second term in the square root function accounts for the additional risk associated with companies that currently have a higher growth rate, g_S , that is expected to decline to a mature rate of g_L over time.

It is interesting to also note the similarity between the declining growth rate formula and the OJ formula for estimating the implied required return on equity. The only difference is the second term in the square root function. Both formulas would yield the same result if these two terms were equal. A comparison of these two terms is as follows:

$$DG \text{ Term} = \frac{D_0}{P_0} (g_S - g_L) \quad (20)$$

$$OJ \text{ Term} = \frac{E_1}{P_0} (g_2 - g_L) \quad (21)$$

The underlying difference in the two approaches relates to dividends vs. earnings (D_0 vs. E_1) and the rate of decline in the growth rates (g_S vs. g_2).

A Comparison Using Value Line Data

The results from using the Holland (2018) declining growth model can be compared to existing valuation approaches for estimating the required return on equity through calculations using Value Line data. As an illustration, Value Line data was collected for dividend paying companies in the Dow 30 stocks from the one-page Value Line data sheets available during the spring of 2017.

Value Line Data

The following data was collected from the one-page Value Line data sheets for the 30 companies in the Dow Jones Industrials Index:

P_0 = VL report of a recent price around the date of the report, summer 2017,

β = VL estimate of beta,

D_{2017} = the dividend per share in 2017, assumed to be D_0 ,

D_{2018} = VL estimate of D_1 , which is the first forecast year,

D_{2021} = VL forecast of the dividend in 2020-22,

E_{2017} = the earnings per share in 2017, assumed to be E_0 ,

E_{2018} = VL estimate of E_1 , which is the first forecast year,

E_{2021} = VL forecast of earnings in 2020-22.

Calculating the Required Return on Equity

For comparison purposes, we calculated an estimate of the required return on equity for each company in the Dow Industrials 30 Index using five of the methods outlined earlier in this paper. Year 2017 was considered Year 0, or the base year. Therefore, the current dividend yield

(D_0/P_0) is calculated as D_{2017}/P_{2017} . The long-term growth rate (g_L) was estimated to be 2% for all calculations.

The data for Johnson & Johnson (JNJ) will be used to illustrate the calculation of the required return on equity using the five different methods. The data for JNJ are

$$P_{2017} = 123.21$$

$$\beta = 0.80$$

$$D_{2017} = 3.32$$

$$D_{2018} = 3.52$$

$$D_{2021} = 4.90$$

$$E_{2017} = 6.45$$

$$E_{2018} = 7.25$$

$$E_{2021} = 9.90$$

$$g_S = (4.90/3.32)^{1/4} - 1 = 10.22\%$$

$$g_2 = (9.90/7.25)^{1/3} - 1 = 10.94\%$$

$$g_L = 2\%$$

$$R_{10\text{-yr}} = 2.5\%$$

$$E(\text{MRP}) = 5\%$$

Declining Growth Model

Using the method for matching cash flows as outlined in Holland (2018), the growth rate in the dividend from 2018 to 2021 would be

$$g_S = \left(\frac{C_t}{C_0} - 1 \right) \frac{g_L}{(1 + g_L)^t - 1} = \left(\frac{4.90}{3.52} - 1 \right) \frac{.02}{(1.02)^3 - 1} = 12.8103\% \quad (22)$$

Table 1 shows how the cash flows with a declining growth would be matched to the data in Value Line from 2018 to beyond 2021. Using the matched data and Equation 19 to calculate the required rate of return on equity as of 2018 yields

$$\begin{aligned}
 R_E &= \frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) + \sqrt{\left(\frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) \right)^2 + \frac{D_0}{P_0} (g_S - g_L)} \\
 &= \frac{1}{2} \left(\frac{3.9709}{123.21} + .02 \right) + \sqrt{\left(\frac{1}{2} \left(\frac{3.9709}{123.21} + .02 \right) \right)^2 + \frac{3.52}{123.21} (.128103 - .02)} \\
 &= 2.505\% + 6.096\% = 8.601\%
 \end{aligned} \tag{23}$$

Using the declining growth model to calculate a stock price as of 2018 yields

$$\begin{aligned}
 V_t &= C_t \left(\frac{\left(\frac{g_S}{g_L} + g_S \right)}{R_E - g_L} - \frac{\left(\frac{g_S}{g_L} - 1 \right)}{R_E} \right) \\
 V_{2018} &= 3.52 \left(\frac{\left(\frac{.1281}{.02} + .1281 \right)}{.08601 - .02} - \frac{\left(\frac{.1281}{.02} - 1 \right)}{.08601} \right) = 127.18
 \end{aligned} \tag{24}$$

In order to exactly match the stock price of 123.21 for 2017, a trial and error procedure is used to find the present value of the 2017 dividend of 3.52 and a terminal value as calculated by Equation 24. By adjusting the discount rate until the present value is equal to the 2017 price, the required rate of return on equity was calculated to be 8.493% from the declining growth model.

Constant Growth Model

$$R_E = \frac{D_1}{P_0} + g_L = \frac{3.52}{123.21} + .02 = 4.86\% \tag{25}$$

H-Model

$$\begin{aligned} R_E &= \frac{D_0}{P_0} [(1 + g_L) + H (g_S - g_L)] + g_L \\ &= \frac{3.32}{123.21} [1.02 + 20 (.1022 - .02)] + .02 = 7.18\% + 2\% \\ &= 9.18\% \end{aligned} \quad (26)$$

CAPM

$$R_E = R_f + \beta_{NJ} E(MRP) = 2.5\% + 0.80 (5\%) = 6.50\% \quad (27)$$

OJ-Model

$$\begin{aligned} R_E &= \frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) + \sqrt{\left(\frac{1}{2} \left(\frac{D_1}{P_0} + g_L \right) \right)^2 + \frac{E_1}{P_0} (g_2 - g_L)} \\ &= \frac{1}{2} \left(\frac{3.52}{123.21} + .02 \right) + \sqrt{\left(\frac{1}{2} \left(\frac{3.52}{123.21} + .02 \right) \right)^2 + \frac{7.25}{123.21} (.1094 - .02)} \\ &= 2.43\% + 7.65\% = 10.08\% \end{aligned} \quad (28)$$

Calculating R_E for the DJIA 30 Stocks

Figure 1 shows the results of calculating the required rate of return for the 30 stocks in the Dow Jones Industrial Index using the five different methods illustrated above, sorted by the results from the declining growth model. Note that the constant growth model yields a result that is consistently lower than the declining growth model, because the short-term growth rate is normally above the long-term growth of 2% per year. The H-Model yields results are

comparable to the declining growth model, but are slightly higher when the required rates are higher. Compared to CAPM, the declining growth model produces estimates of the required return that vary more than estimates from CAPM. For example, the declining growth model produces estimates of the required rate of return that are lower for companies with lower required rates, and higher estimates than CAPM for companies with higher required rates. Finally, the OJ Model produces the highest estimates of the required rate of return on equity, primarily because the calculation is based on earnings rather than dividends. Figure 1 illustrates that the declining growth model provides reasonable estimates of the required return on equity compared to other calculation methods.

Summary

We show that a new declining growth model can be used effectively to calculate an estimate of the required return on equity. A comparison is shown with four other calculation methods. The results illustrate that the declining growth model produces reasonable estimates of the required return, which have the feature of incorporating the effect of a declining growth rate over time. Thus, we show that this new valuation model provides an additional flexible method for estimating the required rate of return on equity.

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Figure 1
 Calculated Required Return on Equity
 For the Dow Industrials 30 Stocks

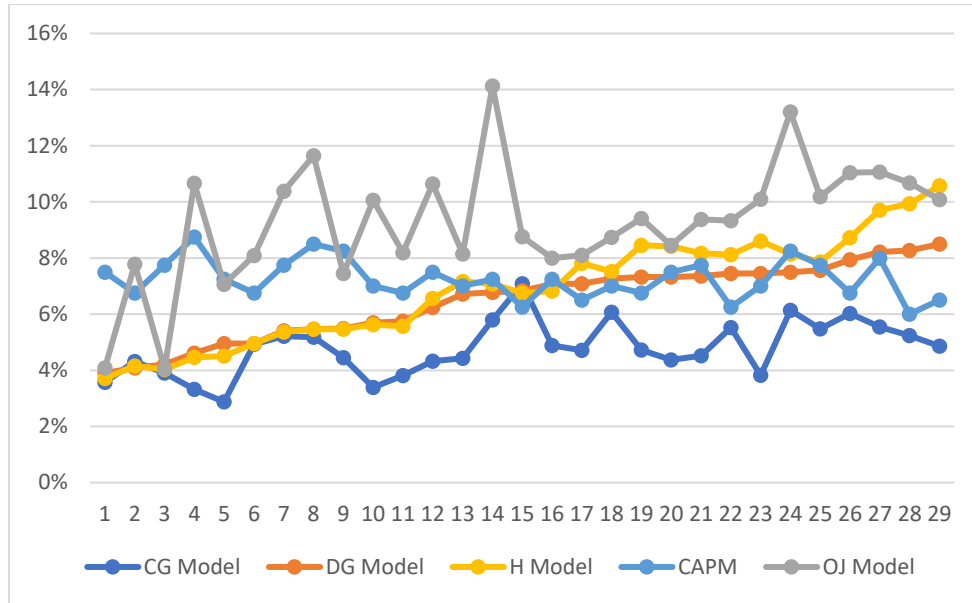


Table 1

Matching Cash Flows Beginning at Year 2018 with a Declining Growth Rate

(First 20 Years Shown)

($g_S = 12.8103\%$, $g_L = 2\%$)

$$H_2 = \frac{g_S}{g_L} = \frac{12.8103\%}{2\%} = 6.4051$$

$$L_2 = \frac{g_S}{g_L} - 1 = 6.4051 - 1 = 5.4051$$

$$C_2 (H_t - L_t) = C_t$$

Year	Grow at 2%/yr H_t	Constant L_t	$H_t - L_t$	Matching Cash Flow C_t	Declining Growth g
2018	6.41	5.41	1.00	3.5200	
2019	6.53	5.41	1.13	3.9709	12.81%
2020	6.66	5.41	1.26	4.4309	11.58%
2021	6.80	5.41	1.39	4.9000	10.59%
2022	6.93	5.41	1.53	5.3785	9.77%
2023	7.07	5.41	1.67	5.8666	9.07%
2024	7.21	5.41	1.81	6.3645	8.49%
2025	7.36	5.41	1.95	6.8723	7.98%
2026	7.50	5.41	2.10	7.3902	7.54%
2027	7.65	5.41	2.25	7.9186	7.15%
2028	7.81	5.41	2.40	8.4575	6.81%
2029	7.96	5.41	2.56	9.0071	6.50%
2030	8.12	5.41	2.72	9.5678	6.22%
2031	8.29	5.41	2.88	10.1397	5.98%
2032	8.45	5.41	3.05	10.7230	5.75%
2033	8.62	5.41	3.22	11.3180	5.55%
2034	8.79	5.41	3.39	11.9249	5.36%
2035	8.97	5.41	3.56	12.5439	5.19%
2036	9.15	5.41	3.74	13.1753	5.03%
2037	9.33	5.41	3.93	13.8193	4.89%
2038	9.52	5.41	4.11	14.4762	4.75%