

Volatility trading and volatility derivatives

Implied volatilities

The only unobservable parameter in the Black-Scholes formulas is the volatility value, σ . By inputting an estimated volatility value, we obtain the option price. Conversely, given the market price of an option, we can back out the corresponding *Black-Scholes implied volatility*.

- Require root finding algorithms for the determination of implied volatilities.
- Several implied volatility values obtained simultaneously from different options (varying strikes and maturities) on the same underlying asset provide the market view about the volatility of the stochastic movement of the asset price.

Black wrote

“It is rare that the value of an option comes out exactly equal to the price at which it trades on the exchange.

... there are three reasons for a difference between the value and price: we may have the correct value, and the option price may be out of line; we may have used the wrong inputs to the Black-Scholes formula; or the Black-Scholes may be wrong. Normally, all three reasons play a part in explaining a difference between value and price.”

Extreme view point: The market prices are correct (in the presence of sufficient liquidity) and one should build a model around the prices.

Different volatilities for different strike prices

Stock options – higher volatilities at lower strike and lower volatilities at higher strikes

- In a falling market, everyone needs out-of-the-money puts for insurance and will pay a higher price for the lower strike options.
- Equity fund managers are long billions of dollars worth of stock and writing out-of-the-money call options against their holdings as a way of generating extra income.

Commodity options – higher volatilities at higher strike and lower volatilities at lower strikes

- Government intervention – no worry about a large price fall. Speculators are tempted to sell puts aggressively.
- Risk of shortages – no upper limit on the price. Demand for higher strike price options.

Volatility smiles

Interest rate options – at-the-money option has a low volatility and either side the volatility is higher

Propensity to sell at-the-money options and buy out-of-the-money options.

For example, in the butterfly strategy, two at-the-money options are sold and one-out-of the-money option and one in-the-money option are bought.

Different volatilities across time

Supply and demand

When markets are very quiet, the implied volatilities of the near month options are generally lower than those of the far month. When markets are very volatile, the reverse is generally true.

- In very volatile markets, everyone wants or needs to load with gamma. Near-dated options provide the most gamma and the resultant buying pressure will have the effect of pushing prices up.
- In quiet markets no one wants a portfolio long of near dated options.

Use of a two-dimensional implied volatility matrix.

Floating volatilities

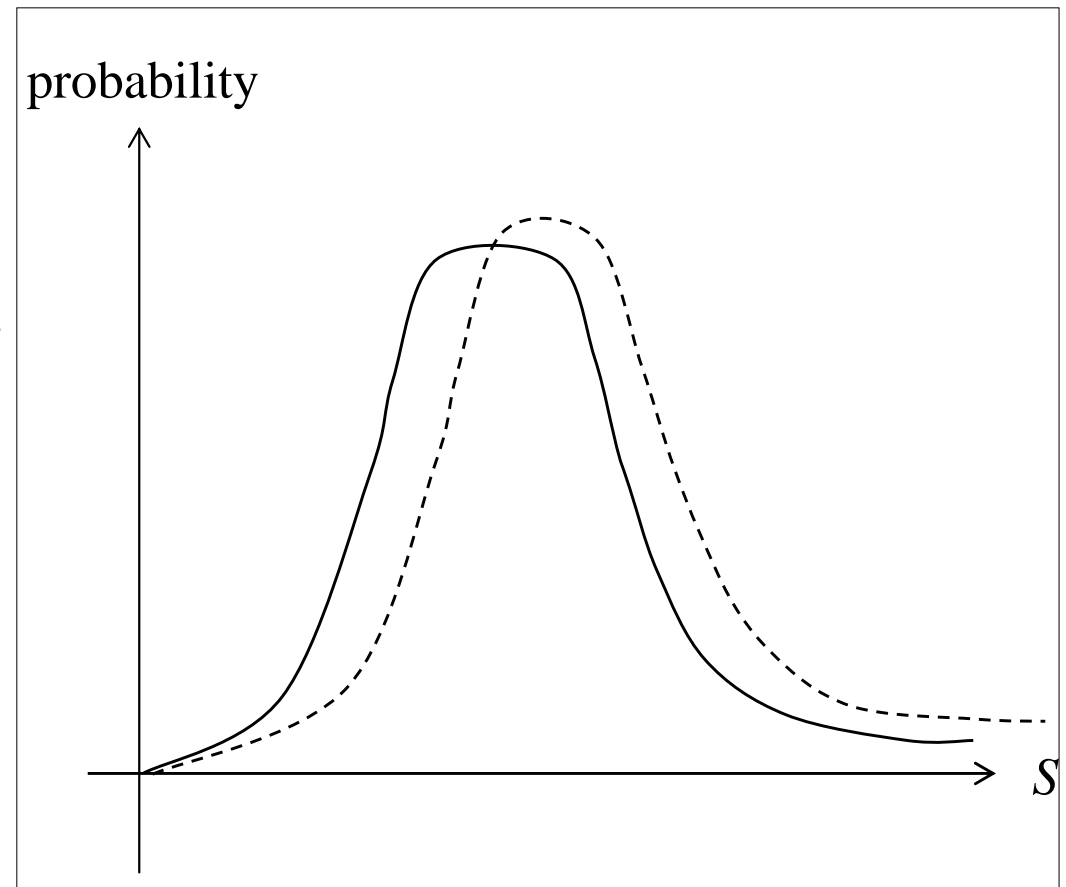
As the stock price moves, the entire skewed profile also moves. This is because what was out-of-the-money option now becomes in-the-money option.

Example

If an investor is long a given option and believes that the market will price it at a lower volatility at a higher stock price then he may adjust the delta downwards (since the price appreciation is lower with a lower volatility).

Terminal asset price distribution as implied by market data

In real markets, it is common that when the asset price is high, volatility tends to decrease, making it less probable for high asset price to be realized. When the asset price is low, volatility tends to increase, so it is more probable that the asset price plummets further down.



solid curve: distribution as implied by market data

dotted curve: theoretical lognormal distribution

Extreme events in stock price movements

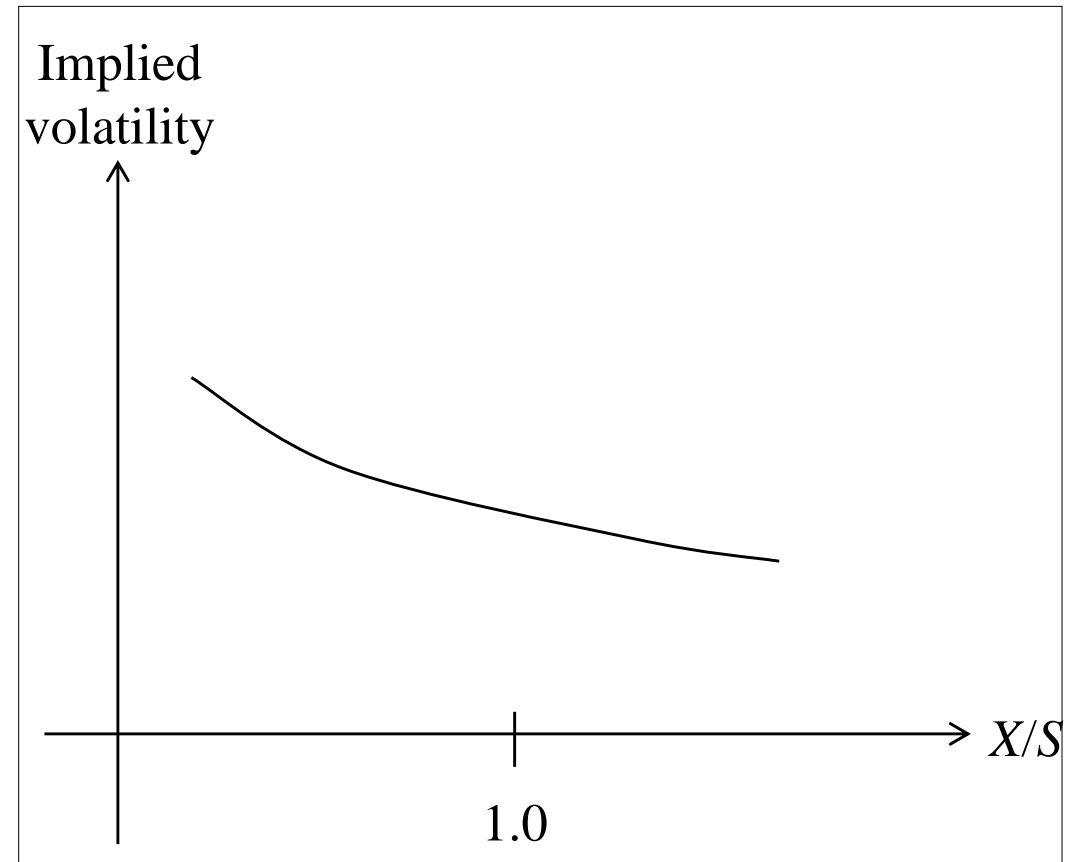
Probability distributions of stock market returns have typically been estimated from historical time series. Unfortunately, common hypotheses may not capture the probability of extreme events, and the events of interest are rare and may not be present in the historical record.

Examples

1. On October 19, 1987, the two-month S & P 500 futures price fell 29%. Under the lognormal hypothesis of annualized volatility of 20%, this is a -27 standard deviation event with probability 10^{-160} (virtually impossible).
2. On October 13, 1989, the S & P 500 index fell about 6%, a -5 standard deviation event. Under the maintained hypothesis, this should occur only once in 14,756 years.

The market behavior of higher probability of large decline in stock index is better known to practitioners after Oct., 87 market crash.

- The market price of out-of-the-money call (puts) has become cheaper (more expensive) than the Black-Scholes theoretical price after the 1987 crash because of the thickening (thinning) of the left-end (right-end) tail of the terminal asset price distribution.



A typical pattern of post-crash smile.
The implied volatility drops against X/S .

Theoretical and implied volatilities

Theoretical volatility

- When valuing an option, a trader's theoretical volatility will be a critical input in a pricing model.
- The strategy of trading on theoretical volatilities involves holding the option until expiry – common strategy of option users.

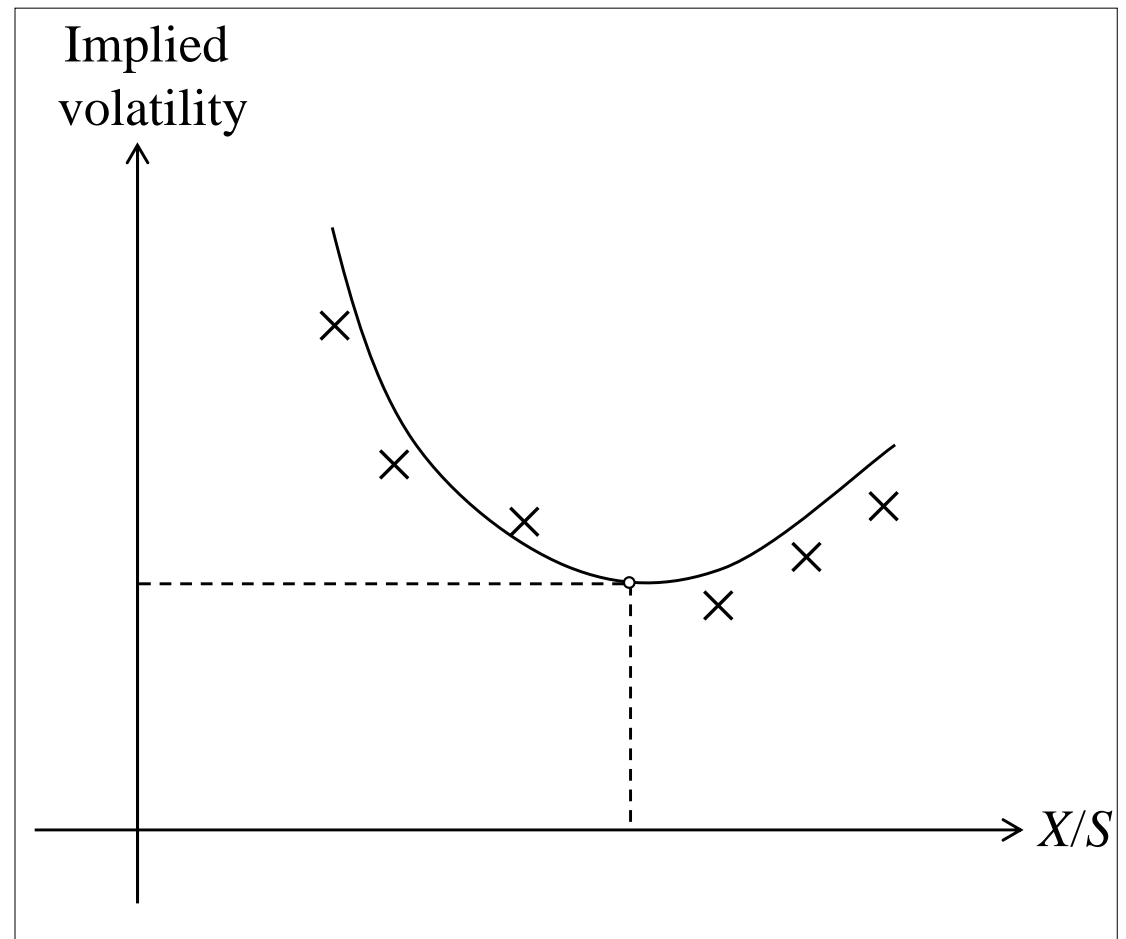
Market implied volatility

- Volatility extrapolated from, or implied by, an option price.
- Trading on implied volatility involves implementing and reversing positions over short time periods.

Preliminary steps

It is always necessary to provide prices of European options of strikes and expirations that may not appear in the market. These prices are supplied by means of *interpolation* (within data range) or *extrapolation* (outside data range).

- A smooth curve is plotted through the data points (shown as “crosses”). The estimated implied volatility at a given strike can be read off from the dotted point on the curve.



Time dependent volatility

- Given the market prices of European call options with different maturities (all have the strike prices of 105, current asset price is 106.25 and short-term interest rate over the period is flat at 5.6%).

<i>maturity</i>	1-month	3-month	7-month
<i>Value</i>	3.50	5.76	7.97
<i>Implied volatility</i>	21.2%	30.5%	19.4%

- Extend the assumption of constant volatility to allow for *time dependent deterministic volatility* $\sigma(t)$.

The Black-Scholes formulas remain valid for time dependent volatility

except that $\sqrt{\frac{1}{T-t} \int_t^T \sigma(\tau)^2 d\tau}$ is used to replace σ .

How to obtain $\sigma(t)$ given the implied volatility measured at time t^* of a European option expiring at time t . Now

$$\sigma_{imp}(t^*, t) = \sqrt{\frac{1}{t - t^*} \int_{t^*}^t \sigma(\tau)^2 d\tau}$$

so that

$$\int_{t^*}^t \sigma(\tau)^2 d\tau = \sigma_{imp}^2(t^*, t)(t - t^*).$$

Differentiate with respect to t , we obtain

$$\sigma(t) = \sqrt{\sigma_{imp}(t^*, t)^2 + 2(t - t^*)\sigma_{imp}(t^*, t) \frac{2\sigma_{imp}(t^*, t)}{\partial t}}.$$

Practically, we do not have a continuous differentiable implied volatility function $\sigma_{imp}(t^*, t)$, but rather implied volatilities are available at discrete instants t_i . Suppose we assume $\sigma(t)$ to be piecewise constant over (t_{i-1}, t_i) , then

$$\begin{aligned} & (t_i - t^*)\sigma_{imp}^2(t^*, t_i) - (t_{i-1} - t^*)\sigma_{imp}^2(t^*, t_{i-1}) \\ &= \int_{t_{i-1}}^{t_i} \sigma^2(\tau) d\tau = \sigma^2(t)(t_i - t_{i-1}), \quad t_{i-1} < t < t_i, \end{aligned}$$

$$\sigma(t) = \sqrt{\frac{(t_i - t^*)\sigma_{imp}^2(t^*, t_i) - (t_{i-1} - t^*)\sigma_{imp}^2(t^*, t_{i-1})}{t_i - t_{i-1}}}, \quad t_{i-1} < t < t_i.$$

Local volatility surface $\sigma(S, t)$

The time dependence in implied volatility can be turned into a volatility of the underlying that is time dependent.

Question: Can we deduce $\sigma(S, t)$ from $\sigma_{imp}(t^*, X, T)$?

Theoretical solution

Suppose a distribution of European call prices of all strikes and maturities are available, as denoted by $V(X, T)$, we then can obtain the dependence of the volatility on strike and expiry, based on the current asset price S^* and current time t^* .

The risk neutral random walk for S is assumed to be

$$\frac{dS}{S} = rdt + \sigma(S, t)dZ .$$

Implied volatility tree

An implied volatility tree is a binomial tree that prices a given set of input options correctly.

The implied volatility trees are used:

1. To compute hedge parameters that make sense for the given option market.
2. To price non-standard and exotic options.

The implied volatility tree model uses all of the implied volatilities of options on the underlying - it deduces the best flexible binomial tree (or trinomial tree) based on all the implied volatilities.

Volatility trading

Trading based on taking a *view on market volatility* different from that contained in the current set of market prices. This is different from *position trading* where the trades are based on the expectation of where prices are going.

Example

A certain stock is trading at \$100. Two one-year calls with strikes of \$100 and \$110 priced at \$5.98 and \$5.04, respectively.

These prices imply volatilities of 15% and 22%, respectively.

Strategy Long the cheap \$100 strike option and short of the expensive \$110 strike option.

Trading volatilities

Short term players

- Sensitive to the market prices of the options.
- This is more of a speculative trading strategy, applicable only to liquid options markets, where the cost of trading positions is small relative to spreads captured in implied volatility moves.

Long term players

- If a trader's theoretical value is higher than the implied volatility, he would buy options since he believes they are undervalued.

Possible Win-Win Scenarios

BUYER

**IMPLIED VOL TRADER:
EXPECTS IMPLIEDS TO GO UP**

OPTION

\$ \$ \$

SELLER

**THEORETICAL VOL TRADER:
SEES IMPLIEDS
AS OVERVALUED**

**THEORETICAL VOL TRADER:
SEES IMPLIEDS
AS UNDERVALUED**

OPTION

\$ \$ \$

**IMPLIED VOL TRADER:
EXPECTS IMPLIEDS
TO GO DOWN**

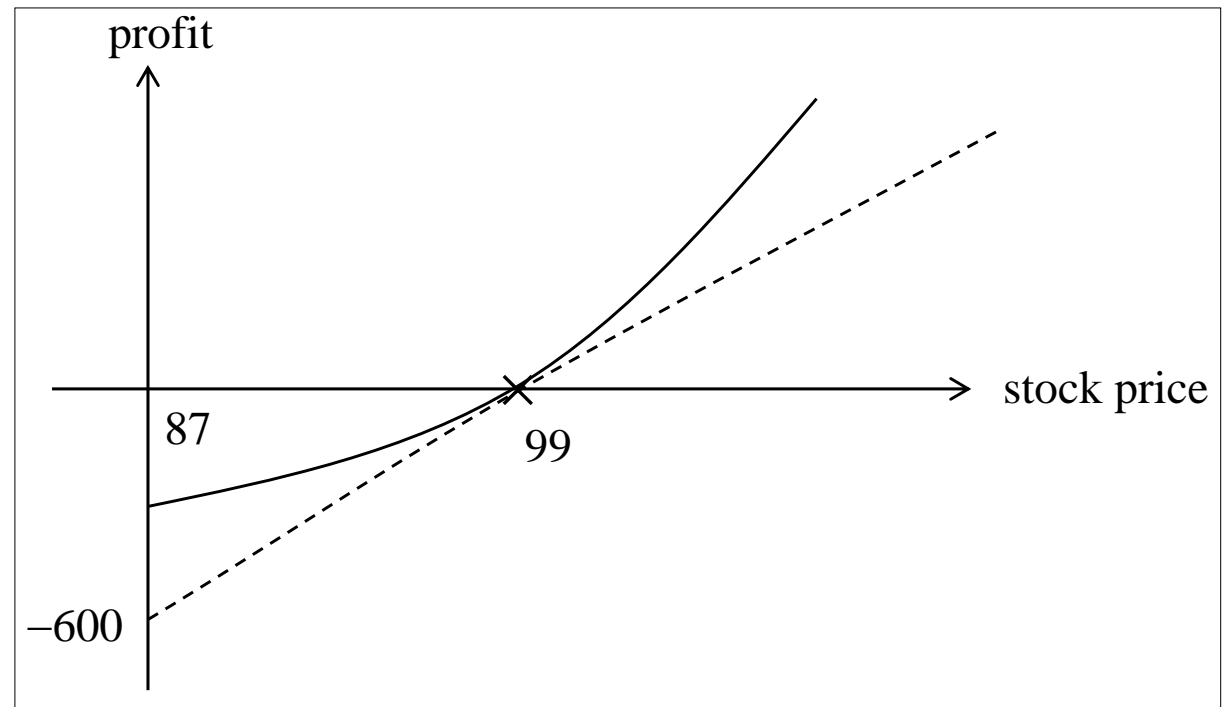
Market data: Stock price = \$99, call price = \$5.46, delta = 0.5

portfolio A: 50 shares of stock; $\pi_A|_{t=0} = \$4,550$

portfolio B: 100 call options; $\pi_B|_{t=0} = \$546$

solid line: option portfolio

dotted line: stock portfolio



Both portfolios are *delta equivalent*.

Since the option price curve is concave upward, the call option portfolio always outperforms the delta equivalent stock portfolio.

Long volatility trade

Whichever way the stock price moves, the holder always make a profit. This is the essence of the long volatility trade.

- By rehedging, one is forced to sell in rising markets and buy in falling market – trade in the opposite direction of the market trend.

Where is the catch

- The option loses time value throughout the life of the option.

Long volatility strategy

Competition between the original price paid and the subsequent volatility experienced. If the price paid is low and the volatility is high, the long volatility player will win overall.

Vega risk

Vega is defined as the change in option price caused by a change in volatility of 1%.

- Shorter dated options are less sensitive of volatility inputs. That is, vega decreases with time.
- Near-the-money options are most sensitive and deep out-of-the-money options are less sensitive.

Gamma trading and vega trading

Time decay profit: position gamma $\times \frac{(\text{asset price})^2}{2} \times (\text{implied volatility})^2$

Gamma trading Net profit from realized volatility

position gamma $\times \frac{(\text{asset price})^2}{2} \times [(\text{realized volatility})^2 - (\text{implied volatility})^2]$

Vega trading Net profit from changes in implied volatility

vega $\times (\text{current implied volatility} - \text{original implied volatility})$

Maturity and moneyness

The ability of individual derivative positions to realize profits from gamma and vega trading is crucially dependent on the average maturity and degree of moneyness of the derivatives book.

- For at-the-money options, long maturity options display high vega and low gamma; short maturity options display low vega and high gamma.
- For out-of-the-money options, long maturity options display lower vega and high gamma, and short maturity options higher vega and lower gamma.

Balance between gamma-based and vega-based volatility trading

1. If a trader desires high gamma but zero vega exposure, then a suitable position would be a large quantity of short maturity at-the-money options hedged with a small quantity of long maturity at-the-money options.
2. If a trader desires high vega but zero gamma exposure, then a suitable position would be a large quantity of long maturity at-the-money options hedged with a small quantity of short maturity at-the-money options.

Long gamma – holding a straddle

A trader believes that the current implied volatility of at-the-money options is lower than he expects to be realized. He may buy a straddle: a combination of an at-the-money call and an at-the-money put to acquire a delta neutral, gamma position.

Trading mispriced options

If options are offered at an implied volatility of 15% and a manager believes that the real volatility is going to be higher in the future, say, 25%. How to profit?

He should set up a delta neutral portfolio.

If his prediction is correct, he can profit in two ways:

1. The rest of the market begin to agree with him, then the option price will mark up. He gains by unwinding his option position.
2. The market continues to price options at 15%,. He keeps the portfolio delta neutral (delta calculated based on market volatility). His rehedging profit will exceed the time decay losses.

Log contract

The settlement price is given by $\log S_T$. The present value of the Log contract during its life is almost equal to

$$\frac{1}{2}(ISD^2 - \sigma^2) \times \text{time to maturity},$$

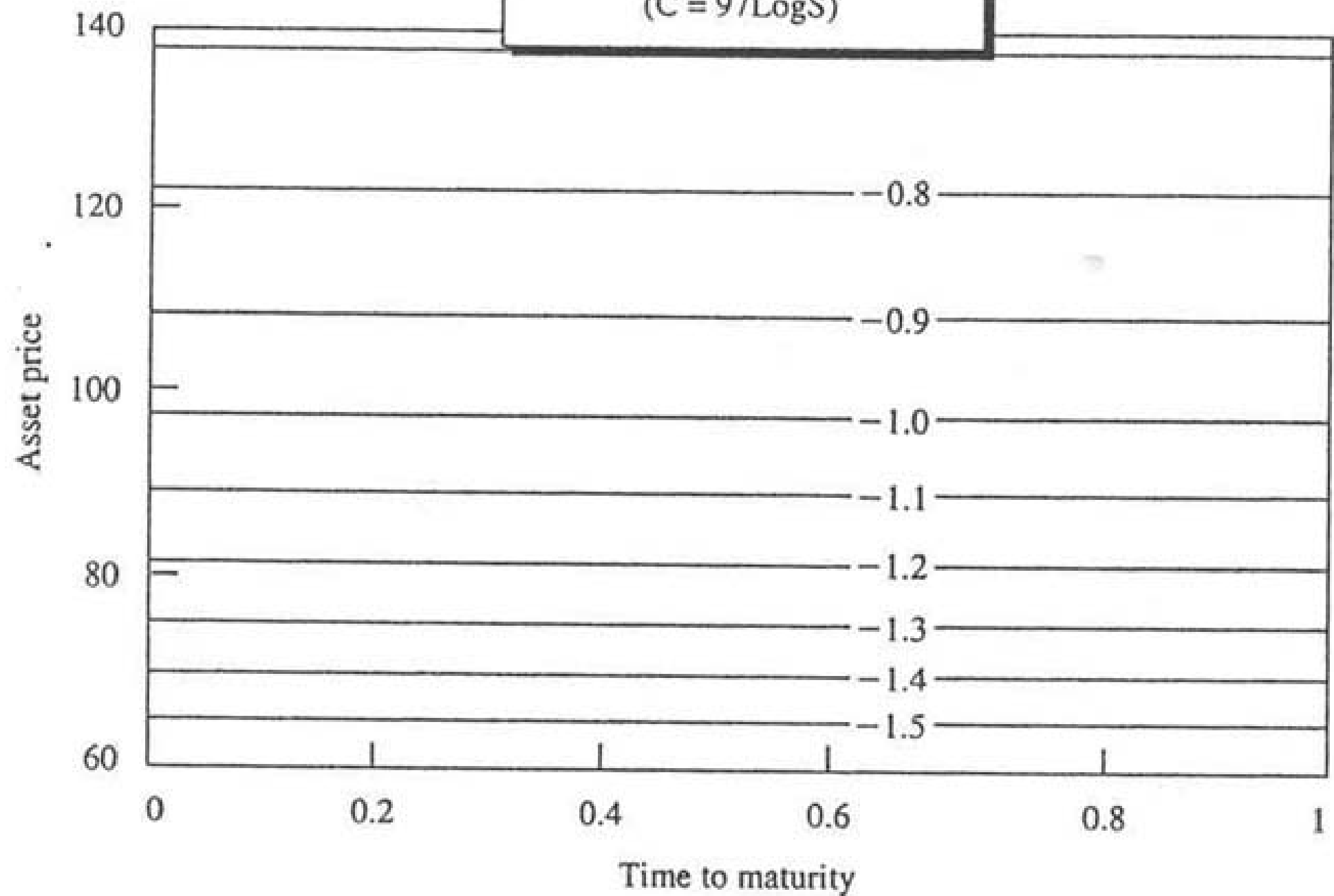
where ISD is the volatility implied in the price of the Log Contract and σ is the realized volatility.

Advantages of the Log contract

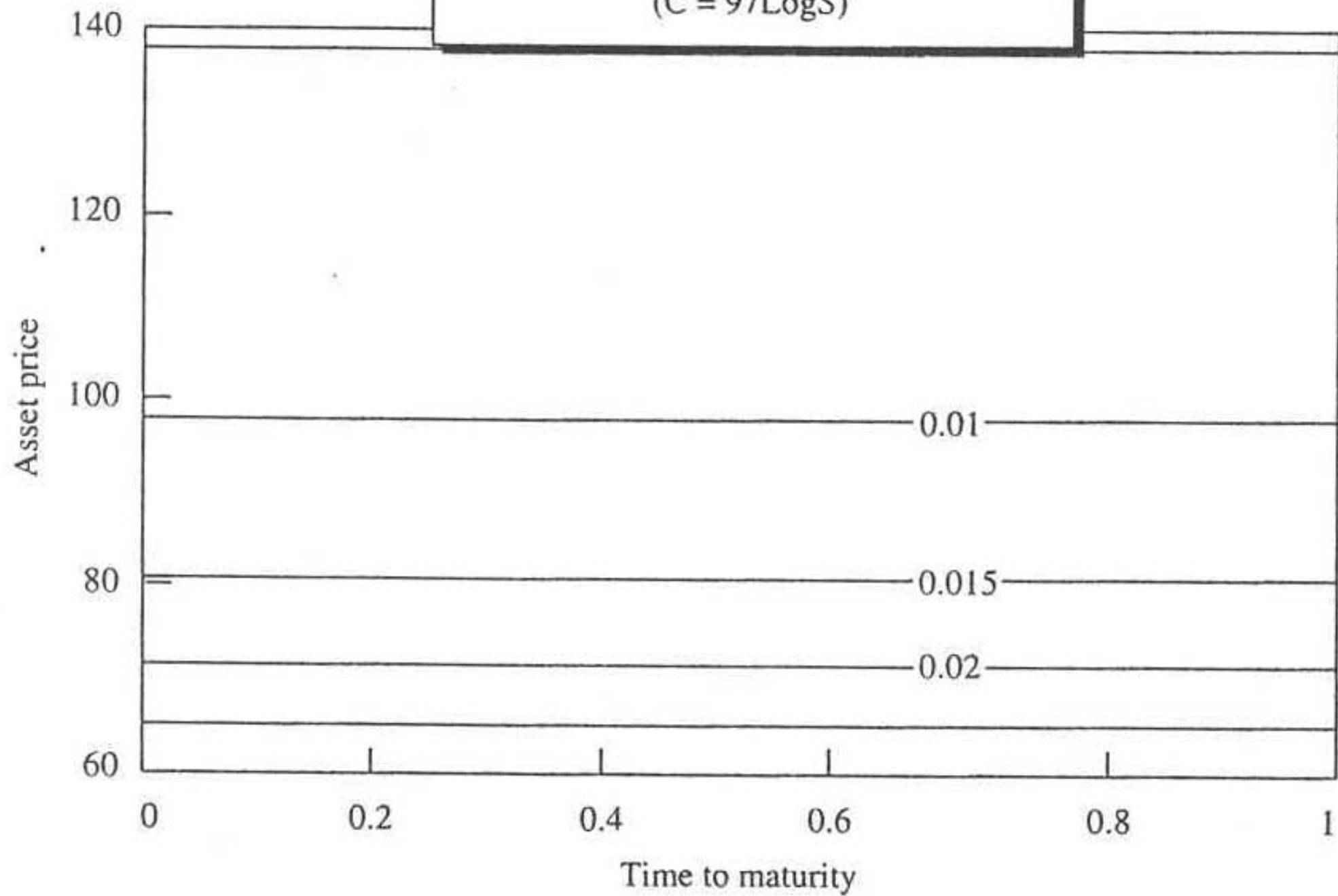
There is a need for simple option product that enable investors to trade views on future volatility. For the Log contract:

- It is easy to delta hedge. A trader who is long one Log contract will delta hedge by shorting \$1 worth of the underlying asset.
- The performance of a delta hedged Log contract depends only on the outcome volatility and not on the hedger's forecast of volatility.
- The dynamic strategy is a stable strategy that depends only in the level of the asset price, not on the time to maturity.
- When delta hedged, it is a pure volatility play, unlike a delta hedged option.

Delta for a Log Contract
($C = 97\text{Log}S$)



Gamma for a Log Contract
($C = 97\text{Log}S$)



Variance swap contract

The terminal payoff of a variance swap contract is

$$\text{notional} \times (v - \text{strike})$$

where v is the realized annualized variance of the logarithm of the daily return of the stock.

$$v = \frac{N}{n-1} \left[\sum_{i=0}^{n-1} \left(\ln \frac{S_{i+1}}{S_i} - \mu \right)^2 \right]$$

Variance swap contract (cont'd)

where n = number of trading days to maturity

N = number of trading days in one year (252)

μ = realized average of the logarithm of daily return of the stock

$$= \frac{1}{n} \sum_{i=0}^{n-1} \ln \frac{S_{i+1}}{S_i} = \frac{1}{n} \ln \frac{S_n}{S_0}.$$

- The payoff could be positive or negative.
- The objective is to find the fair price of the strike, as indicated by the prices of various instruments on the trade date, such that the initial value of the swap is zero.

Observe that

$$v = \frac{N}{n-1} \left[\frac{\sum \left(\ln \frac{S_{i+1}}{S_i} \right)}{n} - \left(\frac{\sum \ln \frac{S_{i+1}}{S_i}}{n} \right)^2 \right].$$