# Indian Institute of Management Calcutta

# Working Paper Series

## WPS No 814/November, 2018

## Evolution of Online Advertising

**Nagandla Chandana** and **Thati Vikram Abhishek**

**PGDM 2017-19**

Indian Institute of Management Calcutta

**Anindya Sen** *

Professor, Indian Institute of Management Calcutta

(* Corresponding Author)

**Indian Institute of Management Calcutta**

**Joka, D.H. Road**

**Kolkata 700104**

URL: http://facultylive.iimcal.ac.in/workingpapers

# Evolution of Online Advertising

**Nagandla Chandana** and **Thati Vikram Abhishek**

**PGDM 2017-19**

Indian Institute of Management Calcutta

**Anindya Sen** *

Professor, Indian Institute of Management Calcutta

(* Corresponding Author)

## Abstract

Today Digital or Online advertising is an important part of any firm's marketing strategy. Within online advertising there are various channels like sponsored search, display ads or contextual ads. Tech giants like Google, Facebook etc. face different challenges in selling ad space from each channel. This is reflected in the auction mechanisms each of these channels employ. In this paper, the evolution of online advertising since the 1990s to the present-day is described. Generalised Second Price (GSP) and Vickrey Clarke Groves (VCG), the two main auction mechanisms in use today, are analysed in detail. Also the reasons for their use in specific online advertising channels or ad platforms are discussed.

**Keywords:** Auctions, Online Ad Auctions, Sponsored Search, Contextual Ads.

# 1. Introduction

With the advent of Internet, traditional advertising has gone through a paradigm shift. Today Online Advertising (online ads) forms an integral part of any firm's marketing efforts. This can be deduced from the fact that online ad spending has crossed television advertising spending in 2017. With online ads expected to grow at a faster rate than television advertising, the gap is only going to widen. Within online advertising, sponsored search advertising contributes to about 42% of the total online ad spending followed by social media (contextual ads) and video ad spending. Most of this spending is done through online ad auctions. With the exponential growth of search engines like Google, Yahoo! and social networking sites like Facebook and Instagram, these auctions facilitate the sale of billions of dollars of worth of advertisements and hence are an important source of revenue for these ad platforms. Most ubiquitous of these auction mechanisms today is the Generalised Second Price (GSP) auction, which we will look at in detail subsequently.

A typical online ad auction, begins with a search engine user querying for a word. The search engine would return with a mix of organic (search algorithm based) and inorganic (sponsored/paid) results. The number of sponsored results (slots) are usually limited and are ranked according to the position in which they appear. Various players (advertisers) who want to be displayed in these sponsored slots bid for them in real time. These players are then ranked in the decreasing order of their bids, with the highest bidder getting the first ranked slot and so on. When a user clicks on any particular sponsored ad, the advertiser is charged an amount as per the auction mechanism determined by the search engine (Ad

platform). These sponsored results can be of value to the search engine user as well depending on the relevance of the advertisements.

Revenue realised by the ad platform and the cost to the advertisers' characteristics are different across various mechanisms. These nuances form the basis for the selection of a particular auction mechanism over the other. In this paper, we conduct a comparative study of most popular auction mechanisms by tracing the evolution of the online advertising market since the 1990s. We also do an in-depth analysis of Generalised Second Price (GSP) auctions and VCG auctions, through which most of the online sponsored search and contextual ad auctions take place. Based on this, we would eventually comment on their attractiveness to various ad platforms.

## 2. Unique features of online advertising

Online advertising has a number of unique features compared to offline traditional auctions of any design. These include:

A. *Dynamic bidding:* The bids by advertisers in online auction can be changed at any time. This can happen when the ad platform conducts an auction every time a user queries for a keyword. For instance, in one user query an advertiser's result might be shown in the first slot, but in the next query (after a while) it might be shown in the second slot if one of the other advertisers changes his/her bid to more than the initial advertiser.

B. *Sponsored Slots are Perishable (for Search engines only):* The advertisement slots available for each search query by the user

of search engine are perishable. If no advertisements are shown in any slot, those slots are wasted and potential revenue is lost by the search engine. This is unlike any traditional auction where a tangible item is being sold rather than some perishable service. Hence, there is an incentive for the search engine to attract different kinds of advertisers so that any kind of keyword is not left non-monetised.

C. *Measure of what is being sold:* In online advertising auctions, there is no clear measure of the value of what is being sold. Each party involved has a different estimate of the value that is being sold. From the advertiser's viewpoint, the value on offer is the cost of acquisition of a customer, i.e. one who transacts on their website. So, he/she would want a pricing model which is dependent on the revenue realised on the website by the traffic generated by the online advertisement on the ad platform. This would be like paying a commission to the ad platform every time a sale is generated due to the advertisement. From the ad platform's standpoint, the unit of value is the number of impressions on the sponsored slot as a final sale depends on number of factors which are not under the ad platform's control like the product quality of the advertiser. Therefore, the ad platform is unconcerned about the revenue generated on the advertisers' website and would like to charge the advertiser every time his/her advertisement is shown (number of impressions) to the search engine user. The online advertisement industry has finally converged to a middle ground of cost-per click model, where in the advertiser pays the search engine every time a user clicks on a sponsored search link.

# 3. Sponsored Search Advertising Timeline

In this section, we trace the evolution of sponsored search auctions through the 90s up until late 2000s.

**1970s to Early 1990s:**

The first online advertising was through spam email on May 3, 1978. The mail was sent by a person named Gary Thuerk, a marketing manager at the Digital Equipment Corporation(DEC) inviting around 400 recipients for the product demonstration by DEC. This email caused huge outrage among some recipients and prompted the Defence Communication Agency(DCA) to take a strong action against spam and kept spam away from the inbox for almost a decade.

In 1980, Usenet, a public discussion forum where people could post question and answers on any topic was launched and was occasionally flooded with advertising spam posts.

In 1984, Prodigy, a joint venture between broadcaster CBS, computer manufacturer IBM and retailer Sears was founded. It offered its users access to news, email, bulletin boards, sports and weather info along with an un-clickable ad at the bottom of each page. The users were more interested in the services rather than ads which resulted in the invention of first ever Ad blocker. The users were more interested in using email services and interactive bulletin board rather than news and other offerings and this was costly to Prodigy as it was charging minimal monthly subscription fee. Later on, Prodigy was not able to handle the traffic on its site and could not survive competition from ISPs. It finally closed down in 1999.

**1994 Banner Ads and cookies**

The first clickable online ad was sold in 1994 by Hotwired, which was a digital publication of wired magazine. Hotwired set aside portions of its website for advertising and sold the ad spaces to advertisers which were called ‘Banner Ads’. AT&T was the first advertiser to place a banner ad on Hotwired’s website. For two years prior to the debut of the banner ad, AT&T was running ad campaigns on TV, radio and print on futuristic technological wonders with the caption “You will”. One such ad by AT&T was a woman shown on video call to her infant on the screen and asked "Have you ever tucked your baby in? From a phone booth?" and the ad ended with the promise saying “You will”. AT&T spent around $50 million per annum on those advertisements. Then AT&T hosted an ad asking users “Have you ever clicked your mouse right HERE?” and with a promise “You will”. When users clicked on that ad, it redirected the user to a tour of few world’s famous museums giving the users a taste of internet’s ability. AT&T paid $30,000 for the banner to be displayed for three months on the website. The click through rate of the ad was 44%. Later, due to the huge success of banner ads, Hotwired signed contracts with fourteen other advertisers like MCI, Club Med, defunct malt beverage company Zima, and Volvo. Here the advertisers paid to use the ad space for a certain time similar to magazines and newspapers.

Around the same time, CMP technology, a B2B company that provided information on tech products and services was launched with banner ads from AT&T, MCI communication and Tandem Computers Incorporation.

Vibe, Time Inc.’s culture magazine was launched with banner ads from MCI communication, Timex Corp, General Motors, Saturn, AirWalk Footwear and Jim Beam Brands Co. It was paid $ 20,000 a month per ad to be displayed on the website.

Global Network Navigator, a commercial web publication was launched in early 1993 by O'Reilly & Associates. The first ad on GNN was sold to a law firm called Heller Ehrman, White & McAuliffe. By May 1994, GNN was being accessed 150,000 times per week, and had more than 30,000 subscribers. The users of GNN increased exponentially during that time. By June 1995, GNN had 400,000 daily viewers of whom 180,000 were subscribers and was able to charge hefty amounts to host banner ads on its website. It charged $110 to $11,000 a week from advertisers like Master Card and Zima. It was sold to AOL in June 1995.

In 1994, cookies were introduced by Netscape as part of its browser Mosaic Netscape browser. Cookies help the advertisers to distinguish online shoppers and track user's behaviour online thus enabling them to target the customers better.

**1995 Measuring Banner Ads Effectiveness by Webconnect:**

As banner ads gained popularity, an ad agency called Webconnect began helping its clients to identify the right websites to place their ads thus enabling the clients to get more visibility among their target demographics. They also came up with a metric to measure the number of times a banner ad was shown to a user. It helped the firms to change their advertisements to the user after he/she had seen it for a certain number of times so that the user would not be bored with repetitive ads. Webconnect enabled the advertisers to measure the number of impressions, clicks and sales of the advertisements through its proprietary ICS tracking system.

**1995 Keyword based advertising by Yahoo:**

In 1995, Yahoo created the first keyword based advertising and first keyword ad was on "Golf". Another search-engine, InfoSeek, began to

target banner-ads to keywords. At the same time, another search engine, OpenText, tried to target text-based ads to user search queries. Around this time, InfoSeek and Netscape changed their advertising model to Cost per Impression where the advertisers are charged on the basis of the number of views a particular ad receives. Microsoft launched MSN online in August 1995. During 1994 - 96, the online advertising started moving from banner ads to keyword based advertising thus enabling the advertisers to reach the target audience and garner high click through rate and conversion rates.

**1996 DoubleClick tools:**

Though the banner ads enabled the online advertisement, advertisers still did not have good tools to measure the tangible results. The ads were not organized and it was difficult for websites to find advertisers interested in buying ad spaces. At this point, a firm called DoubleClick (later acquired by Google in 2007) came up with a service named DART (Dynamic Advertising Reporting & Targeting) which was intended to increase the purchasing efficiency of advertisers and to minimize unsold inventory for publishers. DoubleClick matched the advertisers with websites to reach the target audience which improved the click through rate of the ads. They achieved it through performance tracking of previous ad campaigns. It helped the advertisers to track and customize the ad performances without waiting till the end of the campaign to reflect on the results. This enabled the advertisers to track how many times an ad was clicked and view across multiple websites thus paving the way for Pay Per Impression model. DoubleClick was a huge success at that time.

During this, many firms like New York Times, Wall Street journal started their own websites and earned revenue by venturing into online advertisement. In Feb 1996, Focalink Communications launched a tool on

ad placement and reporting which gained them huge popularity. This helped the advertisers in maximizing the impressions of the ads.

**1997 Pop up Ads**

In 1995 Netscape came up with the programming language JavaScript. This paved the way for the invention of Pop up ads. Pop up ads were originated in Tripod.com in 1997. Ethan Zuckerman who was a developer for Tripod.com created pop up ads which open up a new window in the browser. It was created with the intention to associate an ad with a user's page without putting it directly on the page. But pop ups became an annoyance to users and became the most hated advertising technique. Later multiple ad blocks for the web browsers were created to curtail the pop up ads. Pop up ads are still prevalent though. There are two kinds of pop ads - pop up and pop under. The only difference between those is how the new window opens. In the case of pop up ads, a new browser window opens on top of the current screen whereas in pop under ads, the new browser window opens behind the current screen thus not interrupting the user as much as pop up ads but garnering more views than banner ads. It also reduces the negative reaction to the page that loaded the pop under ad as it gets seen only after the user closes the current screen and less likely to know where the ad came from. During this time, the pop ads have a click through rate of 7% whereas the banner ads have click through rate of 1% (Source: https://blog.adcash.com/pop-under-and-pop-up-ads/). This is mainly due to phenomenon called 'Banner Blindness' where visitors consciously or subconsciously ignore banner-like information.

**1998-99 Pay per click by Overture Services - Generalized First Price auction**

Till 1998 advertisers were paying based on the number of times an ad was shown (Pay per Impression Model). The advertisers were not able to quantify the revenue/traffic driven to their website by placing the ad. Thus, there was a need for a Pay-per-click (PPC) or revenue sharing model (fee based on percentage of revenue). At this point, Overture Services (then called GoTo) offered advertisers the option of bidding on how much they would be willing to pay to appear at the top of results in response to specific keyword searches. In this model, an advertiser comes up with the keywords related to his/her product/service and would bid an amount he/she is willing to pay every time a user clicks on the link to advertiser's website which appears in a position (based on the bid) when a user searches for the keyword. This model enabled the advertisers target their ads to more relevant users and to pay per click instead of paying for ad banner (pay per impression) which was shown to every user.

This model became a huge success and Overture through partnerships enabled portals MSN and Yahoo! to monetize the millions of web searches on their sites. Overture used Generalized First Price (GFP) auction revenue model in which the advertisers' bids for the positions and search rankings were based on the bids (highest bidder gets the top position) and the advertiser paid his/her bid. But the first price auction is unstable and may not have a pure Nash equilibrium. The following example demonstrates how.

**Example:**

Suppose there are 2 slots on a page and 3 advertisers are bidding for these two slots. Advertisers 1, 2, 3 value the click at $4, $3, $2

respectively. Advertiser 1 & 2 will end up getting Slot 1 & 2 respectively. But this first price auction is unstable because if the bids can be changed frequently then the bidders will not bid their true value and instead keep changing their bids in response to other bidders. In this case, the bidder 2 bids $2.01 to get the 2nd slot and bidder 1 bids $2.02 to get slot 1.  Now bidder two bids $2.03(his true value is $3). The bidders keep raising their bids in pennies until bidder 2 pays $3 and bidder 1 pays $3.01. But 2 realises that he can attain the same positions by paying $2.01, so he ends up paying $2.01 and 1 pays $2.02. This competition between advertisers to outbid each other led to investment in bidding robots which keep updating the bid to outbid the competitor. Here the faster the robot, the faster an advertiser can outbid his/her competitor. In this case if 1 has a faster robot then 2 knows that 1 will outbid him till his maximum value of the click. This causes 2 be content with bidding for 2nd position with bid just enough to drive 3 out of the game. So, 2 ends up paying $2.01 and 1 pays $2.02. Again, the robots repeat the same bidding process and the bids tend to form a saw tooth pattern as shown in Figure 1. The revenue does not change even the true values of the clicks are much higher for bidders 1 & 2.

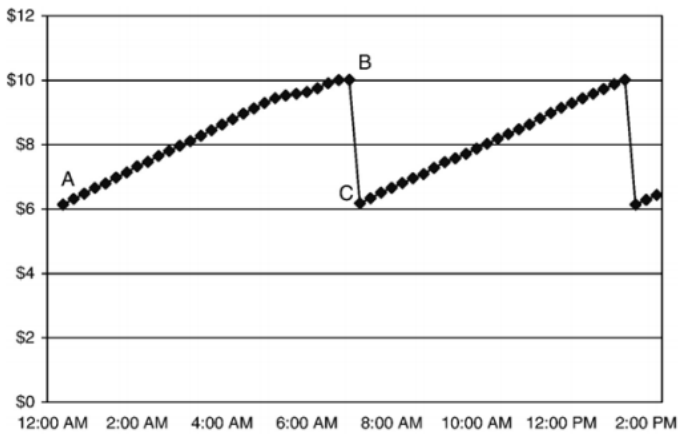


Figure 1 Source: Strategic Bidder Behaviour in Sponsored Search Auctions by Benjamin E., Michael O.

**2002 Pay per Click by Google - Generalized Second Price Auction:**

Google launched AdWords in 2000 with 350 advertisers. Today, Google has millions of advertisers and accounts for 33% of world's ad revenue. Google is the leader in digital advertising followed by Facebook and Alibaba.

At first, Google charged advertisers a monthly fee for ad spaces. During this time Google used to maintain the ad campaign for advertisers. To accommodate the small-scale advertisers, Google started AdWords self-service portal. Later it shifted to cost per thousand impressions model charging the advertiser on basis of number of impressions.

Though in early 2000, Google sold ads on cost per impression basis, in February 2002, it came up with a new auction cost per click model, Generalized Second Price (GSP) Auction, addressing the inefficiencies of GFP Auctions. An interesting thing to note here is how a blunder by GoTo founder helped Google make billions of dollars. According to US regulation in early 2000s, a patent application must be filed within one year of an offer for sale, a sale, or a public disclosure of the invention. But by the time GoTo founder, Bill Gross realized that he had to file for patent for his invention of pay-per-click advertising model, the model had already been disclosed by GoTo for more than a year and so they couldn't file the patent. GoTo was able to file patent for a bunch of unimportant things. Though Google implemented Generalized Second Price (GSP) Auction model, it built up on GoTo's model and improvised it. Thus, Google later went on to make billions of dollars by leveraging GoTo's mistake.

In GSP Auction, advertisers bid for the positions and search rankings were based on the bids (highest bidder gets the top position) and the

advertiser in $i^{th}$ position pays the amount equal to the bid of the advertiser in $i + 1^{th}$ position.

Example: In the above example discussed, the advertiser 1 pays $3 which is bid amount of advertiser 2 and 2 pays $2, the bid of advertiser 1.

Realizing the advantage of GSP, Overture/Yahoo (which was acquired by Yahoo in 2003) also shifted to GSP mechanism. Though Yahoo! and Google use GSP mechanism, Yahoo uses bids for rankings whereas Google in order to make the ads more relevant to users and improve customer experience introduced a subjective qualitative score (Quality Score) which factors in ad relevance and click through rate (CTR) along with bids to determine the rankings. Google AdWords became a huge success and the company made 85% of its revenue from AdWords in 2003. Advertisers paid as high as $200/Click during that period. The following graph shows Google's advertising revenue over the years.

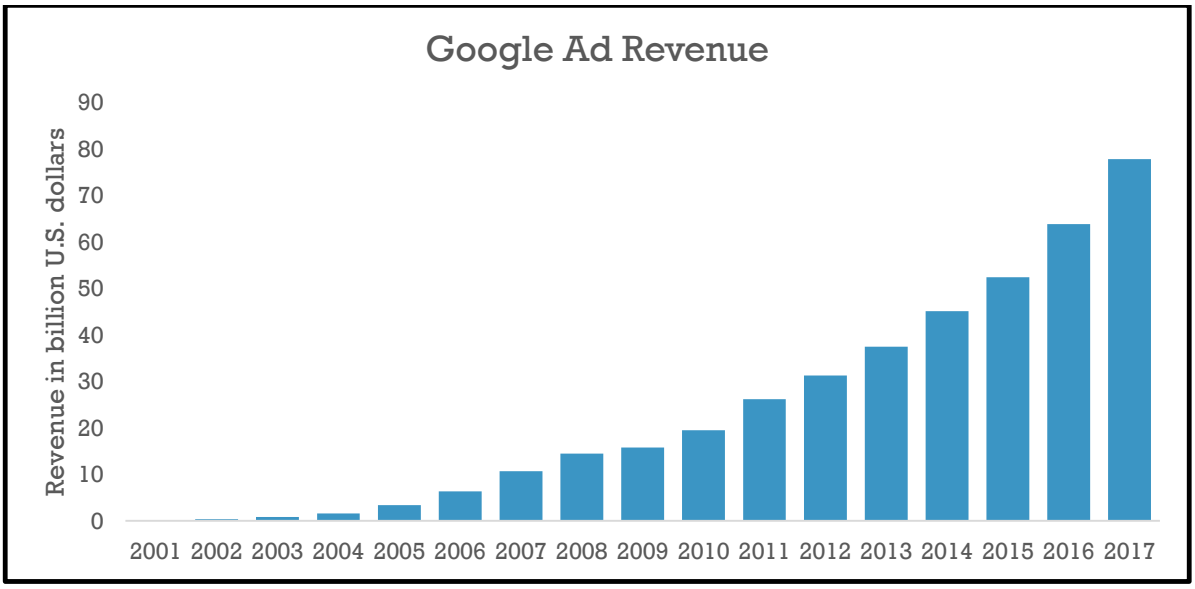


Figure 2 Source: https://www.statista.com/statistics/266249/advertising-revenue-of-google/

In March 2003, Google launched Ad Sense and acquired a firm, Applied Semantics which specialized in Ad Sense technology. Google uses Ad sense technology to place automatic text, image, video, or interactive media advertisements on websites based on the context. AdSense specializes in creating and placing advertisements on a website or blog based on the website's content. This improves the relevance of ad on a website and makes it less intrusive to the user. Many websites use Google AdSense program to generate revenue from their web content. Google uses both Cost-per-impression model and pay-per-click model to charge the advertisers in AdSense program. This program has been particularly helpful for small websites that do not have resources to develop their own advertising programs. Today, Google generates around 24% of its ad revenue from AdSense.

**How Google Adsense works:**

Assume that X owns a website about healthy diet and has approximately 10000 visitors a month. The website is doing well and has a large following; however, X is not making any money from her site.

In order to monetize the traffic coming to her website, X can sign up for Google AdSense account. After installing the code (given by Google) onto her website, dynamic ads that are related to healthy diet will automatically begin to show up on the website. And X gets some percent of cost per click Google charges from the advertiser every time a user clicks on the ad, thus, enabling X make money. This also benefits advertisers from cheaper cost per click than AdWords.

In 2006 Google acquired YouTube for $1.65 billion. YouTube was counting 100 million video views per day by then. By 2007, YouTube was launched in nine countries and YouTube introduced new models of

advertising. It started In-video ads, pre-roll advertising and by 2009 YouTube allowed ads in seven different formats. YouTube has grown rapidly and by October 2009, it had more than 1 billion video views per day. YouTube partnered with 10,000 partners, including Univision, Disney, Turner and Channel 4 and they are making six figures every year from advertising on the site. Similar to AdSense, they started a program to monetize from uploading video content. A user can turn on account monetization in his YouTube account. Through AdSense, YouTube will post the ads that are relevant based on the video content and the demographics it attracts. YouTube has a 45/55 split for the content creators, so Google keeps 45% of Ad revenue and the user gets the remaining 55 percent.

Google has been adding new ad formats and partner programs in Search Engine, YouTube and its other networking sites and improving its different ad platforms by making ads more relevant to the users. Today Google considers 7 qualitative factors - history quality score, landing page experience past and present, Ad relevance past and present, past Click through rate (CTR) and expected CTR in its Quality Score which it uses along with the bids to determine rankings in ad space auction.

**2006 Pay-Per-Click by Facebook - Vickrey–Clarke–Groves (VCG) mechanism:**

Facebook was launched in 2004 by Mark Zuckerberg and co-founders Dustin Moskovitz, Chris Hughes and Eduardo Saverin. In August 2006, Facebook partnered with J.P. Morgan Chase to promote the Chase credit card and posted banner ads on its site inviting users to join a special Chase network. The same year, Facebook and Microsoft formed a strategic relationship for banner ad syndication making Microsoft’s adCenter the exclusive provider of banner ads and sponsored links on the

site. In November 2007, Facebook introduced "Facebook Ads" -pages for brands and businesses, Facebook Insights and "Beacon" that encourages the business owners to use a Bluetooth device Beacon in their office or store which enables Facebook users to get more information about these business or stores under "Place Tips". The following year, it launched the Engagement Ads and self-serve Ads for pages and events giving the capability of engaging users. In 2014, Fb launched Audience Network which serves the same purpose as Google AdSense. Both the companies are fighting tooth and nail to attract the advertisers and in the process kept on adding new Ad features and improvising Ad platforms.

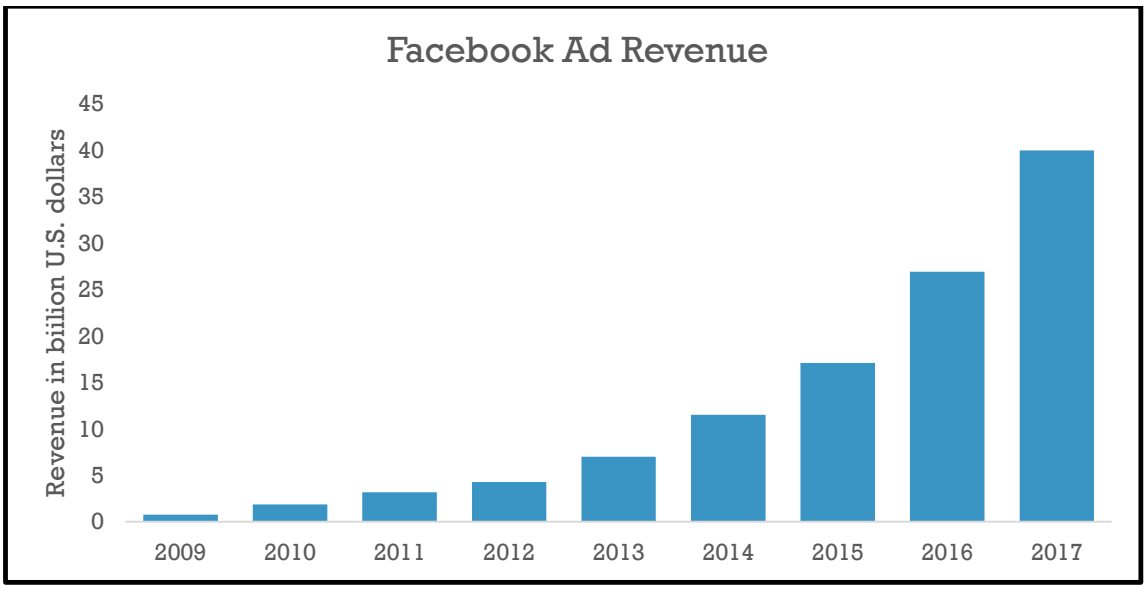


Figure 3 Source: https://www.statista.com/statistics/271258/facebooks-advertising-revenue-worldwide/

Similar to Google, Facebook uses pay-per-click model. But it uses VCG mechanism to charge the advertisers. Facebook with the intent of assigning ad slots in a socially optimal manner (charging each advertiser the cost he/she causes to the other advertisers due to his presence) started using VCG mechanism, a sealed bid auction.

In Vickrey–Clarke–Groves (VCG) auction, each advertiser pays his social cost.

$P_i$ = Optimal welfare for other players if advertiser i is not present - Optimal welfare for other players if advertiser i is present

where

$P_i$ = Price paid by advertiser i

It turns out that it in the VCG auction it is optimal for each advertiser to bid the true value per click as we shall see in a later section.

## 4. Generalised Second Price Auction for Search Engines

### Auction rules GSP

In this section, we describe the framework for examining Generalised Second Price Auction (GSP). The auction is modelled for each time a user queries for a particular keyword. Let there be **K** bidders i.e. advertisers, bidding for **N** sponsored search slots (particular area where sponsored search result can be displayed), with the slot shown in the first position more desirable than the second and so on. The expected number of clicks on a particular sponsored slot is modelled as a constant $\alpha_i$ for each of the N slots. The value derived from each click by $k^{th}$ advertiser is $s_k$. The value realised by the $k^{th}$ advertiser from the i-th slot is $\alpha_i s_k$.

These assumptions implicitly imply that the value derived by the advertiser from a click is independent of the position in which the click is received and the expected number of clicks received on a particular slot

is independent of the advertisement shown in that or any other slot. Let $\alpha_i > \alpha_j$, if $i < j$. The slot with lower number, i.e. those shown higher, have a higher click-through rate $\alpha_i$.

The Generalised Second Price (GSP) auction is modelled as follows. At time $t$, let a search engine user query for a particular keyword. Let $b_k$ be the bid submitted by the $k_{th}$ advertiser for the above keyword before time $t$. Let $b_{(j)}$ and $j$ be the $j_{th}$ highest bid and bidder respectively. The bidders are ranked in the descending order of their bids. Then the GSP mechanism allocates the first slot (with the highest click through rate) to the bidder ranked 1, with the highest bid i.e. $b_{(1)}$. Similarly, the second slot is allotted to the bidder with the second highest bid and so on. Any ties are resolved randomly.

The payment made by each advertiser per click is equal to the bid of the next highest advertiser i.e. $p^i = b_{(i+1)}$. The value derived by advertiser in each slot I is given by $\alpha_i s_i$ and the payment is given by $\alpha_i b_{(i+1)}$. Thus, an advertiser $i's$ net payoff is given by $\alpha_i(s_i - b_{(i+1)})$. If $N \geq K$ then the last advertiser's payment is equal to zero.

**Locally envy free equilibrium**

Advertisers on a sponsored search are allowed to change their bids frequently. Therefore, these auctions can be thought of as infinitely repeated games which start with each advertiser having only private information. In the course of time, as each of the advertisers change their bids, they learn the values of other advertisers as well. In such repeated games, the set of equilibria are very large and the strategies that can support such equilibria tend to be quite complex. Hence, in theory although such strategies can be implemented through automated

algorithms, they are seldom approved by the search engines as they can potentially allow the advertisers to collude and reduce the search engine's revenues.

Therefore, the focus is on simple strategies and rest points in the bidding process, i.e. the vector of bids to which the advertisers converge to. To study such equilibrium we make few assumptions, which would allow us to look at these auctions as one-shot simultaneous move game of complete information. First, all values pertaining to each of the advertisers and the search engine are common knowledge. This can be assumed without any loss of generality as over time each of the advertisers learn the values of other advertisers through their bidding behaviour. Secondly, the convergence vector of bids must be best responses to each other. If that is not the case, as the bids can be changed at any time, any one of the bidders might have an incentive to change the bids, thus breaking the equilibrium.

One simple strategy by a particular advertiser can be to upgrade to the slot above him. Suppose advertiser $k$ bids $b_k$ and therefore is assigned to the $i^{th}$ slot. If he keeps increasing his bids in small increments, this wouldn't change his payoff (which is dependent on the bidder below him) but this will reduce the payoff of the advertiser $k'$ with bid $b_{k'}$ above him. If advertiser $k'$ retaliates by reducing his bids slowly, eventually the two bidders will switch their slots with each other. Thus, if the vector of bids converges to a stable point, no advertiser would want to switch slots with the advertiser above him. Such vector of bids is called "locally envy free" equilibrium. More formally, an equilibrium in one-shot simultaneous move games of complete information induced by GSP is locally envy-free if a bidder cannot improve his payoff by exchanging his slot with a bidder

ranked one place above him. Therefore, for a bidder in any position $i$ $\forall$ $i \leq Min(N+1, K)$,

$$\alpha_i(s_i - p^{(i)}) \geq \alpha_{i-1}(s_i - p^{(i-1)})$$

Here $p^{(i)}$is the price per click paid by the advertiser in the $i^{th}$ slot and is equal to $b^{(i+1)}$. We can view a locally envy free equilibrium as a prediction of the vector of bids to which a GSP auction tends to converge to.

**Equilibrium Revenue to Search Engine**

In this section, we calculate upper and lower bounds of the revenues that the search engine can realise when vector of bids converges to a locally envy free equilibrium based on the above discussion. In a locally envy free equilibrium, an advertiser in slot $i+1$, would not want to switch slots with the advertiser immediately above him in slot $i$, therefore we have the equation,

$$\alpha_{i+1}(s_{i+1} - p^{i+1}) \geq \alpha_i(s_{i+1} - p^i)$$

Rearranging, we get

$$\alpha_i p^i \geq \alpha_{i+1} p^{i+1} + s_{i+1}(\alpha_i - \alpha_{i+1})$$

This inequality intuitively shows that the payment made by an advertiser for a slot has to be weakly greater than the payment made by an advertiser one slot below him plus the value of the extra clicks to the advertiser.

If we denote the price per click of the last slot (say Kth slot) as $p^K$, and list down the above recursion for all slots, we get

$$\alpha_1 p^1 \geq s_2(\alpha_1 - \alpha_2) + s_3(\alpha_2 - \alpha_3) \ldots \quad \ldots \quad \ldots \ldots \ldots \ldots \ldots + \alpha_K p^K$$

$$\alpha_2 p^2 \geq \qquad s_3(\alpha_2 - \alpha_3) + s_4(\alpha_3 - \alpha_4) \ldots \ldots + \alpha_K p^K$$

$$\alpha_3 p^3 \geq \qquad s_4(\alpha_3 - \alpha_4) \ldots \ldots + \alpha_K p^K$$

Summing up over all the advertisers,

$$\sum_k \alpha_k p^k \geq s_2(\alpha_1 - \alpha_2) + 2s_3(\alpha_2 - \alpha_3) \ldots \ldots + (K-1)\alpha_K p^K$$

This gives us the lower bound on the equilibrium revenue. Similarly, to calculate the upper bound we argue that an advertiser in slot $i$, wouldn't want to switch to a slot below him. So, for him/her we can write the inequality as,

$$\alpha_i(s_i - p^i) \geq \alpha_{i-1}(s_i - p^{i-1})$$

Rearranging we get,

$$\alpha_i p^i \leq \alpha_{i+1} p^{i+1} + s_i(\alpha_i - \alpha_{i+1})$$

Listing down the recursion for all the slots as above, we get

$$\alpha_1 p^1 \leq s_1(\alpha_1 - \alpha_2) + s_2(\alpha_2 - \alpha_3) \ldots \quad \ldots \quad \ldots \ldots \ldots \ldots \ldots + \alpha_K p^K$$

$$\alpha_2 p^2 \leq \qquad s_2(\alpha_2 - \alpha_3) + s_3(\alpha_3 - \alpha_4) \ldots \ldots + \alpha_K p^K$$

$$\alpha_3 p^3 \leq \qquad s_3(\alpha_3 - \alpha_4) \ldots \ldots + \alpha_K p^K$$

Summing up over all advertisers,

$$\sum_k \alpha_k p^k \leq s_1(\alpha_1 - \alpha_2) + 2s_2(\alpha_2 - \alpha_3) \ldots \ldots + (K-1)\alpha_K p^K$$

This gives us an upper bound on the equilibrium revenue. If the value per click of each advertiser if assumed to be drawn from a distribution, and if the number of slots N is much lesser than number of bidders (i.e. K is large), then the two bounds converge. We would then have an estimate of equilibrium revenue.

**GSP of Real Search Engines**

In the above discussion, we have assumed that all the advertisers are similar to each other, except in their value per click. But in reality, there is also a difference in the click through rates realised by them in any slot. So, any advertiser in the $i^{th}$slot, receives $\alpha_i\beta_k$ clicks, where $\alpha_i$ is the click through generated due to the position the advertiser is in and $\beta_k$is the click through generated due to the intrinsic quality of the advertiser.

In Yahoo!'s version of GSP, the bidders are still ranked by their bids. And these bids form a locally envy free equilibrium for an advertiser in $i^{th}$ slot when

$$\alpha_i\beta_i(s_i - b_{(i+1)}) \geq \alpha_{i-1}\beta_i(s_i - b_i)$$

Dividing both sides by position number $\beta_i$, we get

$$\alpha_i(s_i - b_{(i+1)}) \geq \alpha_{i-1}(s_i - b_i)$$

This is same as the condition obtained for a locally envy free equilibrium when all $\beta_i s$ are equal to 1. Thus, in Yahoo!'s version of GSP $\beta_i s$ do not affect the equilibrium.

Google handles ranking a little differently than Yahoo!. It assigns a quality score $\gamma_k$ to each advertiser. This score is dependent on the relevance of the ad text to the keyword and the quality of the advertiser's

webpage. Google ranks advertisers by a product of bid and quality score $b_k\gamma_k$. And the payment made by the advertiser when he wins slot $i$ is,

$$p_i = b_{i+1}\gamma_{i+1}/\gamma_i$$

Here the bids would form a locally envy free equilibrium when for any $i$ and j,

$$\alpha_i\beta_i(s_i - b_{i+1}\gamma_{i+1}/\gamma_i) \geq \alpha_j\beta_i(s_i - b_{j+1}\gamma_{j+1}/\gamma_i)$$

Diving both sides by $\beta_i$ and multiplying by $\gamma_i$, we get

$$\alpha_i(s_i\gamma_i - b_{i+1}\gamma_{i+1}) \geq \alpha_j(s_i\gamma_i - b_{j+1}\gamma_{j+1})$$

From the above equation we can conclude that a set of bids is a locally envy free equilibrium in Google's version of GSP with slots having click through rates $\{\alpha_i\}$, advertisers with quality scores $\{\gamma_i\}$ and values $\{s_i\}$ when the set of adjusted bids $\{\gamma_i b_i\}$ form an equilibrium in the basic model with slot click through rates $\{\alpha_i\}$, per click user values $\{s_i\gamma_i\}$ with no quality scores or any advertiser specific factors.

**Truth telling is not a dominant strategy of GSP**

In this section, we see how truth telling need not be a dominant strategy in GSP auction through a simple example.

Consider an auction where there are 3 advertisers who are bidding for the same keyword which has 2 slots available. Let the click through rates of these slots be 200 and 199. Let the values per click of the 3 advertisers be \$10, \$4 and \$2. If all the advertisers bid truthfully, then advertiser 1 and advertiser 2 win the auction with their price per click being \$4 and \$2 respectively. Advertiser 1's payoff would be $(10 - 4) * 200 = \$1200$ and advertiser 2's payoff would be $(4 - 2) * 199 = \$398$. If advertiser 1 was

to bid untruthfully and reduce his bid to $3 while the other advertisers bid truthfully, he/she would win the second slot paying a price of $2 per click where in his payoff would be $(10 - 2) * 199 = \$1592 > \$1200$.

Hence, he has an incentive to reduce his bid and play untruthfully to enhance his payoff. Thus, we can see that truth telling is not a dominant strategy in GSP.

## 5. Vickrey Clarke Groves (VCG) Auction

### Auction Rules VCG

The way in which a VCG auction is conducted is very similar to GSP auction. Let us consider an auction where there are 4 advertisers bidding for a particular keyword with 3 sponsored slots. Let their bids be $b_1 > b_2 > b_3 > b_4$with the first, second and third advertiser winning respective slots. In GSP, the price of each winning advertiser would have been the bid of the advertiser below them, so advertiser 1 would have a price per click of $b_2$,advertiser 2 would pay $b_3$ and advertiser 3 would pay $b_4$.

Unlike GSP, in a VCG auction an advertiser in any position would be charged the externality he imposes on the other advertisers/bidders due to his presence. Thus, payment made by an advertiser $i$ is equal to the aggregate value of clicks to the other advertisers if the advertiser was not present minus the aggregate value of the clicks for other advertisers when the advertiser is present. In the above described 4-advertiser 3-slot instance, the payment made by advertiser 1 would thus be,

$$Aggregate\ Value\ if\ advertiser1\ absent:\ \ b_2\alpha_1 + b_3\alpha_2 + b_4\alpha_3$$

$$Aggregate\ Value\ if\ advertiser1\ present: b_2\alpha_2 + b_3\alpha_3$$

$$Advertiser\ 1's\ payment:\ b_2(\alpha_1 - \alpha_2) + b_3(\alpha_2 - \alpha_3) + b_4\alpha_3$$

Similarly, we can calculate the payments for other advertisers as well. Results are listed below

$$Advertiser\ 2's\ Payment: b_3(\alpha_2 - \alpha_3) + b_4\alpha_3$$

$$Advertiser\ 3' Payment: b_4\alpha_3$$

**Truth-telling a dominant Strategy in VCG auctions**

Truth telling is a dominant strategy in VCG auctions. It is easy to see this once we consider each advertiser's payment. From the above section, if assume that each player is bidding truthfully then $b_i = v_i$ and we note that each advertiser's payment then becomes,

$$p^1\alpha_1 = s_2(\alpha_1 - \alpha_2) + s_3(\alpha_2 - \alpha_3) + s_4\alpha_3$$

$$p^2\alpha_2 = s_3(\alpha_2 - \alpha_3) + s_4\alpha_3$$

$$p^3\alpha_3 = s_4\alpha_3$$

Here we note that the payment any advertiser makes is independent of his own value per click but is dependent on other advertiser's value/bid per click. The advertiser's value per click only helps him gain the slot he deserves. But, this might lead to an argument that in bid to gain top slot an advertiser might bid abnormally high above his truthful value. To counter this, we argue that such bids might get the advertiser a better slot but the value he gains due to the incremental clicks would be less than the price he pays for these clicks. Thus, in a VCG auction it is in the best interest of the advertiser to bid truthfully.

**Revenue in an VCG auction**

In this section, we calculate equilibrium revenue in a VCG auction. Earlier we have noted that payment by each advertiser in a VCG auction for a 4-advertiser 3-slot case. Now consider $N$ advertisers and $K$ slots with $K < N$ (without loss of generality), then the payments become (putting $s_{K+1} = p_K$),

$$p^1\alpha_1 = s_2(\alpha_1 - \alpha_2) + s_3(\alpha_2 - \alpha_3) + s_4(\alpha_3 - \alpha_4) \dots\dots.. + p^K\alpha_K$$

$$p^2\alpha_2 = \qquad s_3(\alpha_2 - \alpha_3) + s_4(\alpha_3 - \alpha_4) \dots\dots\dots + p^K\alpha_K$$

$$p^3\alpha_3 = \qquad s_4(\alpha_3 - \alpha_4) \dots\dots\dots\dots\dots. + p^K\alpha_K$$

Summing this over all the advertisers up to $p_K\alpha_K$ we get,

$$\sum_k \alpha_k\, p^k = s_2(\alpha_1 - \alpha_2) + 2s_3(\alpha_2 - \alpha_3) \dots\dots \; + (K-1)\alpha_K p^K$$

This is equal to the lower bound on the equilibrium revenue for a GSP auction as seen in section *2.3*. Hence, we see that when a vector of bids by advertisers for a particular keyword converge, the revenue realised by the search engine is weakly greater in a GSP auction than in a VCG auction. This forms the basis of our arguments in the next section as to why GSP has seen more adoption in sponsored search auctions than VCG.

## 6. Conclusion

Online advertisement started when a firm's marketing manager sent spam e-mails to about 400 recipients. This eventually evolved into advertisers spamming online user forums of various kinds. Then world's

first un-clickable display ad was used on an online news portal. Later on, a need was felt for more focussed targeting of advertisement which lead to Yahoo! pioneering Keyword based online ads in 1995. From here on, innovation in the online advertisement industry was directed towards charging the advertisers better. This led to the formulation of pay-per thousand impressions & pay-per click models. As the industry matured further, it converged to pay-per-click model with variations in the payment as decided by auctions predominantly Generalised Second Price (GSP) and Vickrey Clark Groves (VCG) mechanisms.

Most of the search engines today have converged to using GSP auctions with few minor improvements. A major motivation for such improvements has been to improve the relevance of sponsored search to the search engine user. One such modification by Google is to improve the constituents of Quality Score (QS) in the ranking mechanism in auctions. So instead of ranking advertisements purely based on their bids, they are ranked by a product of their bid and an improved Quality Score. The way various search engines define this Quality score is not the same. The success of Google’s auction model has led to its widespread adoption in all kinds of sponsored search including e-commerce (Amazon Ad badger).

But Facebook with the same motivation to show contextually relevant ads, has been experimenting with VCG auctions. VCG allows Facebook to rank the ads not only with other ads buy also with the user’s stories, to improve its relevance. Although both Facebook and Google are aiming to achieve similar objectives, they continue to use different auction mechanisms. VCG has the added incentive of truthful bidding, making it more efficient so that the advertisers can concentrate on their campaigns rather than trying to rig the auctioning system.

But Google has not made it clear why it is sticking to GSP, even though VCG mechanism encourages truthful bidding. One major reason for this may be the revenues realised by the search engine in the two auction mechanisms. As seen earlier, every equilibrium revenue of GSP is weakly greater than the equilibrium revenue of VCG auctions. Hence, by shifting to VCG auctions a firm of the size of Google adds a lot of uncertainty to its future revenues. Another reason is the auction mechanism. VCG is not very easy to explain to the advertisers unlike GSP which is very simple both to comprehend and to implement. Other reason could be the network effect, Google started with GSP in 2002 and since then a whole ecosystem has been created around GSP like Google training kits, third-party bidding software, advertisers bidding strategies et cetera which could be difficult to dislodge from their status quo at least as far as sponsored search is concerned.

Today therefore, most of the sponsored search advertisements feature a GSP auction or some variation of it. But for contextual advertisements on web pages (display ads) whose auction mechanisms were deliberated much later in 2012, VCG was reconsidered and implemented.

Hence, we see, although both the major mechanisms have their advantages and disadvantages, due to the timing and the way in which they have evolved they have found users in different forms of online advertising.

**References**

1. Benjamin Edelman, Michael Ostrovsky, and Michael Schwartz. Internet advertising and the Generalized Second Price auction. American Economic Review Vol. 97, No. 1, March 2007

2. Hal Varian, Christopher Harris. VCG in Theory and Practise. American Economic Review Vol. 104, No. 4, May 2014
3. Hal R. Varian. Online ad auctions. American Economic Review, Vol. 99, No. 2, May 2009
4. Hal R. Varian. Position auctions. International Journal of Industrial Organization Vol. 25, No. 6, 2007
5. Bernard J. Jansen and Tracy Mullen. Sponsored search: an overview of the concept, history, and technology. Int. J. Electronic Business Vol. 6, No. 2, 2008
6. Lawrence M. Ausubel and Paul Milgrom. The Lovely but Lonely Vickrey Auction. Stanford Institute for Economic Policy Research Discussion Paper No. 03-36, August 2004
7. Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay Vazirani. Algorithmic Game Theory. Cambridge University Press New York, 2007
8. Ben Greiner, Axel Ockenfels, and Abdolkarim Sadrieh. Internet Auctions. The Oxford Handbook of the Digital Economy, Oxford University Press, 2012
9. www.worldata.com/wdnet8/articles/the_history_of_Internet_Advertising.htm
10. https://www.recode.net/2017/12/4/16733460/2017-digital-ad-spend-advertising-beat-tv
11. www.zakon.org/robert/internet/timeline
12. https://blog.hubspot.com/marketing/history-of-online-advertising
13. https://blog.hubspot.com/marketing/history-of-online-advertising
14. https://www.shootorder.com/blog/history-pay-per-click/

15. https://publishing2.com/2008/05/27/google-adwords-a-brief-history-of-online-advertising-innovation/
16. http://www.oxfordhandbooks.com/view/10.1093/oxfordhb/9780195397840.001.0001/oxfordhb-9780195397840-e-14?print=pdf
17. https://blog.adcash.com/pop-under-and-pop-up-ads/