A Formal Model for Investment Strategies to Enable Automated Stock Portfolio Management

Completed Research Paper

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Abstract

In this paper, we develop a formal model to specify a stock investment strategy. Based on an extensive review of investment literature, we identify determinants for portfolio performance – such as risk attitude, rebalancing interval or number of portfolio positions – and formalize them as model components. With this model, we aim to bridge the gap between pure decision support and algorithmic trading systems by enabling the implementation of investment approaches into an executable specification which forms the foundation of an automated portfolio management system. Such a system helps researchers and practitioners to specify, test, compare and execute investment approaches with strong automation support. To ensure the technical applicability of our model, we implement a prototype and use it to show the effectiveness of the model components on portfolio performance by running several investment scenarios.

Keywords: Investment Decision Support, Investment Strategy, Stock investment, Portfolio Management, Investment Strategy Model

Introduction

IT increasingly dominates international financial markets and stock exchanges. While stock trading *transactions* have taken place via electronic networks for quite some time, recent developments show an increasing automation of the *decision making* processes when it comes to stock investments. The advantages are obvious: machines are able to conduct large-scale analyses 24/7 and neither ask for commissions nor do they have hidden agendas. Once an investment strategy is defined, they conduct it rigorously, which might sometimes lead to undesired results such as the Flash Crash on May 6, 2010 (Patterson 2012). But compared to human investors, machines are immune against the influence of emotions which have shown to spoil the objective judgment of humans (Lucey and Dowling 2005).

However, the human investor is still needed to identify investment opportunities and define strategies to exploit those opportunities. It seems reasonable to combine the strengths of both actors: use the human

creativity and experience to identify and define investment strategies and then let a machine execute this strategy automatically to avoid human emotions conflicting with rational strategic decisions. Not surprisingly, research suggests a range of system designs to support investment decisions (see the following section for an overview). However, we did not find a flexible and comprehensive framework to specify an abstract investment strategy in a form machines can understand and execute. Such a generic model serves researchers as well as practitioners as a conceptual foundation for portfolio management systems. For researchers, an implementation of the model helps to build a laboratory environment to analyze investment strategy patterns and test new approaches for stock investment analysis (e.g. using indicators from Twitter or News analyses). By allowing batch runs on historic stock data with different parameter settings, researchers can estimate parameter effects on portfolio performance by conducting sensitivity analysis. For practitioners, the model allows a precise specification of an investment idea including back-testing and provides strong support to automate portfolio management.

In this paper, we develop a formal model to specify a stock investment strategy. Based on an extensive review of investment literature, we identify determinants for portfolio performance and describe them formally to fit in our model. We build the model with the intention to enable the implementation of an automated portfolio management system, i.e. a system that is able to receive a formal strategic description and act upon it autonomously for a specified period of time (either a simulation on historic data or a real-time application with current stock market data). The challenge is to find a way of specification which is universal and flexible enough to cover the broad range of possible approaches for investment strategies, but yet formal enough to be executed by a machine. To ensure a maximum flexibility, we integrate a generic stock ranking mechanism (cf. Gottschlich and Hinz 2014 and section below) based on user-defined metrics derived from stock analysis (e.g. momentum or volatility). Such a metric or a combination of several metrics is used as a score to bring the stocks of the investment universe in an order of preference. Hence, we are able to include a flexible way to express for any investment strategy what makes a stock more preferable over another form the investor's point of view. We ensure that our model can be implemented in software by implementing our model specification as a prototype system and use the results to show the effectiveness of the model components.

Previous approaches focus on specific aspects of investment decision making, e.g. security price analyses or News effects, but an overarching framework to integrate those techniques in a flexible way and apply them for automated portfolio management is not yet available. Application scenarios for such a model are plentiful though: Investors can simulate and test strategic approaches to stock market investment based on a variety of indicators on historic data. Once they find a working strategy, they can enable automatic strategy execution for future transactions within the parameters specified by the investment strategy. Our approach targets at the gap between professional algo- or high-frequency trading of large institutions and manual stock selection of private investors. The main intention is to make the analytical capabilities (and to a less extent the speed advantages) of information systems accessible for stock market investment decisions in a flexible and agile way to provide a laboratory environment and decision support system for investors (cf. Gottschlich and Hinz 2014). This paper introduces the model as a "language" to express the reasoning and restrictions of an investment strategy, but does not aim at finding optimal parameter settings in terms of portfolio performance. In fact, as we designed the model to be generic for a large diversity of investment approaches, this would result in the search for an optimal investment approach to stock markets – though desirable, this exceeds the scope of this paper.

The remainder of the paper is organized as follows: the upcoming section provides an overview of previous works in the area of investment decision support. Subsequently, we introduce our methodology. The section "Model Development" introduces all model components based on an extensive body of investment literature and closes with a formal specification of our model. The extensive evaluation of the model functionality based on six scenarios follows in the "Model Evaluation" section. We conclude our work with the final section showing venues for further improvement of this approach.

Investment Decision Support

The majority of previous research on investment decision support strives to provide better insights to investors by improved information support. Commonly used are methods to model stock price development utilizing optimization or machine-learning approaches. Specifically, artificial neural networks show broad coverage in investment decision support literature, often in combination with other

approaches. Tsaih et al. (1998) combine a neural network approach with a static rule base to predict the direction of daily price changes in the S&P 500 stock index futures which outperforms a passive buy-and-hold strategy (Tsaih et al. 1998). Chou et al. (1996) follow a similar approach for the Taiwanese market (Chou et al. 1996). Liu and Lee (1997) propose an Excel-based tool for technical analysis (Liu and Lee 1997). Kuo et al. (2001) show that they reach a higher prediction accuracy of stock development when including qualitative factors (e.g. political effect) in addition to quantitative data (Kuo et al. 2001).

More interactive forms of decision support have appeared, e.g. systems providing a laboratory-like environment for investors to conduct standard as well as customized analyses. Dong et al. (2004) suggest a framework for a web based DSS which implements a comprehensive approach including support for rebalancing an investor's portfolio according to his risk/return profile. They integrated On-Line Analytical Processing (OLAP) tools for customized multidimensional analyses (Dong et al. 2004). Another approach for interactive investor support provides the possibility of stepwise model generation such that investors can start with simple models from a toolbox and incrementally add more building blocks to arrive at more complex prediction models (Cho 2010).

The impact of news on stock prices can lead to distinct market reactions and hence support for their evaluation has been subject to several works. Mittermayer (2004) suggests a system which processes news to predict stock price movements taking a three step approach: extract relevant information from news by text processing techniques, assign them into three categories (good, bad neutral) and then turn those into trading recommendations (Mittermayer 2004). Schumaker and Chen (2009) use a similar approach of news analysis applying different linguistic textual representations to identify their value for investment decisions (Schumaker and Chen 2009). Muntermann (2009) focuses on more actionable support for private investors and suggests a system design that estimates price effects of ad hoc notifications for public companies and sends out text messages to mobile phones including predicted effect size and time window (Muntermann 2009). This helps private investors to decrease disadvantages in speed or awareness they usually suffer compared to institutional investors.

Other approaches try to exploit collective intelligence for stock investment decisions. Stock recommendations from stock voting platforms on the Internet have shown to be a valuable source for investment decisions (Avery et al. 2009; Hill and Ready-Campbell 2011) and can even be superior to the advice of institutional analysts (Nofer and Hinz 2014). Therefore, Gottschlich and Hinz (2014) suggest a decision support system which uses the wisdom-of-crowd concept to automatically select stocks from a German market and transform this selection into a target portfolio based on a formal investment strategy. While their approach is close to an automated portfolio management system, their specification of a formal investment strategy is still at an early stage and not extensively rooted in investment literature inducing a need for further development.

Surprisingly, there is little coverage of algorithmic trading system designs in scientific literature. This might be explained by the distinct value such designs have to their (commercial) developers, who have little incentive to spread this valuable knowledge. However, some recent works also shed some light onto this field (e.g. Kissel 2013; Narang 2009).

Based on these streams of literature, we are not aware of an integrated model to express investment strategies in a formal way while allowing for generic and hence flexible investment potential identification. In this paper, we want to address this gap by providing a formal model which an investor can use to specify an investment strategy in a formal way so it can be processed by an automated portfolio management system. We strive for a systematic approach of combining decision support of investors (i.e. providing insight or information) and stock investment execution (i.e. buying and selling selected stocks). Such a system derives a target portfolio layout by applying the formal investment strategy on specific stock data and provides simulation of an investment strategy on historic data or trading a strategy on current data. Our core question is: How can we formally specify investment strategies to enable automatic strategy execution? This model is supposed to be a major building block for the development of an automated portfolio management system.

Methodology

We conduct an extensive literature review of investment literature. From literature, we identify factors that drive portfolio performance which we formalize and integrate into our model. The goal is to create a

universal model which bridges the gap between the domains of investment strategy formulation and the detailed instruction level a machine needs to process.

To ensure our model is really executable by a machine, we implement a prototype in the validation section and analyze the effectiveness of each model component using a scenario analysis. Before we begin to develop the model, we introduce two central terms needed for understanding throughout this paper.

Risk measure

The risk of a portfolio is the probability of missing an expected return. We can split the total risk of a portfolio into two components: systematic and unsystematic risk (Evans and Archer 1968; Li et al. 2013). Unsystematic risk evolves from each of the individual positions in the portfolio and is related to the specific company (e.g. management errors). An investor can reduce this kind of risk by diversifying his portfolio (Li et al. 2013; Statman 1987). In contrast, systematic risk affects the market as a whole (e.g. change in interest rates) and is not affected by portfolio diversion.

We model the risk associated with an investment in a certain security as the volatility of its past returns measured by the standard deviation. This is a common approach in literature (Markowitz 1991; Sharpe 1992) and easy to understand: if we have two securities yielding the same return, the one with the lower volatility showed a more steady development and hence a more reliable return potential throughout the holding period.

Profit/Return measure

The profit or return of an investment is the percentage of value increase (or decrease, if negative) within a specific period. Formally, we measure profit R_t as:

$$R_t = \left(\frac{p_t}{p_0} - 1\right) * 100$$

with p_t = price of stock at end of period t, p_0= price of stock at beginning of period t. Intuitively, we measure return as the difference of buying and selling price, while disregarding distributed dividends.

Model Development

In this section we are going to introduce the components of our formal investment strategy model based on an extensive investment literature research to identify factors relevant for the performance of a portfolio (cf. Table 1). These components serve as a parameter set which jointly describes an investment strategy in a formal way. One instance tuple of these parameters describes a specific strategy which an appropriate software implementation can interpret and process. We integrate each of the identified components into our formal model which we specify subsequently. Figure 1 shows an overview of the investment process steps that implement such a strategy and the application of the model components in each step. The process starts with a computation of a ranking metric from stock data which is subsequently used to rank the stocks of the investment universe considered by the investor. Based on this order of preference, the system splits the available capital, determining the sizes of each portfolio position. Finally, the portfolio is rebalanced such that it reflects the new layout defined in the process.

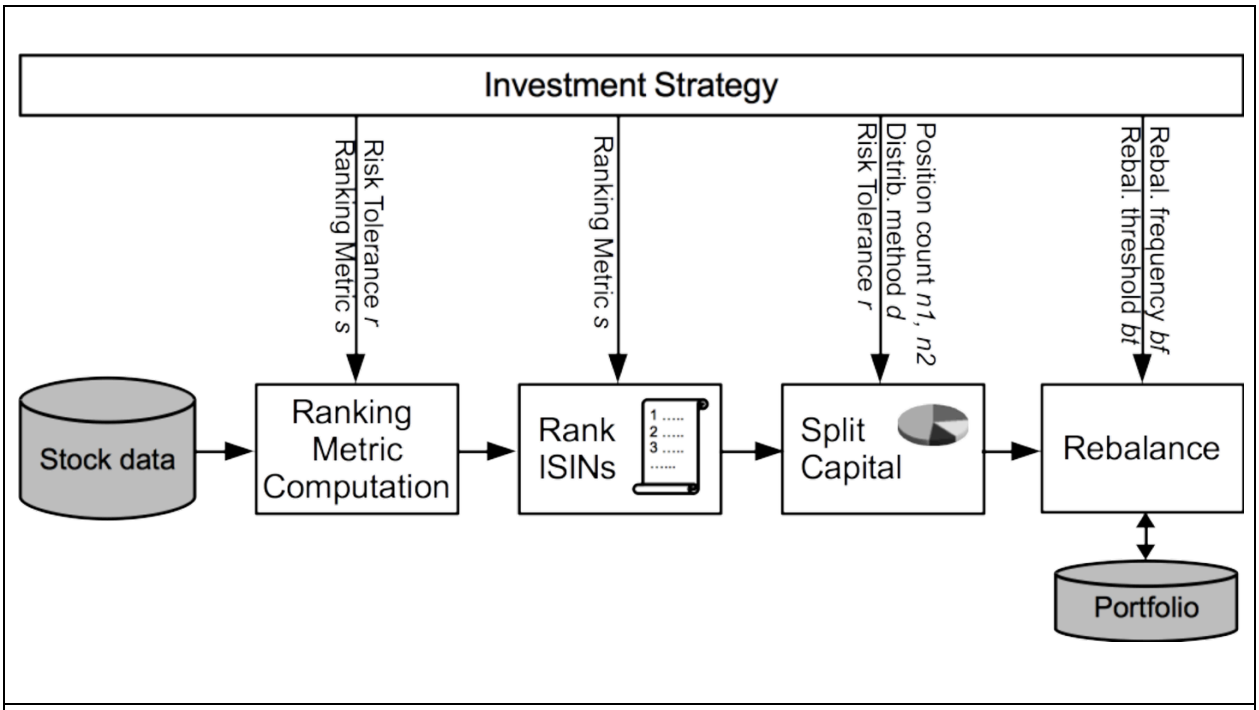

Figure 1. Overview of Investment Process and Influence of Model Components on Process Steps

Investor's Risk Tolerance

One of the fundamental parts of an investor's strategy is the definition of the target risk-return profile. The relationship between risk and return has been extensively discussed in literature (cf. Guo and Whitelaw 2006) and there is evidence of a trade-off between risk and return (Ghysels et al. 2005; Guo and Whitelaw 2006). Intuitively, it makes sense that investors demand a higher potential return to invest in a stock with a higher risk. Modern Portfolio Theory states that investors strive to create a portfolio that maximizes returns at a given risk level or to minimize risk at a given return target (Markowitz 1952). Hence, it is sufficient to define either a target risk or a target return.

We decide to model the risk-return profile as a maximum acceptable risk level and strive to optimize portfolio return. This is more intuitively to investors and more convenient when it comes to implementation because we can compute a good estimate of a security's risk based on past development (compared to the difficulty of forecasting expected returns precisely). Of course, the risk computed from the security's historic development might not be an appropriate predictor for its future risk – but this is a general problem in investing which we cannot address ex ante with our model.

Technically, the risk level specified with the investment strategy by the investor serves as the target risk for the portfolio creation/rebalancing step. Based on this restriction, the goal is to maximize the prospected return. At the current stage, we model the risk tolerance as the amount of volatility (based on historic figures) an investor is willing to take when a portfolio is created. When using the Markowitz portfolio optimization (see section Portfolio Optimization (Markowitz)), we use the risk tolerance as target risk level which forms a restriction for the portfolio optimization algorithm. Doing so, the risk level of the target portfolio should likely be in the desired range of the investor. Formally, we denote the investor's risk tolerance with $\boldsymbol{r}$.

This modeling of risk as threshold to volatility is a quite technical and simple approach. More sophisticated methods for risk management have been proposed and enable investors to express the risk they take more precisely. Specifically, Value-at-Risk (VaR) is a common measure to express risk associated with an investment (see e.g. (Linsmeier and Pearson 2000) for an introduction). VaR is the amount of loss which will be exceeded only with a specified probability p within an observation period t.

There are several common methods to determine VaR: The Historical, the Delta-Normal and the Monte Carlo Approach (for details see Linsmeier and Pearson 2000). They have in common, that they apply a distribution of profits/losses of an investment to determine an amount of loss that is only exceeded with a defined probability, usually 5% or less. For example, if we look at the historic distribution returns for an investment within the period *t*, the amount at the 5% quantile is what an investor would lose if the return would be as bad as it has only been in 5% of historic cases. The rationale is that under "normal circumstances", i.e. within the 95%, the loss will be less than the VaR and hence within that confidence, risk is under control. The definition of *p* demarks the border between "normal" and "abnormal" circumstances and depends on the investment universe and the attitude towards risk of the investor. For more details on how to use VaR for portfolio management, see e.g. (Krokhmal et al. 2002) or (Ogryczak and Sliwinski 2010).

To apply VaR for our model as an alternative to the target risk level ***r***, we need to exchange *r* with the parameters necessary to determine VaR: ***p*** which is the quantile to determine the VaR value from the profit/loss distribution and ***t***, the holding period. In our model, the holding period is connected to the rebalancing interval (see section Rebalancing Interval or Frequency) because in between rebalancing events, no changes in the portfolio will occur. This has to be taken into account when using the model for implementation.

Regarding the method of VaR calculation, we suggest using historical VaR if the necessary data is available as it should deliver highest accuracy for the price of computation effort – which is convenient for our intention to create a machine executable model for investment strategies. Alternatively, Delta-Normal VaR is a simplified method of computation which should be preferred in a portfolio management system if performance is an issue.

Rebalancing Interval or Frequency

Markowitz initially specified his model only for the case of one period to set up an optimal portfolio at the beginning (Elton and Gruber 1997), but disregards actions necessary to contain risk or secure intermediate returns over the course of time, such as checking and re-adjusting portfolio positions (Cohen and Pogue 1967). However, in reality, when portfolio positions shrink or grow over time due to their price development, frequent checking and rebalancing of portfolio positions is important to restore the initial asset allocation and meet the target profile of the portfolio regarding diversification and risk (cf. Buetow et al. 2002). Such adjustments lead to sell or buy orders for portfolio positions which cause transaction cost. Hence, an investor needs to maintain a balance between necessary rebalancing and cost for adjustment (Woodside-Oriakhi et al. 2013).

There are two basic approaches to portfolio rebalancing: *Calendar-/Frequency*-Rebalancing and *Range*-Rebalancing (cf. Buetow et al. 2002; Plaxco and Arnott 2002). Using Calendar-Rebalancing, an investor checks frequently, e.g. every month, quarter or year, the positions of his portfolio and adjusts them according to the target layout. This is an easy approach which does not depend on any external triggers, but is also passive towards dramatic changes in market environment as long as they happen in between rebalancing intervals.

Range-Rebalancing, in contrast, defines thresholds which specify the tolerance of an investor towards changes in position size. For example, a tolerance of 5% allows position sizes to deviate up to 5% from the initial asset allocation (up or down) before a rebalancing of the portfolio is triggered (Buetow et al. 2002). This allows investors to react immediately on possibly serious changes in portfolio positions and thus limit losses close to the tolerance interval. However, in reality, this approach needs support by suitable automatic portfolio surveillance to enable timely reaction.

Plaxco and Arnott (2002) show how important rebalancing is to maintain a defined risk profile: starting from a portfolio split of 50% in equities and 50% in bonds in 1950 and following a *drifting mix* strategy (i.e. re-investing any dividends in equities and interest in bonds), the portfolio would end up at a split of 98% in equities and 2% in bonds over the course of 50 years – resulting in an obviously different risk profile (Plaxco and Arnott 2002). Buetow et al. (2002) tested a combined approach of Calendar- and Range-Rebalancing. They defined a threshold for position size deviation and checked this threshold frequently after a defined time period. They found that especially when markets are turbulent, frequent

rebalancing is profitable (Buetow et al. 2002). In addition, in most cases range-based rebalancing had positive effects on portfolio performance over the last five decades (cf. Plaxco and Arnott 2002).

Thus, rebalancing is important to maintain the intended risk profile of an investor over time. We introduce both Calendar- and Range-Rebalancing into our model and denote the rebalancing frequency with ***bf*** and the threshold for deviations of portfolio size in % with ***bt***.

Number of Portfolio Positions

The number of positions in a portfolio is another determinant of portfolio diversification. Statman (1987) found that a portfolio's risk – measured as the standard deviation of returns – drops with every additional random stock that is added (Statman 1987). Elton and Gruber (1977) found that that 51% of a portfolio's standard deviation can be eliminated by increasing the number of positions from 1 to 10. Additional 10 positions only eliminate another 5%. Figure 2 shows the decline of standard deviation (risk) with an increasing amount of stocks in the portfolio based on a correlation between stocks of 0.08 (as measured by Campbell et al. (2001)). It confirms that the major part of standard deviation reduction can be achieved with 10-20 stocks. Statman (1987) recommends 30-40 different positions.

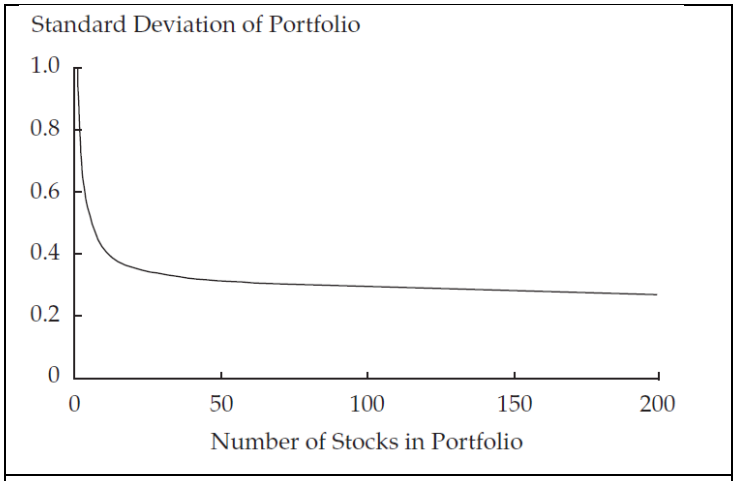

Figure 2. Expected Standard Deviation with Portfolio Diversification (all stocks have equal weight). The correlation between the returns of two stocks is 0.08, and the standard deviation of any stock is 1.0. (Statman 2004)

But an increasing number of portfolio positions increases the average transaction cost per position as well (Konno and Yamamoto 2003). Hence, to reach at the optimal number of portfolio positions, an investor should add stocks as long as the marginal transaction cost is lower than the marginal benefit, i.e. the reduction in risk by diversification (cf. Evans and Archer 1968; Statman 1987). This relationship has been subject to an extensive body of research. Evans and Archer (1968) found that adding more than 10 stocks to a portfolio cannot be economically justified (Evans and Archer 1968). In 2004, Statman revised this analyses and found based on current data that a portfolio can contain up to 300 positions before marginal cost outweigh marginal benefit (Statman 2004). Shawky and Smith (2005) analyzed U.S. domestic funds and found they hold a number of 40 to 120 positions and a positive correlation between fund size and number of positions (Shawky and Smith 2005).

Literature shows that the recommended number of positions increases during the last decades. Campbell et al. (2001) show that an investor needed 50 positions during 1986-1997 to reach an equal portfolio standard deviation than one could achieve in 1963-1985 with 20 positions. One reason is that the standard deviation between stocks dropped from 0.15 in 1984 to 0.08 in 1997 (Statman 2004). In addition, by the extended use of information technology financial markets became more efficient and hence transaction cost could be decreased (Hendershott et al. 2011).

Obviously, spreading stock investments over industries is specifically important for diversification. But nevertheless, risk reduction is actually driven to a larger extent by the number of positions than by diversification over industries (e.g. Domian et al. 2007; Statman 2004).

Considering this state of research, we include a range for the number of portfolio positions into our model. Thus, investors are able to express their view on the number of portfolio positions which is an integral part of an investment approach and hence should be included in an investment strategy. We denote the minimum number of portfolio positions with ***n1*** and the maximum number with ***n2***.

Stock Ranking Mechanism

At the heart of an investment strategy, and often even considered identical, is the decision which stocks to choose for a portfolio. When it comes to stock selection, there is a huge variety of approaches derived from different investment philosophies like fundamental investment, technical analysis and behavioral finance.

The question is: How can we integrate flexible support for such diverse approaches into our model while preserving its formal character to enable automatic execution of investment strategies? We embrace an approach suggested in Gottschlich & Hinz (2014), using a scoring mechanism to rank the stocks in the investment universe by preference. By incorporating the score, or metric, in the investment strategy specification, we include the knowledge or decision rule which stocks are preferable over others from the investor's viewpoint. Depending on the investor's view of the world, this could be a technical (e.g. momentum of previous week or moving average for last 200 days) or fundamental (e.g. price/earnings ratio) indicator or any other metric for which data can be provided. In fact, it is a function that states which stock is preferable over another based on selected criteria. Combination of metrics, e.g. by building a weighted sum of several scores, allows for more complex ranking mechanisms. Investors can create a library as a directed graph of metrics which they can employ in their investment strategies. This graph of metrics represents the investor's knowledge and analytical capability for stock selection.

A system implementing our model would compute the score(s) when a portfolio layout needs to be determined (e.g. at a rebalancing event) and rank all stocks according to their score (cf. Figure 1). For example, a possible (simple) metric would be the price/earnings ratio. The system would then compute this metric for every stock at a specific day (when a rebalancing occurs) and sort all stocks by this value such that stocks with low price/earnings ratios are ranked better than those with higher ratios (assuming a long strategy)[1] and hence preferred for portfolio selection. Ranking can also comprise filtering, i.e. stocks which should not be taken into account receive prohibitive scores. In the example above, stocks with price/earnings ratios of e.g. above 50 might be excluded.

By introducing a metric for stock order preference in our model, we can still provide an abstract formal model description, but allow for flexible specification of stock preference in an application scenario. For an application example, see the Validation section. We denote the stock preference score in our model with ***s***.

Distribution of Capital

To arrive at a final portfolio layout from a list of ranked stocks, we need to decide how to split the available capital among those stocks. How many stocks starting at the top of the list should be considered and which amount of money should be distributed to each of them? While the first part of the question can be answered with the help of the model components ***n1*** and ***n2***, specifying the desired range of portfolio positions, the question of how the capital should be split among selected stocks forms another dimension of investment decision.

Naïve Diversification (1/N)

The easiest way to split a certain amount of capital to N selected stocks is an equal distribution. With this method, at every rebalancing event, each portfolio position is adjusted to the same share of total capital C

[1] *This is a simplified example for illustration purposes which does not necessarily implement a very successful strategy.*

(which is C/N) (cf. DeMiguel et al. 2007; Tu and Zhou 2011). Advantages of this approach are an easy implementation and independence from biases potentially caused by estimation or optimization techniques for returns or weights. Also winning and losing positions can be identified at a glance. Practitioners do use such easy methods of capital spread for their investments and they are able to compete with more sophisticated methods (DeMiguel et al. 2007).

Portfolio Optimization (Markowitz)

Given a set of stocks the portfolio selection approach as proposed by Markowitz (1952) determines, based on their historic development, weights for each of the stocks to derive an efficient portfolio. Efficient means that there is no other stock selection that beats the given solution in terms of risk/return ratio. Technically, the approach minimizes variance (= risk) for a given return level or maximizes return for a given risk level. The approach received criticism, partly due to the fact that it maximizes errors in the estimation of expected return or stock correlation. Those estimates are necessary for the method to determine position weights (Chopra and Ziemba 1993). In addition, portfolios built using Markowitz' optimization approach did not necessarily outperform other methods of portfolio construction (cf. Tu and Zhou 2011). But nevertheless, it is a reasonable approach to determine an optimal portfolio regarding risk/return trade-off. For our purposes, having an already pre-ranked list of stocks and trying to determine how much capital should go into which position, the method is a convenient way to determine optimal weights based on risk/return figures.

Apart from the two capital distribution methods mentioned, investors can specify their own methods, too. As we have a ranked list of stocks, another reasonable distribution method would be e.g. a diminishing distribution, assigning most weight to the best-ranked stock and reducing the share while descending the list. This enables investors to reap the benefits of the ranking to a greater extent than with naïve diversification. It is obvious that the decision on how to distribute the available capital on the stocks selected has influence on the portfolio performance outcome and should be included in our investment strategy model.

We model the capital distribution method as a function that receives the ranked list of stocks and returns a number of weights for those stocks. For naïve diversification, the function could simply take the top 10 stocks and assign a weight of 10% to each of them. The optimization approach after Markowitz is more complex and involves running an optimization algorithm. Other approaches are possible, by implementing an appropriate function and including it in the investment strategy model. We denote such a function of capital distribution with ***d***.

Model specification

Based on an extensive body of investment literature, we identified several components (cf. Table 1) which affect portfolio performance and hence are determinants for portfolio creation that should be included in a formal model for investment strategies. Formally, we specify an investment strategy ***IS*** as an instance of the following tuple of model components:

$$IS = \langle r, bf, bt, n1, n2, s, d \rangle$$

Table 1 summarizes the model components. This model provides a way to specify investment strategies in a way machines can interpret and execute and hence is an important step towards an automated portfolio management system (e.g. Gottschlich and Hinz 2014). In the upcoming Evaluation section, we implement the model and show how different values for strategy parameters affect portfolio performance, thereby showing the effectiveness of the model.

Table 1. Model Components	
IS	Investment strategy
r	Investor's risk tolerance
bf	Rebalancing frequency
bt	Rebalancing threshold
n1	Minimum number of portfolio positions
n2	Maximum number of portfolio positions
s	Stock ranking metric (score)
d	Capital distribution method

Model Evaluation

In this section, we want to show how the model is applied to formulate an investment strategy which can be executed by a system to yield a target portfolio layout based on the strategy parameters. To show that our model provides the intended functionality, we introduce a base scenario and a number of test scenarios which differ in one of the model parameters to isolate the effect of the specific parameter. We compare the resulting portfolio of each test scenario with the base scenario to observe the results of the parameter adjustment. The purpose of this evaluation is not to identify the most profitable parameter settings, but rather show that changes of parameters have an effect on portfolio performance and hence there inclusion in the model is justified. The search for optimal parameter settings is subject to further research which finds strong support by an automated laboratory environment based on our formal strategy model.

Implementation

To evaluate the functionality of our model, we implemented a prototype system to execute an investment strategy which is specified using our model. The system takes a specified investment strategy and executes it over a specified time period while tracking the portfolio layout and performance development. In fact, the system converts the specified investment strategy into a portfolio layout for a specific day. Besides the portfolio, the system tracks the amount of cash available for a simulation run. We conduct our simulation with an initial cash value of 100,000 EUR.

We implemented the prototype in JAVA using the Spring Framework and a MySQL database to store stock quotes, investment strategies and portfolio layouts. To compute the complex Markowitz calculations, we integrated the statistical software R which is executed and controlled by the system.

We ran all the scenarios in a time period of two years: January 2009 to December 2010. Stock quotes were closing prices at Frankfurt Stock Exchange. For transaction cost, we used the cost model of a large German retail broker who charges 4.90 EUR per transaction plus 0.25% of transaction volume; minimum fee is 9.90 EUR and maximum fee is 59.90 EUR. Transaction costs were aggregated to an external account and hence had no effect on the available investment capital. Further, we did not consider payment of dividends or taxes. One scenario run took approximately between 40 and 70 minutes on an Intel Core 2 Duo with 2.53 GHz and 4 GB RAM.

Scenarios

We defined 6 scenarios for evaluation purposes (see Table 2). The first scenario serves as a base scenario. The other scenarios each vary one parameter of the model to show the resulting effect on portfolio performance. By varying only one parameter at a time, we ensure that observed effects were caused by the specific manipulated parameter. Thus, we can evaluate the effectiveness of the model parameters. We keep one parameter fixed: stock selection. For stock selection, we use the score from Gottschlich and Hinz (2014) (GH). This score is based on a collective estimate of a large crowd on a stock voting platform and

measures the potential price increase or decrease that the crowd assigns to a certain security on a certain day. That means for a long strategy (which we apply here), we use this score in a descending order to rank the stocks with the highest potential first (for details cf. Gottschlich and Hinz 2014). We do not change the stock selection parameter throughout the test run, because it is not our focus to compare stock selection mechanisms. We could use any other score as well with its respective results on portfolio performance to evaluate the effects of the other parameters.

For capital distribution, we use the Markowitz portfolio selection theory (PST) approach to arrive at a portfolio with optimized risk/return profile in two variants: PST(12) uses price history of the last 12 months to compute the variance of a portfolio position, while PST(6) only uses past 6 months. Thus, PST(12) should be more stable and react slower to changes of volatility in a security's development, while PST(6) reacts quicker, but also more volatile.

Table 2: Model Evaluation Scenarios

Scenario	Risk Tolerance ***r***	Rebalancing Frequency ***bf***	Rebalancing Threshold ***bt***	Min. # Positions ***n1***	Max. # Positions ***n2***	Stock selection score ***s***	Capital Distri-bution ***d***
S0	30%	weekly	0%	0	10	GH	PST(12)
S1	60%	weekly	0%	0	10	GH	PST(12)
S2	30%	monthly	0%	0	10	GH	PST(12)
S3	30%	weekly	10%	0	10	GH	PST(12)
S4	30%	weekly	0%	0	2	GH	PST(12)
S5	30%	weekly	0%	0	10	GH	PST(6)

Results

As an external benchmark for performance comparison, we show the development of the DAX stock index which captures the 30 largest German companies (based on market capitalization and turnover). Table 3 shows an overview of all scenario portfolio results, while Figure 3 shows a plot of the scenario portfolio performances over the whole period. At a first glance, we see that the performances of the different scenarios differ, giving a first indication that the parameters included in the model are indeed determinants of portfolio performance and hence should be contained in our model. An exception is the result of Scenario S3 which performs identical to S0. We will discuss this observation in detail in the subsection of Scenario S3.

Scenario S0 – Base scenario

The base scenario (cf. Table 2) applies a rather conservative risk tolerance of 30% with a weekly rebalancing frequency. The rebalancing threshold is 0% which means that every deviation from the position target weights leads to an adjustment of portfolio position size. We specify no required minimum of portfolio positions, letting the system decide to invest or keep cash when a rebalancing event occurs. For this test run, we want to maintain a simple portfolio and hence set the maximum number of positions to 10. In formal terms, S0 can be specified as:

$$S0 = \langle 0.3, Weekly, 0\%, 0, 10, GH, PST(12) \rangle$$

Looking at the results (Table 3 or Figure 3, respectively), we see that the DAX benchmark develops positively with a return of approx. 40%, while S0 clearly outperforms the DAX benchmark with a return

after transaction costs (TC) of app. 126%. These are the absolute results, but for our subject, we are more interested in the relative results between scenarios.

Table 3: Overview on the Model Evaluation Results (rounded)

Initial Capital	Resulting Capital	Return Rate (rounded)	Transaction Costs (TC)	Result – TC	Return Rate – TC (rounded)	Risk
Dax Benchmark						
100,000€	140,551.81€	40.44%	n/a	n/a	n/a	13.92%
Base Scenario S0						
100,000€	241,836.86€	141.84%	16,103.97€	225,732.89€	125.73%	27.25%
Evaluation Scenario S1						
100,000€	300,544.64€	200.54%	17,187.04€	283,357.6€	183.36%	35.74%
Evaluation Scenario S2						
100,000€	181,097.76€	81.10%	4,825.14€	176,272.62€	76.27%	18.46%
Evaluation Scenario S3						
100,000€	241,836.86€	141.84%	16,103.97€	225,732.89€	125.73%	27.25%
Evaluation Scenario S4						
100,000€	207,683.79€	107.68%	7,293.08€	200,390.7€	100.39%	19.30%
Evaluation Scenario S5						
100,000€	370,286.18€	270.29%	21,877.22€	348,408.96€	248.41%	33.01%

Scenario S1 – Risk tolerance

In scenario S1, we change the investor's risk tolerance to 60%. As we expect a higher risk to yield a higher return (Ghysels et al. 2005), the Scenario S1 should outperform the base scenario. Formally, we specify S1 as:

S1 = <0.6, Weekly, 0%, 0, 10, GH, PST(12)>

From Table 3, we see that the S1 portfolio indeed outperforms the S0 portfolio by almost 60 points while the risk associated with the portfolio also increased to 35.74%. As we altered no other parameter except the risk tolerance, we conclude that the increased risk tolerance led indeed to a higher portfolio performance at the price of a higher risk. These observations confirm the results of (Ghysels et al. 2005; Guo and Whitelaw 2006) and show the functionality of this parameter in our model.

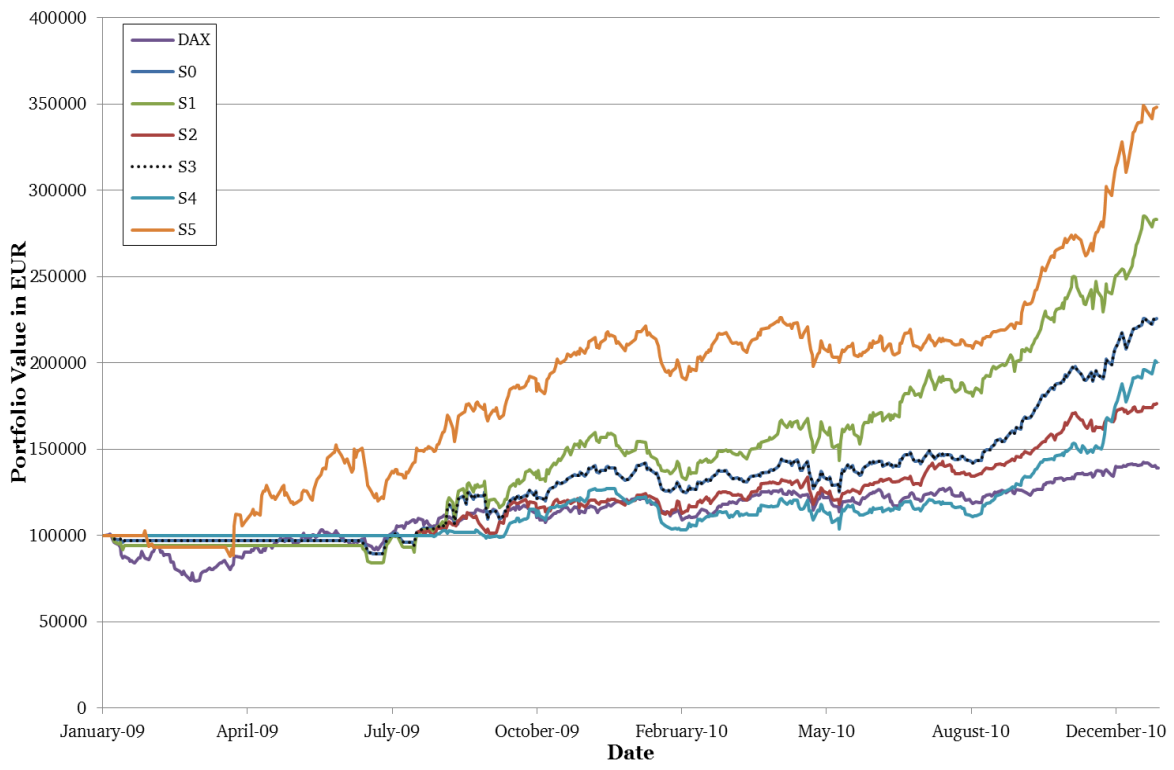

Figure 3. Performance of Scenario Portfolios with Transaction Costs Deducted

Scenario S2 – Rebalancing Frequency

In this scenario, we change the rebalancing frequency from a weekly to a monthly portfolio check and adjustment. Since Buetow et al. (2002) showed that a smaller rebalancing intervals increase returns, we expect a negative impact of this parameter change compared to the base scenario S0. The full specification of S2 is:

S2 = <0.3, Monthly, 0%, 0, 10, GH, PST(12)>

Table 3 shows a performance for the S3 portfolio of 81.10% compared to 141.84% of the base scenario S0. So changing the rebalancing interval alone from weekly to monthly and keeping everything else equal, the performance drops by app. 60 points. In addition, due to the less frequent rebalancing interval, there are less trades to be made (123 trades in contrast to 442 trades in the base scenario), resulting in lower transaction costs, as Figure 4 shows. However, the lower transaction costs cannot compensate the loss in price development. All in all, these findings show that the rebalancing interval is an important determinant of portfolio performance and a necessary part of an investment strategy specification.

Scenario S3 – Rebalancing Threshold

The Rebalancing Threshold defines the maximum deviation a portfolio position may show against the target weights before its size is re-adjusted. Buetow et al. (2002) reported a positive impact by increasing the Rebalancing Threshold from 0% to 10%. Another positive effect could arrive from lower transaction costs, as a higher tolerance towards target weight deviation can lead to a lower number of trades and hence reduce transaction costs. The formal specification of S3 is as follows:

S3 = <0.3, Weekly, 10%, 0, 10, GH, PST(12)>

The results of Scenario S3 are identical to those of the base scenario S0. Why is that? This is due to a conflict of parameters: the applied metric for stock selection (GH) is very volatile in its recommendations leading to a very different list for every rebalancing event. So a rebalancing based on this metric is rather fundamental exchange of portfolio positions. Because the target weights of portfolio positions and the positions themselves change so much, this parameter is masked by the stock selection metric and shows no effect in the current test run. Future evaluations of the model should analyze if this parameter is effective with different evaluation data.

Scenario S4 – Maximum number of portfolio positions

Because the number of portfolio positions affects diversification of a portfolio which is connected to portfolio performance and risk, we adjust the maximum number of portfolio positions in Scenario S4 from 10 to 2 and evaluate the effect. Due to the lower diversification, we would expect a higher risk associated with the portfolio. As we decrease portfolio size by a large extent, we also expect transaction costs to be lower than in the base scenario. Scenario S4 is specified as follows:

S4 = <0.3, Weekly, 0%, 0, 2, GH, PST(12)>

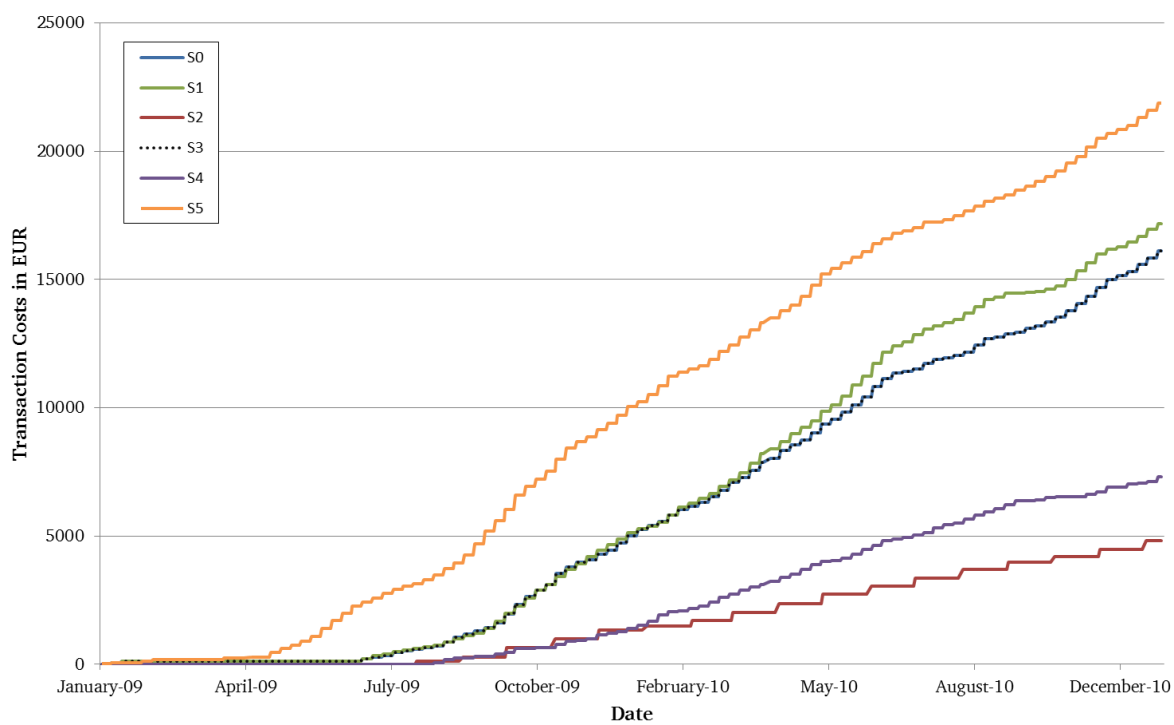

Figure 4. Development of Transaction Costs

S4 has a return rate before TC of 107.68% compared to 141.84% in the base scenario (cf. Table 3). Surprisingly, the risk is not increased compared to the base scenario, but instead dropped to a value of 19.30% compared to 27.25% in the base scenario. This is against the expectations from previous literature, which predict a higher risk with less diversification. We conclude that this is a random effect with our evaluation data set. However, in accordance with previous research is the drop in performance compared to the base scenario which comes with the reduction of risk. In this respect, our results are consistent with literature.

As expected, the transaction costs also drop from 16,104 EUR in the base scenario to 7,293 EUR (cf. Table 3 and Figure 4) – less than half. However, these savings cannot over-compensate the loss caused by lower diversification.

Scenario S5 – Capital distribution

For the last evaluation scenario S5, we modify the method of capital distribution. In all previous scenarios, including the base scenario, we took a history window of 12 months to compute the volatility and correlation metrics for stocks which are needed for the Markowitz portfolio selection. Now we shorten this window to 6 months. By doing so, the investment behavior of the system should be more responsive to recent market developments and act more agile on market changes. Formally, we specify S5 as:

S5 = <0.3, Weekly, 0%, 0, 10, GH, PST(6)>

Scenario S5 shows the strongest performance of all portfolios – 270.29% return compared to 141.84% in the base scenario. Looking at Figure 3, we see that all other scenarios show no trading activity during the first few months of the evaluation period. The reason is a rather volatile bear market in 2008, which ended in 2009 and turned into an upwards trend. The scenarios which use the past 12 months to estimate stock risk, stick longer to the negative evaluation of stocks before the positive developments allow the system to invest again instead of keeping a 100% cash position. With a 6-month time window for risk assessment, the positive market development leads to a quicker pick-up of the bull market by Scenario S5 and hence explains its superior performance compared to the other scenarios.

We confirm that the capital distribution mechanism is a crucial part of an investment strategy and even slight modification can have large impact on portfolio performance. Hence, we are confident that the capital distribution method should be an integral part of our formal model for investment strategies.

Conclusion

In this paper, we introduced a formal model to specify investment strategies in a generic way. Based on an extensive review of current investment literature, we identified determinants of stock portfolio performance and formalized them as components in our model. To the best of our knowledge, no such universal and integrated approach of formalizing investment strategies existed before. However, this is a crucial ingredient to bridge the gap between pure decision support systems, which support a human decision maker who then executes the decision and fully automated trading systems that are closed (and often secret) systems and do not allow for an interactive exploration of strategies by an investor. In contrast, our model serves as a language to express investment strategies by investors and still enable execution to derive a target portfolio layout automatically when implemented in an appropriate portfolio management system (cf. Gottschlich and Hinz 2014). This fundamental conceptual framework serves both researchers and practitioners by providing a generic laboratory environment to model and analyze new investment ideas and test them in a comparable way with strong automation support. By providing a "language" to describe an investment strategy formally, we also create a precise way to express, communicate and store investment approaches. Further, a portfolio management system which implements the model provides generic support for a wide range of investment approaches which are applicable to automatic portfolio management. Alternatively, such a system provides decision support up to the derivation of stock orders which can still be controlled by human investors before execution.

While we provide a first version of an integrated investment strategy model, there is still room for improvement. First, there might be (and certainly are) other factors which affect portfolio performance which we have not yet addressed with our model. But we are confident that our suggestion in this paper already covers the most widely accepted and most common components of investment strategies. In terms of risk modeling, Value-at-Risk as a common approach used by practitioners should be included in the implementation of the model to make it more applicable for practical use.

Second, the results of our evaluation were not final with respect to the effect of the rebalancing threshold (Scenario S3) and the effect of restricting the number of portfolio positions (Scenario S4). While the effects of these parameters are well founded in theory, based on our data set, we were not able to find support for this theory. Further improvement efforts of the model should include a re-evaluation of these parameters to see if their presence in the model is justified. A third venue for improvement is the handling of conflicts between parameters – as we discovered in Scenario S3, when the stock ranking mechanism made the rebalancing threshold ineffective. The investor could specify a priority of parameters in case of conflict to overcome this limitation.

But most exciting future uses will probably comprise a massive sensitivity testing of investment strategies to apply statistical methods on the significance of strategy parameters. By executing a large number of slightly different parameter settings and analyze their effect on portfolio performance by applying statistical methods, new levers for successful investment approaches could be identified and evaluated. This is an exciting outlook for both, practitioners as well as researchers, as our model would provide the fundamental building block for such a batch testing of investment approaches in a laboratory environment or for real money investments.

In conclusion, we are confident that our proposition is a valid and valuable approach to enable a formal specification of a wide range of investment strategy approaches. We showed by our prototypical implementation that such specified strategies can be executed by an appropriate portfolio management system and that our model is suitable to narrow the gap between pure decision support systems and automated trading systems.

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