Partial Vertical Integration, Ownership Structure and Foreclosure*

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Abstract

We study the incentive of firms to acquire partial, controlling, stakes in vertically related firms and then foreclose a downstream rival. We show that partial acquisition of an upstream supplier (partial backward integration) is more likely to occur and lead to foreclosure than full vertical merger, especially when initially, the upstream supplier is held by dispersed shareholders. By contrast, partial acquisition of a downstream customer (partial forward integration) is less profitable than full vertical merger, especially when initially, the downstream customer has a controlling shareholder whose controlling stake is small, and is never profitable when the downstream customer is initially held by dispersed shareholders. We also show that partial backward integration, followed by foreclosure of a downstream rival is less likely when the downstream customer holds a toehold in the upstream supplier, more likely when the acquisition is made by the controlling shareholder of the downstream customer rather than the firm itself, and is as likely when two downstream customers compete for the acquisition of control in the upstream supplier.

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1 Introduction

One of the main antitrust concerns that vertical mergers raise is that the merger will result in the foreclosure of either upstream or downstream rivals. While most of the discussion on vertical foreclosure has focused on full vertical mergers, in reality, many firms acquire partial stakes in suppliers (partial backward integration) or in buyers (partial forward integration). A case in point is the U.S. cable industry where several operators acquired partial ownership stakes in cable or television networks (see Waterman and Weiss, 1997, p. 24-32). This situation has raised the concern that non-integrated networks will be denied access to cable systems or will obtain access at unfavorable terms.[1]

In this paper we study partial vertical integration and examine the circumstances under which it may occur and lead to "input foreclosure" i.e., the foreclosure of a downstream rival. The main question that we ask is whether firms have an incentive to acquire a partial conrolling stake in an upstream supplier or a downstream buyer in the first place, and how this incentive depends on the initial ownership structure of the target firm (the upstream supplier in case of partial backward integration or the downstream buyer in case of partial forward integartion).

To address this question we consider a model in which two downstream firms buy inputs from several upstream suppliers. Following integration between one of the downstream firms and one of the upstream suppliers, the upstream supplier may wish to foreclose the remaining downstream rival in order to weaken it and thereby boost its own downstream profit. This input foreclosure however lowers the profit of the integrated upstream supplier, because it now forgoes sales to the downstream rival. Under partial backward integration, part of the upstream loss from input foreclosure is borne by the minority shareholders of the integrated upstream supplier. Since the downstream buyer captures the entire associated gain, input foreclosure is more profitable under partial backward integration than under full vertical integration. We show that partial backward integration, which

[1] Recent prominent examples include News Corp.'s (a major owner of TV broadcast stations and programming networks) acquisition of a 34% stake in Hughes Electronics Corporation in 2003, which gave it a de facto control over DirecTV Holdings, LLC (a direct broadcast satellite service provider which is wholly-owned by Hughes), and the 2011 joint venture agreement between Comcast, GE, and NBCU, which gave Comcast (the largest cable operator and Internet service provider in the U.S.) a controlling 51% stake in a joint venture that owns broadcast TV networks and stations, and various cable programming. In the UK, BSkyB (a leading TV broadcaster) acquired in 2006 a 17.9% stake in ITV (UK's largest TV content producer). The UK competition commission found that the acquisition gave BSkyB effective control over ITN and argued that BSkyB would use it to "reduce ITV's investment in content" and "influence investment by ITV in high-definition television (HDTV) or in other services requiring additional spectrum."

leads to input foreclosure, is particularly profitable when the upstream supplier is initially held by dispersed shareholders. In that case, the downstream buyer acquires the minimal stake that ensures control over the upstream supplier at a price that reflects the supplier's pre-acquisition value. The downstream buyer then internalizes only the reduction in the value of the stake it acquires. The rest of the upstream loss is borne by the remaining shareholders of the upstream supplier, who effectively subsidize the input foreclosure.[2] However, when the upstream supplier has initially a controlling shareholder, the downstream buyer needs to compensate this shareholder for reduction in the value of his entire stake in order to induce him to sell a controlling stake to the downstream buyer. Since this stake may well exceed the minimal stake that ensures control, backward interagration is more costly when the upstream supplier has a conrolling shareholder and therefore it is less likely to occur and lead to input foreclosure.

By contrast, input foreclosure is less profitable under partial forward integration than under full vertical integration because part of the downstream gain from foreclosure is now captured by the minority shareholders of the integrated downstream buyer, while the upstream supplier bears the entire cost. We show that this transfer of wealth to the monority shareholders of the upstream supplier renders partial firward integration unprofitable when the downstream buyer is initially held by dispersed shareholders, although it may be profitable when the downstream buyer has a controlling shareholder whose controlling stake is sufficiently large (i.e., there are relatively few minority shareholder who receive a subsidy).

We also consider a few extensions of our basic setup. First, we consider the possibility that the downstream buyer holds a toehold (i.e., an initial non controlling stake) in the upstream supplier before it has the opportunity to acquire full control. We show that the toehold weakens the incentive of the downstream buyer to acquire control over the upstream supplier and use it to foreclose a downstream rival whenever the upstream supplier is initially held by a controlling shareholder, but not if the upstream suppliers is initially held by dispersed shareholders. Second, we show that the controlling shareholder of a downstream firm will prefer to acquire control over an upstream supplier through some firm under his control in which he holds a small stake, rather

[2] Of course, if the rights of the passive shareholders of the upstream supplier are protected effectively, the downstream buyer would be unable to use its control to foreclose the downstream rival. In reality, however, it may be very hard to prevent such foreclosure on the grounds that it expropriates the wealth of passive shareholders. Indeed had the protection of passive shareholders been perfect, antitrust authorities would have no reason to be concenred about the possibility of input foreclosure following partial backward integration. The cases mentioned above indicate however that antitrust authorities are concened about this possibility.

than acquire control directly. In particular, an acquisition by a controlling shareholder expands the range of parameters for which input foreclosure occurs. Finally, we show that competition between the two downstream firms for the acuisition of control over an upstream supplier boosts the price paid for the acquired shares, although it does not alter the range of parameters for which foreclosure takes place.

There are three strands in the literature on input foreclosure.[3] Bolton and Whinston (1993) consider a model in which two downstream firms invest in order to boost the quality of their products before they trade with an upstream supplier. Vertical integration strengthens the incentive of the integrated downstream firm to invest and weakens the incentive of nonintegrated firm to invest. As a rseult, the nonintegrated firm has a lower chance to buy the input when its supply is limited. Bolton and Whinston interpret this situation as input foreclosure. Unlike in Bolton and Whinston (1993), where foreclosure is a by-product of the effect of integration on downstream investments, foreclosure in the other two strands of the literature is due to a deliberate refusal of an upstream supplier to supply the input. In Hart and Tirole (1990), an upstream supplier prefers to deal exclusively with a single downstream firm in order to allow this firm to monopolize the downstream market. The upstream supplier in turn extracts the monopoly downstream profit via a non linear contract. The third strand in the literature, due to Ordover, Saloner and Salop (1990) and Salinger (1988), considers models in which the vertically integrated firm deliberately forecloses downstream rivals in order to raise their costs of buying the input from alternative suppliers. This benefits the integrated downstream firm, who now faces weaker rivals in the downstream market.[4]

Our model is closely related to the third, raising rival cost, strand. Similarly to these models, the upstream supplier in our model also forecloses the downstream rival in order to give the integrated donwstream firm a strategic advantage in the downstream market. In our model though, foreclosure does not raise the costs of downstream rivals but rather it dimishes the value that the downstream rival can offer consumers. More importantly, our paper focuses on partial, rather than full vertical integration.

There are only few papers which consider the competitive effects of partial vertical integration. Greenlee and Raskovitch (2006) and Hunold, Röller, and Stahl (2012) consider passive

[3] See Rey and Tirole (2007) and Riordan (2008) for literature surveys.

[4] The assumption that the upstream firm can commit to foreclose the downstream rival.has crticized as problematic, see Hart and Tirole (1990) and Reiffen (1992) and see Ordover, Salop, and Saloner (1992) for a response. Several papers, including Ma (1997), Chen (2001), Choi and Yi (2001) and Church and Gandal (2000), have proposed models that are immune to this criticism.

acquisitions, which affect the incentive of the acquirer, but do not have a direct effect on the target's strategy like in our paper. We are aware of only two papers which consider the acquisition of partial controling stakes. Baumol and Ordover (1994) show that when a downstream firm controls a bottleneck owner with a partial ownership stake, it has an incentive to divert business to itself, even if downstream rivals are more efficient, because it fully captures the benefits from this diversion, but bears only part of the associated upstream loss.[5] Spiegel (2013) examines a model in the spirit of Bolton and Whinston (1993), in which foreclosure arises due to the efefct of vertical integration on the incentives of the downstream firms to invest in the quality of their products. He shows that relative to full vertical integration, partial vertical integration may either alleviates or exacerbate the concern for vertical foreclosure and examine the implications for consumer welfare. Neither one of these papers, however, examines how the incentive to integrate depends on the owneship structure of the target, which is the main focus of the current paper.

The rest of the paper is organized as follows. The basic model is presented in Section 2. In Section 3 we examine the incentive to engage in "input foreclosure" following partial backward or forward inetgartion. Our main results appear in Section 4, where we examine how the incentive to partially integrate and engage in input foreclosure depend on the initial ownership structure of the target firm. In Section 5 we study three extensions of our basic setup, and in Section 6 we concude. In the Appendix we show

2 The model

Consider two downstream firms, D_1 and D_2, that use up to $N \geq 1$ differentiated inputs to provide a final good/service to consumers. Each input $i = 1, 2, \ldots, N$ is produced by a single upstream supplier U_i. To simplify matters, we assume that the cost that an upstream supplier incurs when it serves a downstream firm is c.[6] Let $\Pi(k, l)$ denote the (reduced form) profit of a downstream firm when it uses k inputs and its rival uses l inputs, before any payments to upstream suppliers. Throughout the analysis we will impose the following assumption:

[5] Reiffen (1998) examines the stock market reaction to Union Pacific (UP) Railroad's attempt in 1995 to gain effective control over Chicago Northwestern (CNW) Railroad with a partial ownership stake. However, he finds that CNW's stock price reacted positively, rather than negatively, to events that made the merger more likely to be consumeated. This finding is inconsistent with the idea that UP would have diverted profits from CNW to itself by foreclosing competing railroads.

[6] We can easily modify this linearity and assume that the cost of serving only one downstream firm is c_1 and the cost of serving two is $c_2 > c_1$.

A1 $\Pi(k,l)$ *is increasing with* k *at a decreasing rate and decreasing with* l

For example, D_1 and D_2 can be two cable or satellite TV providers, which buy content TV channels in the upstream content market, or online retailers, who sell different brands on their websites. Assumption A1 is then natural in these cases since, other things being equal, a cable TV provider faces a higher demand when it offers more channels while his rival offers less and likewise, an online retailer faces a higher demand when it offers more brands and its rival offers fewer brands.

The sequence of events is as follows. At the outset, all firms are independently owned. Then, either one downstream firm, D_1, acquires a controlling stake in upstream supplier U_1 (backward integration), or U_1 acquires a controlling stake in D_1 (forward integration); we denote the minimal ownership stake that gives the acquirer control over the target by $\underline{\alpha}$.[7] We will say that integration is partial if $\alpha < 1$. Hence, under partial backward integration, D_1 acquires a stake $\underline{\alpha} \leq \alpha < 1$ in U_1, while under partial forward integration U_1 acquires a stake $\underline{\alpha} \leq \alpha < 1$ in D_1.

Given the new ownership structure, each of N upstream suppliers decides whether to supply the input to both downstream firms or to only one. These decisions are publicly observable and irreversible.[8] The upstream suppliers then make take-it-or-leave-it offers to the downstream firms. Inputs are then procured, and the final product is produced and payoff are realized.

3 Incentives for input foreclosure

In this section, we examine the incentive of U_1 to vertically foreclose D_2 under four scenarios: (i) U_1 is independent of D_1, (ii) U_1 and D_1 are fully integrated (full integration), (iii) D_1 has a partial ownership stake in U_1 (partial backward integration), and (iv) U_1 has a partial ownership stake in D_1 (partial forward integration).

[7] Typically the assumption in the literature is that $\underline{\alpha} = 50\%$. In reality, though, $\underline{\alpha}$ can be well below 50%. For example, News Corp. acquired a de facto control in Hughes Electronics Corporation in 2004 by acquiring a 34% stake; see FCC, MB Docket No. 03-124, Memorandum Opinion and Order, January 14, 2004. In the UK, the Competition Commission concluded that BSkyB's acquisition of a 17.9% stake in ITV in 2006, gave BSkyB "the ability materially to influence the policy of ITV which gives rise to common control" (see Paragraph 2 in Competition Commission, 2007, "Acquisition by British Sky Broadcasting Group plc of 17.9 per cent of the Shares in ITV plc").We will therefore not impose restrictions on $\underline{\alpha}$.

[8] This assumption can be justified as in Church and Gandal (2000) and Choi and Yi (2000), where each upstream firm needs to adapt the input to the special needs of each downstream firm. The assumption allows us to sidestep the "commitment problem," which arise for example in Ordover, Salop, and Saloner (1990).

Before we turn to the different scenarios, we first consider the second stage of the game, in which the N upstream suppliers make simultaneous take-it-or-leave-it offers to the two downstream firms. To fix ideas, suppose that downstream firm D_i already buys $k-1$ inputs and downstream firm D_j buys l inputs. The marginal willingness of D_i to pay for the k'th input is

$$\Delta_1(k,l) \equiv \Pi(k,l) - \Pi(k-1,l).$$

This expression represents the incremental profit of D_i from adding the k'th input, given that the rival, D_j, uses l inputs. Assumption A1 implies that $\Delta_1(k,l)$ is positive but decreasing with k. For later use, let us denote the externality that an increase in the number of inputs used by D_j imposes on D_i's profit by

$$\Delta_2(k,l) \equiv \Pi(k,l) - \Pi(k,l-1).$$

By Assumption A1, $\Delta_2(k,l) < 0$ for all k and l.

To ensure that selling N inputs is profitable, we will make the following assumption:

A2 $\Delta_1(N,N) > c$

While Assumption A2 ensures that selling inputs is profitable if both downstream firms buy all N inputs, it is possible that an upstream supplier may prefer to sell its input to only one of the two downstream firms. To see why, note that when D_j increases the number of inputs it uses from $l-1$ to l, the marginal willingness of D_i to pay for input l changes by

$$\Delta_{12}(k,l) \equiv \Delta_1(k,l) - \Delta_1(k,l-1).$$

Although in general $\Delta_{12}(N,N)$ could be either positive or negative, the example that we present in the Appendix shows that it is reasonable to assume that $\Delta_{12}(N,N) < 0$. That is, the marginal willingness of D_i to pay for inputs decreases when D_j is using an extra input. For the sake of concreteness, we will assume that this is indeed the case:

A3 $\Delta_{12}(k,l) \leq 0$ for all k,l

Given that selling an extra input to D_j depresses the price that D_i is willing to pay, at least in principle, it could be that an upstream firm may be unwilling to supply both downstream firms. The following assumption rules out this possibility and ensures that under non-integration, both downstream firms buy all N inputs:

A4 $\Delta_1(k,l) - c > -\Delta_{12}(k,l)$ for all k, l

Assumption A4 implies that the maximal profit that an upstream supplier can make by selling an extra input to D_i, $\Delta_1(k,l) - c$, exceeds $-\Delta_{12}(k,l)$ which is the associated loss of profit from selling to D_j. With Assumption A4 in place, we prove the following result, which establishes the equilibrium behavior of non-integrated upstream suppliers:

Lemma 1: *In equilibrium, non-integrated upstream suppliers sell to both D_1 and D_2, irrespective of whether D_1 and U_1 are partially or fully integrated and irrespective of whether U_1 forecloses D_2 or not. If D_1 and U_1 are integrated and U_1 forecloses D_2, upstream suppliers $2, \ldots, N$ charge D_1 a price $\Delta_1(N,N)$ for the input and charge D_2 a price of $\Delta_1(N-1,N)$. If D_2 is not foreclosed, all upstream suppliers charge D_2 a price of $\Delta_1(N,N)$ and all non-integrated upstream suppliers charge D_1 a price of $\Delta_1(N,N)$.*

Proof: By Assumption A2, in equilibrium each supplier sells to at least one downstream firm. Now suppose by way of negation that there exists an equilibrium in which k_1 suppliers sell exclusively to D_1, k_2 suppliers sell exclusively to D_2, and $N - k_1 - k_2 \geq 0$ suppliers sell to both downstream firms. In this equilibrium, D_1 buys $N - k_2$ inputs and D_2 buys $N - k_1$ inputs. Hence, the marginal willingness of D_1 to pay for inputs is $\Delta_1(N-k_2, N-k_1)$, while the marginal willingness of D_2 to pay for inputs is $\Delta_1(N-k_1, N-k_2)$. Since the upstream suppliers make take-it-or-leave-it offers to the two downstream firms, in equilibrium, each downstream firm pays a price equal to its marginal willingness to pay. Consequently, the profit of each supplier that sells exclusively to D_1 is

$$\Delta_1(N-k_2, N-k_1) - c.$$

If the supplier also sells to D_2, its profit becomes:

$$\underbrace{\Delta_1(N-k_2, N-k_1+1)}_{\text{The price that D}_1\text{ pays}} + \underbrace{\Delta_1(N-k_1+1, N-k_2)}_{\text{The price that D}_2\text{ pays}} - 2c.$$

Selling to both D_1 and D_2 is more profitable since

$$\begin{aligned}
&[\Delta_1(N-k_2, N-k_1+1) + \Delta_1(N-k_1+1, N-k_2) - 2c] - [\Delta_1(N-k_2, N-k_1) - c] \\
&= \Delta_1(N-k_1+1, N-k_2) - c - \underbrace{[\Delta_1(N-k_2, N-k_1) - \Delta_1(N-k_2, N-k_1+1)]}_{-\Delta_{12}(N-k_2, N-k_1+1)} > 0,
\end{aligned}$$

where the inequality follows from Assumption A4. A similar argument applies when suppliers sell exclusively to D_2. Hence, in equilibrium, suppliers $2, \ldots, N$ sell to both D_1 and D_2.

The last part of the lemma follows because D_1 and D_2 pay input prices that reflect their marginal willingness to pay. ■

Given Lemma 1, we only need to consider the equilibrium behavior of U_1. Clearly, if U_1 is not integrated with D_1 (fully or partially), then its equilibrium behavior is no different than that of other upstream suppliers. Hence, we can report the following Corollary to Lemma 1.

Corollary 1: *Under non-integration, both D_1 and D_2 buy all N inputs at a price of $\Delta_1(N,N)$. The resulting profit of each downstream firm is*

$$V_0^D = \Pi(N,N) - N\Delta_1(N,N), \tag{1}$$

while the profit of each upstream supplier is

$$V_0^U = 2\left[\Delta_1(N,N) - c\right]. \tag{2}$$

Note that the equilibrium profit of each downstream firm is positive since $\Pi(N,N) = \sum_{k=1}^{N} \Delta_1(k,N)$, so $\Pi(N,N) - N\Delta_1(N,N) = \sum_{k=1}^{N} \left[\Delta_1(k,N) - \Delta_1(N,N)\right] > 0$, where the inequality follows because $\Delta_{11}(\cdot,\cdot) < 0$ implies that $\Delta_1(k,N) > \Delta_1(N,N)$ for all $k < N$. The equilibrium profit of upstream suppliers is profitable by Assumption A2.

We next characterize the equilibrium under no integration, full integration, partial backward integration, and partial forward integration. As we shall see, in this cases, foreclosure might arise in equilibrium.

3.1 Full vertical integration

Under full vertical integration, D_1 and U_1 fully merge to create a new firm, which we call DU_1. Given Lemma 1, we only need to check whether the integrated firm, DU_1, is interested in selling the input to D_2 and if so, at which price.

To this end, let w denote the price that D_1 pays U_1 for the input. Since D_1 and U_1 are fully merged, w is merely a transfer payment within the same organization, and hence it is irrelevant. However, under partial integration, w matters. In particular, when D_1 partially controls U_1, it would like to set w as low as possible, in which case, D_1 essentially expropriates the wealth of U_1's

non-controlling shareholders. In the opposite case where U_1 partially controls D_1, U_1 would like to set w as high as possible, in order to expropriate the wealth of D_1's non-controlling shareholders. This means that in principle, there are two channels through which partial ownership matters: (i) it can lead to the foreclosure of D_2, and (ii) it can lead to a distortion of w and hence to a transfer of wealth from U_1 to D_1 or vice versa. The second channel however can arise even if D_1 were a monopoly in the downstream market and hence is not directly related to the interaction between vertical integration and competition which is our main focus. We will therefore "shut down" this channel by assuming that U_1 cannot discriminate in favor of or against D_1 and hence w must be equal to the price that all other upstream suppliers charge for their inputs.[9] Using Lemma 1, this price is $w = \Delta_1(N, N-1)$ if D_2 is foreclosed, and $w = \Delta_1(N, N)$ if D_2 is not foreclosed.

If DU_1 sells to D_2, then both downstream firms buy all N inputs, so the marginal willingness of each of them to pay for inputs (and hence the price of each input) is $\Delta_1(N, N)$. Hence, the resulting profit of DU_1 is $V_0^D + V_0^U$. If DU_1 forecloses D_2, then the willingness of D_1 to pay for inputs increases to $\Delta_1(N, N-1)$, so the downstream profit of DU_1 becomes

$$V_1^D = \Pi(N, N-1) - N\Delta_1(N, N-1), \tag{3}$$

while the upstream profit of DU_1 becomes

$$V_1^U = \Delta_1(N, N-1) - c. \tag{4}$$

Overall, the profit of DU_1 when D_2 is foreclosed is $V_1^D + V_1^U$.

Foreclosing D_2 is therefore optimal if and only if $V_1^D + V_1^U \geq V_0^D + V_0^U$. Using equations (1), (2), (3), and (4), this is true whenever

$$L < G, \tag{5}$$

where

$$L \equiv V_0^U - V_1^U = \Delta_1(N, N) - c + \Delta_{12}(N, N),$$

and

$$G \equiv V_1^D - V_0^D = -\Delta_2(N, N) + N\Delta_{12}(N, N).$$

To interpret (5), note that L represents the loss of upstream profit due to foreclosure: $\Delta_1(N, N) - c$ represents the forgone upstream profit due to the foreclosure of D_2, and $\Delta_{12}(N, N) \equiv \Delta_1(N, N) -$

[9] For instance, discriminaton against D_1 could be deemed as an outright violation of the fiduciary duties that D_1's management has towards D_1's non-controlling shareholders. Likewise, discriminaton in favor of D_1 would be deemed as an outright violation of the fiduciary duties that U_1's management has towards U_1's non-controlling shareholders.

$\Delta_1(N, N-1) < 0$ is the increase in the price that the upstream unit of DU_1 charges the downstream unit of DU_1 when D_2 is foreclosed (in which case the willingness of DU_1's downstream unit to pay for inputs increases from $\Delta_1(N, N)$ to $\Delta_1(N, N-1)$).[10] This price increase cuts the upstream loss from foreclosure, though by Assumption A4, overall, $L > 0$.

The effect of foreclosure on the downstream profit of DU_1 is represented by G. The first term, $-\Delta_2(N, N) \equiv -(\Pi(N, N) - \Pi(N, N-1))$, is the extra downstream profit that DU_1 makes due to the foreclosure of D_2. The second term reflects the idea that once D_2 is foreclosed, the willingness of D_1 to pay for inputs increases from $\Delta_1(N, N)$ to $\Delta_1(N, N-1)$; since D_1 buys N inputs and since we assume that upstream suppliers have all the bargaining power, the total increase in DU_1's payment for inputs is $N\Delta_{12}(N, N)$. As far as know, this adverse effect of foreclosure on input prices has not been identified earlier in the literature. Of course, this effect is extreme in our model due to our assumption that upstream suppliers have all the bargaining power when they negotiate with downstream firms. This effect will be less extreme if downstream firms were to have some bargaining power too and it would disappear altogether if the downstream firm were to make the upstream suppliers take-it-or-leave-it offers.

We can now summarize the discussion as follows:

Lemma 2: *Suppose that D_1 and U_1 fully integrated. Then there exists a unique equilibrium in which the integrated firm DU_1 forecloses D_2 if and only if $G \geq L$.*

Since $L > 0$ by Assumption A4 (foreclosure entails a loss of upstream profits), Lemma 2 implies that a necessary (though not sufficient) condition for foreclosure under full integration is that $G > 0$. That is, foreclosure must boost the downstream profit of DU_1. When $G < 0$, foreclosure never arises in equilibrium. To make the analysis interesting, we will therefore impose the following assumption:

A5 $G \equiv -\Delta_2(N, N) + N\Delta_{12}(N, N) > 0$

3.2 Partial backward integration

Now suppose that D_1 acquires a fraction $\alpha \in [\underline{\alpha}, 1)$ of U_1's shares, where $\underline{\alpha}$ is the minimal equity stake that gives D_1 full control over U_1. The remaining $1 - \alpha$ stake in U_1 is held by dispersed,

[10] As mentioned earlier, although payments within DU_1 are mere transfers and hence wash out, it is instructive to write them explicitly since these payments will play an important role when we consider partial vertical integration.

passive, shareholders.

To characterize the equilibrium, we will follow the same steps as in the full integration case. The only difference is that now, U_1's decisions are taken by D_1 with the objective of maximizing D_1's profit, plus α times U_1's profit, instead of maximizing the joint profit of D_1 and U_1 (the difference then is that now $\alpha < 1$, while under full integration, effectively $\alpha = 1$).

Given Lemma 1, we only need to consider D_1's decision on whether to use its control over U_1 to foreclose D_2. If D_2 is not foreclosed then by corollary 1, the profits of D_1 and U_1 are V_0^D and V_0^U, so D_1's overall profit, including its stake in U_1's profit, is $V_0^D + \alpha V_0^U$. By contrast, if D_1 uses its control over U_1 to foreclose D_2, then, the profits of D_1 and U_1 becomes V_1^D and V_1^U, so D_1's overall profit becomes $V_1^D + \alpha V_1^U$.

Clearly, U_1 will foreclose D_2 if and only if $V_1^D + \alpha V_1^U \geq V_0^D + \alpha V_0^U$, which implies in turn that

$$\alpha \underbrace{\left(V_0^U - V_1^U\right)}_{L} < \underbrace{V_1^D - V_0^D}_{G}. \tag{6}$$

Since $L > 0$ by Assumption A4, (6) is more likely to hold when α is small. This implies in turn that D_1 would like to use its control over U_1 to foreclose D_2 only when α is sufficiently small. Intuitively, under partial backward integration, D_1 bears only a fraction α of the loss L to U_1 from foreclosing D_2, but it captures the entire downstream gain, G. Hence, D_1 has a stronger incentive to use its control to foreclose D_2, relative to the full information case. Moreover, this incentive becomes stronger as α decreases.

To reduce the number of cases we need consider, we now impose the following assumption:

A6 $G > \underline{\alpha} L$

Assumption A6 implies that there exists a range of ownership stakes, $\left[\underline{\alpha}, \min\left\{\frac{G}{L}, 1\right\}\right]$, that give D_1 control over U_1 and induce D_1 to use its control over U_1 to foreclose D_2. Without this assumption, D_1 will never find it optimal, when it controls U_1, to foreclose D_2. We can now summarize the discussion in the following lemma:

Lemma 3: *Suppose that D_1 owns a controlling stake $\alpha \geq \underline{\alpha}$ in U_1. Then there exists a unique equilibrium in which D_1 uses its control over U_1 to foreclose D_2 if and only if $\alpha \in \left[\underline{\alpha}, \min\left\{\frac{G}{L}, 1\right\}\right]$.*

3.3 Partial forward integration

Next, we consider the case where U_1 acquires a fraction $\alpha \in [\underline{\alpha}, 1)$ of D_1's shares. As before, $\underline{\alpha}$ is the minimal equity stake that gives the acquirer, U_1 in this case, full control over the target's operating decisions. The remaining $1-\alpha$ stake in D_1 is held by dispersed, passive, shareholders.

To characterize the equilibrium, note from Corollary 1 that under non-foreclosure, the profits of D_1 and U_1 are V_0^D and V_0^U, so U_1's overall profit, including its stake in D_1, is $\alpha V_0^D + V_0^U$. If U_1 forecloses D_2, the profits of D_1 and U_1 become V_1^D and V_1^U, so U_1's overall profit becomes $\alpha V_1^D + V_1^U$. Clearly, U_1 will foreclose D_2 if and only if $\alpha V_1^D + V_1^U \geq \alpha V_0^D + V_0^U$, which holds whenever

$$\underbrace{\left(V_0^U - V_1^U\right)}_{L} < \alpha \underbrace{\left(V_1^D - V_0^D\right)}_{G}. \tag{7}$$

Equation (7) shows that U_1 would like to foreclose D_2 only when α is sufficiently large. Intuitively, under forward integration, U_1 bears the entire upstream loss from foreclosing D_2, but since it only holds a fraction of D_1's ownership, it captures only a fraction α of the associated downstream gain (the rest of the gain accrues to the passive shareholders of D_1). Clearly then, U_1 has a stronger incentive to foreclose D_2 when α increases.

The next lemma summarizes the discussion:

Lemma 4: *Suppose that U_1 owns a controlling stake $\alpha \geq \underline{\alpha}$ in D_1. Then there exists a unique equilibrium in which U_1 forecloses D_2 if and only if $\alpha G \geq L$. Clearly, if $L > G$, there is no foreclosure in equilibrium.*

3.4 Comparison

Having characterized the equilibrium under the full integration, partial backward integration, and partial forward integration, the following proposition follows immediately from Lemmas 2-4:

Proposition 1: *Partial backward integration expands the range of parameters for which D_2 is foreclosed relative to full integration, while partial forward integration shrinks it. Moreover, if foreclosure is profitable under full integration, i.e., $G \geq L$, then partial backward integration always leads to foreclosure, while partial forward integration leads to foreclosure only when α is sufficiently close to 1. And, if foreclosure is not profitable under full integration, i.e., $G < L$, then partial backward integration leads to foreclosure only when α is sufficiently small, while partial forward integration never leads to foreclosure.*

Proposition 1 is reminiscent of Baumol and Ordover (1994). It shows that partial backward integration raises the concern for input foreclosure, while partial forward integration alleviates this concern.

We conclude this section by examining how our results change when we relax the assumption that the upstream suppliers can make the two downstream firms take-it-or-leave-it offers. To this end, suppose that if D_i buys k inputs and D_j gets l inputs, D_i pays each upstream supplier a price of $\mu\Delta_1(k,l)$ for the input, where $\mu \in \left[\frac{c}{\Delta_1(N,N)}, 1\right]$ measures the bargaining power of upstream suppliers. We assume that $\mu \geq \frac{c}{\Delta_1(N,N)}$ to ensure that the marginal willingness of D_i to pay for inputs exceeds their cost. With these assumptions in place, the post-acquisition values of D_1 and U_1 are

$$V_1^D = \Pi(N, N-1) - N\mu\Delta_1(N, N-1), \qquad V_1^U = \mu\Delta_1(N, N-1) - c,$$

while their pre-acquisition values are

$$V_0^D = \Pi(N, N) - N\mu\Delta_1(N, N), \qquad V_0^U = 2\left[\mu\Delta_1(N, N) - c\right].$$

As a result, the upstream loss from foreclosure becomes

$$L_\mu \equiv V_0^U - V_1^U = \mu\Delta_1(N, N) - c + \mu\Delta_{12}(N, N),$$

and the downstream gain from foreclosure becomes

$$G_\mu \equiv V_1^D - V_0^D = -\Delta_2(N, N) + N\mu\Delta_{12}(N, N).$$

By Assumptions A3 and A4, L_μ is increasing, while G is decreasing with μ. Hence,

Proposition 2: *An increase in the bargaining power of upstream suppliers vis-a-vis downstream firms shrinks the range of parameters for which D_2 is foreclosed.*

The intuition for Proposition 2 is simple: an increase in the bargaining power of upstream suppliers vis-a-vis downstream firms boosts the upstream profits and depresses the downstream profits. Since input foreclosure shifts profits from the upstream firm to the downstream firm, an increase in the bargaining power of upstream suppliers makes foreclosure less attractive.

4 Input foreclosure under endogenous ownership structure

So far we considered the incentive of a vertically integrated firm to foreclose a downstream rival. We now examine the incentive to vertically integrate in the first place. To this end, we assume

that initially D_1 and U_1 are not integrated, and then we ask whether D_1 would like to acquire a controlling stake, $\alpha \geq \underline{\alpha}$, in U (backward integration), or U_1 would like to acquire a controlling stake, $\alpha \geq \underline{\alpha}$, in D_1 (forward integration), and examine how this incentive depends on the initial ownership structure of the target firm.

Specifically, we consider two cases:

(i) Initially, the target (U_1 in the case of partial forward integration and D_1 in the case of partial backward integration) has a single controlling shareholder whose stake is $\alpha_C \in [\underline{\alpha}, 1]$; the remaining $1 - \alpha_C$ stake in U_1 (if any) is held by passive shareholders.

(ii) Initially, the target is owned by a mass 1 of atomistic, dispersed, shareholders.

4.1 Backward integration: U_1 has initially a single controlling shareholder

In this section we examine the incentive of D_1 to acquire a controlling stake $\alpha \in [\underline{\alpha}, \alpha_C]$ in U_1 from U_1's initial controller (whose initial stake is α_C). To acquire α, D_1 makes U_1's controller a take-it-or-leave-it offer b, although as we shall see below, the assumption that D_1 has all the bargaining power vis-a-vis U_1's controller is not essential. If D_1's offer is accepted, D_1 becomes the new controlling shareholder in U_1.[11] We begin with the following lemma.

Lemma 5: *Acquisition of a partial controlling stake in U_1, which is not followed by the foreclosure of D_2, is profitable for D_1 if and only if the post-acquisition price w that D_1 pays U_1 for the input is below $\Delta_1(N, N)$, which is the price that all other upstream suppliers charge for the input.*

Proof: We begin by considering the price that D_1 needs to pay in order to acquire the controlling stake, α, in U_1. If D_2 is not foreclosed following the acquisition, then the post-acquisition value of U_1 becomes

$$\widetilde{V}_1^U = \Delta_1(N, N) + w - 2c, \tag{8}$$

where $w - c$ is U_1's profit from selling to D_1 and $\Delta_1(N, N) - c$ is the profit from selling to D_2. By Corollary 1, the pre-acquisition value of U_1, V_0^U, is given by (2).

[11] Since the value of U_1's shares is the same to all shareholders other than D_1, it is immaterial for the analysis whether the initial controller of U_1 retains a minority stake $\alpha_C - \alpha$, or sells this minority stake (fully or partially) to disperssed shareholder.

Using these expressions, the minimal acceptable bid, b^U, must leave U_1's controller indifferent between accepting and rejecting the offer:

$$\underbrace{b^U + (\alpha_C - \alpha)\,\widetilde{V}_1^U}_{\text{Accepting the offer}} = \underbrace{\alpha_C V_0^U}_{\text{Rejecting the offer}} .$$

Hence,

$$\begin{aligned} b^U &= \alpha_C \left[V_0^U - \widetilde{V}_1^U\right] + \alpha \widetilde{V}_1^U \\ &= \alpha_C \left[\Delta_1 (N,N) - w\right] + \alpha \widetilde{V}_1^U, \end{aligned} \tag{9}$$

where the second equality follows from (2) and (8). Consequently, if D_1 acquires a controlling stake, α, in U_1, its payoff becomes:

$$\begin{aligned} & \left[\Pi (N,N) - (N-1)\,\Delta_1 (N,N) - w\right] + \alpha \widetilde{V}_1^U - \underbrace{\left(\alpha_C \left[V_0^U - \widetilde{V}_1^U\right] + \alpha \widetilde{V}_1^U\right)}_{b^U} \\ = \; & \Pi (N,N) - (N-1)\,\Delta_1 (N,N) - w - \alpha_C \left[\Delta_1 (N,N) - w\right], \end{aligned} \tag{10}$$

where $\Pi (N,N)$ is the downstream profit of D_1 and $(N-1)\,\Delta_1 (N,N)$ is the total payment of D_1 to non-integrated upstream suppliers. If D_1 does not acquire a controlling stake in U_1, then by Corollary 1, its payoff, V_0^D, is given by (1).

The difference between D_1's payoff with and without the acquisition is given by the difference between (10) and (1):

$$\Delta_1 (N,N) - w - \alpha_C \left[\Delta_1 (N,N) - w\right] = (1 - \alpha_C) \left[\Delta_1 (N,N) - w\right].$$

The last expression is positive if and only if $w < \Delta_1 (N,N)$. ■

The idea behind Lemma 5 is simple: so long as $w < \Delta_1 (N,N)$, D_1 uses its control over U_1 to force U_1 to sell it the input at an artificially low price. This is profitable for D_1 because it expropriates some of the wealth of U_1's passive shareholders. As mentioned in Section 3.1, this motive for partial vertical integration exists even if there is no competition with rival firms either in the downstream or the upstream markets; we will therefore abstract from this consideration by assuming that under integration (full or partial), D_1 pays U_1 the same input price that it pays other upstream suppliers. That is, $w = \Delta_1 (N, N-1)$ if D_2 is foreclosed, and $w = \Delta_1 (N,N)$ if D_2 is not foreclosed.

Now, consider the case where D_2 is foreclosed after D_1 acquires control over U_1. Then, the post-acquisition value of U_1 is V_1^U, so analogously to (9), the minimal acceptable offer is given by,

$$\begin{aligned} b^U &= \alpha_C V_0^U - (\alpha_C - \alpha) V_1^U \\ &= \alpha_C \underbrace{\left[V_0^U - V_1^U\right]}_{L} + \alpha V_1^U. \end{aligned} \tag{11}$$

That is, b^U is equal to the post-acquisition value of the acquired shares, αV_1^U, plus a premium, $\alpha_C L$, which compensates the initial controller of U_1 for the loss in the value of his initial stake due to the foreclosure of D_2.

But will D_1 agree to pay b^U for a controlling stake in U_1? To answer this question, note that since $w = \Delta_1(N, N-1)$ when D_2 is foreclosed, D_1's payoff when it acquires a controlling stake in U_1 and then forecloses D_2 is:

$$V_1^D + \alpha V_1^U - \underbrace{\left(\alpha_C L + \alpha V_1^U\right)}_{b^U} = V_1^D - \alpha_C L.$$

Notice that this expression depends on α_C, which is the size of the controlling stake that the initial controller of U_1 holds, but not on the actual size of the acquired stake, α. The reason is that D_1 pays U_1's controller a fair price for the shares it acquires, plus a premium that fully compensates U_1's initial controller for the drop in the value of the stake that he retains in U_1. Since D_1 bears the loss of value on the stake it acquires, it fully internalizes the loss to the entire stake α_C due to foreclosure (a fraction $1 - \alpha_C$ of the loss is borne by the passive shareholders of U_1).

D_1 will therefore acquire a controlling stake in U_1 and use it to foreclose D_2 if and only if its post-acquisition payoff exceeds its pre-acquisition payoff, which by Corollary 1 equals V_0^D:

$$V_1^D - \alpha_C L - V_0^D = G - \alpha_C L \geq 0. \tag{12}$$

Since this condition is independent of the acquired stake, α, D_1 is indifferent to size of its controlling stake (provided of course that it is above $\underline{\alpha}$).[12] Since $L > 0$ by Assumption A4, it is clear that (12) is more likely to hold when α_C is small.

Proposition 3: *Suppose that initially, U_1 has a single controlling shareholder. Then, in equilibrium, D_1 will acquire a controlling stake in U_1 and use it to foreclose D_2 if and only if $G \geq \alpha_C L$. In*

[12] To the extent that foreclosure is easier when the initial controller of U_1 is out of the picture, D_1 might as well acquire the entire stake α_C of U_1's initial controller.

particular, the acquisition takes place for all $\alpha_C \in [\underline{\alpha}, 1]$ if $G \geq L$, it never takes place if $G < \underline{\alpha}L$, and whenever $\underline{\alpha}L \leq G < L$, it takes place only when α_C is sufficiently close to $\underline{\alpha}$.

Proposition 3 implies that if U_1 is initially controlled by a single shareholder and if full vertical integration leads foreclosure, i.e., $G \geq L$, then D_1 will surely acquire control in U_1 and use it to foreclose D_2. On the other hand, when full vertical integration does not lead to foreclosure, i.e., $G < L$, then D_1 may still acquire control over U_1 and then foreclose D_2, but only when the initial stake of U_1's controller is sufficiently small. In other words, the proposition suggests that antitrust authorities should be more concerned with backward integration when the controlling stake is acquired from an initial controller whose controlling stake is relatively small.

Finally, it should be emphasized that the above results would continue to hold even if D_1 does not have all the bargaining power vis-a-vis U_1's initial controller. To see why, note that the joint payoff of D_1 and U_1's initial controller if the acquisition goes through is $V_1^D + \alpha_C V_1^U$, whereas their joint payoff without an acquisition is $V_0^D + \alpha_C V_0^U$. Hence, their joint surplus from partial backward integration is

$$S^{BI} = \underbrace{V_1^D - V_0^D}_{G} - \alpha_C \underbrace{\left[V_0^U - V_1^U\right]}_{L}.$$

Hence, transferring control over U_1 to D_1 is jointly profitable if and only if (12) holds (otherwise, either D_1 is not interested in acquiring control over U_1, or U_1's initial controller is not interested in selling it).[13]

4.2 Backward integration: U_1's ownership is initially dispersed

We now turn to the case where U_1 is initially held by dispersed shareholders. Specifically, we will follow Grossman and Hart (1980) and assume that U_1 is initially held by a continuum of atomistic shareholders, whose total mass is 1. In order to acquire a controlling stake $\underline{\alpha}$ in U_1, D_1 makes a tender offer to U_1's initial shareholders at a price that reflects a value V for the entire firm. Below we solve for the equilibrium value of V and also determine whether D_1 would wish to make the offer restricted and specify a limit on the stake it is willing to acquires (if the tendered stake exceeds the limit, the submitted shares of each tendering shareholder are prorated). We will say that the

[13] The relative bargaining power of D_1 vis-a-vis U_1's initial controller would matter however if D_1 has some fixed cost associated with initiating a takeover. In that case, the lower D_1's bargaining power, the less likely the takeover is.

tender offer succeeds if D_1 manages to acquire at least a stake of $\underline{\alpha}$ (and gains control over U_1), and we will say that the tender offer fails if D_1 does not acquire a stake of at least $\underline{\alpha}$.

To characterize the equilibrium, recall that the pre-acquisition value of U_1 is V_0^U, and its post-acquisition value is V_1^U, and note that if $V_1^U < V < V_0^U$, then it is optimal for each shareholder to tender his shares if the tender offer succeeds (and then get $V > V_1^U$ for the tendered shares), but not if it fails (in which case the shareholder gets $V < V_0^U$ for the sold shares).[14] Hence, there exist multiple equilibria in this case. For example, it is an equilibrium for all shareholder to tender their shares (the offer succeeds even if a single shareholder deviates and does not tender his shares), and it is also an equilibrium for all shareholders not to tender their shares (the offer fails even if a single shareholder deviates and does tender his shares). However, since $V_0^U \geq V_1^U$, equilibria in which the tender offer fails Pareto dominate equilibria in which the tender offer succeeds. We will therefore assume that whenever $V_1^U < V < V_0^U$, a non-tendering equilibrium is played. With this assumption in place, we now prove the following lemma.

Lemma 6: *Suppose that if $V_1^U < V < V_0^U$, then U_1's initial shareholders do not tender their shares. Then in equilibrium, $V = V_0^U$.*

Proof: First, notice that if $V \leq V_1^U$ (the price per share is below the post-acquisition value), then it is a dominant strategy for each shareholder not to tender. And, given the assumption in the lemma, shareholders also do not tender if $V_1^U < V < V_0^U$. Hence, the tender offer fails for sure if $V < V_0^U$. By contrast, if $V \geq V_0^U$, then it is a weakly dominant strategy for each shareholder to fully tender his shares: if the tender offer succeeds, the shareholder gets V_0^U on the sold shares, but gets only $V_1^U < V_0^U$ on retained shares; if the tender offer fails, the value of the shares is V_0^U regardless of whether they are tendered. Since the tender offer surely succeeds, it is optimal for D_1 to set $V = V_0^U$, which is the lowest offer that ensures that the tender offer succeeds. ■

Lemma 6 implies that D_1 has no incentive to acquire control over U_1 if D_2 is not foreclosed following the acquisition. This is because absent foreclosure, D_1's profit remains V_0^D and since D_1 breaks even on the acquisition (the acquisition price absent foreclosure is αV_0^U, which is also the post-acquisition value of D_1's stake in U_1). The lemma also implies that if D_2 is foreclosed, D_1 wishes to buy the minimal stake, $\underline{\alpha}$, which ensures control, since the price of the acquired shares exceeds their post-acquisition value.

[14] If the offer is conditional on success, the shareholder is indifferent about submitting shares when the offer fails.

The remaining question now is whether the acquisition is profitable for D_1 if it does lead to the foreclosure of D_2.[15] We address this question in the next proposition:

Proposition 4: *Suppose that initially, U_1's ownership is dispersed. Then, in equilibrium, D_1 will acquire a controlling stake in U_1 and will use it to foreclose D_2 if and only if $G \geq \underline{\alpha}L$. If this condition holds, D_1 will acquire the minimal stake that ensures control over U_1, i.e., $\underline{\alpha}$, by making a restricted tender offer.*

Proof: Since $V = V_0^U$, acquiring a controlling stake α in U_1 costs D_1 a total of αV_0^U. Recalling that when D_2 is foreclosed $w = \Delta_1(N, N-1)$, the post-acquisition payoff of D_1 is

$$V_1^D + \alpha V_1^U - \alpha V_0^U = V_1^D - \alpha L.$$

Recalling that the pre-acquisition payoff of D_1, V_0^D, is given by (1), and recalling that $G \equiv V_1^D - V_0^D$, the difference between D_1's post- and pre-acquisition payoff is

$$\left[V_1^D - \alpha L\right] - V_0^D = G - \alpha L.$$

Since $L > 0$ by Assumption A4, it is clear that D_1 would never acquire more than the minimal stake that ensures control, namely, $\underline{\alpha}$. Acquiring a stake $\underline{\alpha}$ in U_1 is profitable, however, if and only if $G \geq \underline{\alpha}L$. To ensure that the acquired stake does not exceed $\underline{\alpha}$, D_1 will make his offer restricted and specify that if the tendered stake exceeds $\underline{\alpha}$, the submitted shares of each tendering shareholder will be prorated such that D_1 acquires exactly a stake of $\underline{\alpha}$. ■

Since $\alpha_C \geq \underline{\alpha}$, Propositions 3 and 4 imply that there is a wider range of parameters for which D_1 acquires a controlling stake in U_1 and uses it to foreclose D_2 when U_1 is initially owned by dispersed shareholders, than when U_1 has initially a single controlling shareholder. Intuitively, the acquisition of U_1 by D_1 lowers U_1's value since D_1 uses U_1 to foreclose D_2 and hence it effectively diverts profits from U_1 to D_1. Although in both cases D_1 needs to pay the sellers a price that reflects the pre-acquisition value of U_1, in the case of an initial controller, D_1 must also compensate the controller for his remaining stake in U_1, $\alpha_C - \alpha$. By contrast, in the case of dispersed shareholders,

[15] One may wonder whether a large shareholder may wish to acquire a sufficiently large stake from the dispersed shareholders of U_1 and use it to oppose D_1's decision to foreclosure D_2. Such an action would raise the value of U_1 from V_1^U to V_0^U. But since the dispersed shareholders of U_1 are atomistic, the large shareholder would have to pay them the post-acquisition value of their shares (as in Grossman and Hart, 1980) in order to induce them to submit their shares. As a result, the acquisition is not profitable for the large shareholder.

D_1 can acquire a minimal stake, $\underline{\alpha}$, that ensures control and then effectively expropriate the wealth of the remaining passive shareholders of U_1 by foreclosing D_2. So long as $\alpha < 1$, part of the upstream loss from foreclosure is borne by the passive shareholders of U_1, while the entire downstream gain accrues to D_1.

4.3 Forward integration: D_1 has initially a single controlling shareholder

We now consider the case where U_1 integrates forward by acquiring a controlling stake $\alpha \in [\underline{\alpha}, \alpha_C)$ in D_1 from the initial controller of D_1. Again, we will assume that the acquirer, here U_1, makes a take-it-or-leave-it offer b for the stake α, but as in the partial backward integration case, this assumption is not essential. We begin with the following lemma.

Lemma 7: *Acquisition of a partial controlling stake in D_1, which is not followed by a foreclosure of D_2, is profitable for U_1 if and only if the post-acquisition price, w, that D_1 pays U_1 for the input exceeds $\Delta_1(N,N)$, which is the price that all other upstream suppliers charge for the input.*

Proof: We begin by considering the price that U_1 needs to pay D_1's controlling shareholder in order to acquire a stake $\alpha \in [\underline{\alpha}, \alpha_C)$ in D_1. If U_1 does not foreclose D_2, then the post-acquisition value of D_1 is

$$\widetilde{V}_1^D = \Pi_1(N,N) - (N-1)\Delta_1(N,N) - w, \tag{13}$$

where w is the price that D_1 pays U_1 for the input. By Corollary 1, the pre-acquisition value of D_1, V_0^D, is given by (1).

Analogously to Lemma 5, the minimal acceptable bid is

$$\begin{aligned} b^D &= \alpha_C V_0^D - (\alpha_C - \alpha)\widetilde{V}_1^D \\ &= \alpha_C\left[V_0^D - \widetilde{V}_1^D\right] + \alpha\widetilde{V}_1^D \\ &= \alpha_C\left[-\Delta_1(N,N) + w\right] + \alpha\widetilde{V}_1^D, \end{aligned} \tag{14}$$

where the third equality follows from (1) and (13).

Therefore, if U_1 acquires a controlling stake in D_1, its payoff becomes:

$$\begin{aligned} &\left[\Delta_1(N,N) + w - 2c\right] + \alpha\widetilde{V}_1^D - \underbrace{\alpha_C\left[V_0^D - \widetilde{V}_1^D\right] + \alpha\widetilde{V}_1^D}_{b^D} \\ = \;& \Delta_1(N,N) + w - 2c + \alpha_C\left[\Delta_1(N,N) - w\right]. \end{aligned} \tag{15}$$

If D_1 does not acquire a controlling stake in U_1, then by Corollary 1, its payoff is V_0^U, given by (2). The difference between U_1's post- and pre-acquisition payoff is therefore:

$$\Delta_1(N,N) + w - 2c + \alpha_C[\Delta_1(N,N) - w] - 2[\Delta_1(N,N) - c] = -(1-\alpha_C)[\Delta_1(N,N) - w].$$

The expression is positive if and only if $w > \Delta_1(N,N)$. ■

Lemma 7 implies that it pays U_1 to acquire a controlling stake in D_1 without foreclosing D_2 only when it can expropriate the wealth of D_1's passive shareholders by selling the input to D_1 at a price that exceeds the price at which it sells the input to D_2 and above the price that D_1 pays all other upstream suppliers. As before, we shall rule out this possibility; this implies in turn that forward integration is feasible only if it leads to the foreclosure of D_2.

We now consider the possibility that U_1 acquires a controlling stake α in D_1, and then uses it to foreclose D_2. Analogously to (14), the minimal b needed to induce D_1's initial controller to accept U_1's offer is,

$$\begin{aligned} b^D &= \alpha_C V_0^D - (\alpha_C - \alpha) V_1^D \\ &= \alpha_C \left[V_0^D - V_1^D\right] + \alpha V_1^D \\ &= -\alpha_C G + \alpha V_1^D, \end{aligned} \tag{16}$$

where the third equality follows because $G \equiv V_1^D - V_0^D$. Equation (16) shows that b^D is equal to the post-acquisition value of the acquired shares, αV_1^D, minus a discount, $-\alpha_C G$, which is equal to the appreciation in the value of the initial stake of D_1's initial controller due to the foreclosure of D_2.

To determine if offering b^D is profitable for U_1, notice that U_1's payoff if it gains control over D_1 and then forecloses D_2 is:

$$V_1^U + \alpha V_1^D + \underbrace{\alpha_C G - \alpha V_1^D}_{-b^D} = V_1^U + \alpha_C G.$$

If U_1 does not acquire a controlling stake in D_1, its payoff, V_0^U, is given by (2). In equilibrium, U_1 will acquire a controlling stake in D_1 and then foreclose D_2 if and only if its post-acquisition payoff exceeds its pre-acquisition payoff:

$$V_1^U + \alpha_C G - V_0^U = \alpha_C G - L \geq 0. \tag{17}$$

Since $L > 0$ by Assumption A4, this condition is more likely to hold when α_C is large. Since the condition is independent of the acquired stake, α, U_1 is indifferent to its final stake in D_1, provided that it is high enough to ensure control.

Notice that the joint payoff of U_1 and D_1's initial controller if the acquisition goes through is $V_1^U + \alpha_C V_1^D$, whereas their joint payoff without an acquisition is $V_0^U + \alpha_C V_0^D$. Hence, the joint surplus from partial backward integration is,

$$S^{FI} = \alpha_C \underbrace{\left[V_1^D - V_0^D\right]}_{G} - \underbrace{\left[V_0^U - V_1^U\right]}_{L}.$$

Condition (17) then says that U_1 will acquire a controlling stake in D_1 if and only if the acquisition is jointly profitable for U_1 and D_1's controller. This implies in turn that the relative bargaining powers of the two parties only determine how the joint surplus is divided between them, but not whether the acquisitions will take place.

Proposition 5: *Suppose that initially, D_1 has a single controlling shareholder. Then, in equilibrium, U_1 will acquire a controlling stake in D_1 and will foreclose D_2 if and only if $\alpha_C G \geq L$. In particular, the acquisition takes place for all $\alpha_C \in [\underline{\alpha}, 1]$ if $\underline{\alpha} G \geq L$, never takes place if $G < L$, and if $\underline{\alpha} G \leq L < G$, the acquisition takes place only when α_C is sufficiently close to* 1.

Proposition 5 implies that if full vertical integration does not lead to foreclosure, i.e., $G < L$, then it never pays U_1 to integrate forward when D_1 is initially controlled by a single shareholder. On the other hand, when full vertical integration does lead to foreclosure, i.e., $G \geq L$, then U_1 may wish to integrate forward and then foreclose D_2, provided that the ownership stake of D_1's initial controller is sufficiently large.

4.4 Forward integration: D_1's ownership is initially dispersed

When D_1's shareholders are atomistic, no shareholder is pivotal; since the acquisition boosts the value of D_1, it is a dominant strategy for each shareholder to hold on to his share if V is below the post-acquisition value of D_1.[16] Hence, in equilibrium, U_1 must set V equal to the post-acquisition value of D_1, implying that it breaks even on the share it acquires.[17] Consequently, the acquisition

[16] The situation is then different from the one considered in Section 4.2 where the acuisition lowered the target's value. In that case, whether it is optimal to submit shares or not depends on whether other shareholders submit their shares.

[17] This result is just the Grossman and Hart (1980) free rider problem: when a takeover is value-increasing, the acquirer must offer a price that reflects the post-acquisition value of the target, otherwise it is a dominant strategy for

affects U_1 only through its direct effect on U_1. But since U_1's value either decreases by L if D_2 is foreclosed, or does not change if D_2 is not foreclosed, U_1 has nothing to gain by acquiring control over D_1. Hence,

Proposition 6: *Suppose that initially, D_1's ownership is dispersed. Then forward integration does not take place in equilibrium.*

5 Extensions

5.1 Toeholds

We now examine what happens when, at the outset, D_1 already holds a non-controlling stake, $\alpha_1 < \underline{\alpha}$, in U_1 (i.e., a toehold). To gain control over U_1, D_1 must acquire an additional stake $\alpha - \alpha_1$ in U_1, such that after the acquisition, its controlling stake in U_1 is $\alpha \geq \underline{\alpha}$.

Proposition 7: *Suppose that initially, D_1 holds a non-controlling stake (toehold), α_1, in U_1. Then, the toehold has no effect on the equilibrium if U_1 is initially held by dispersed shareholders. When U_1 has initially a single controlling shareholder, then, in equilibrium, D_1 will acquire a controlling stake in U_1 and will use it to foreclose D_2 if and only if $G \geq (\alpha_C + \alpha_1) L$; in particular, D_1's toehold shrinks the range of parameters for which D_2 is foreclosed.*

Proof: First, suppose that U_1 is initially held by a continuum of atomistic shareholders. By Lemma 6, D_1 will offer these shareholders a price that reflects a value of V_0^U for the entire firm and will therefore pay a total of $(\alpha - \alpha_1) V_0^U$ for the acquired stake. If D_2 is not foreclosed after the acquisition, then the post-acquisition values of D_1 and U_1 are equal to their pre-acquisition values, V_0^D and V_0^U. Hence, D_1's post-acquisition payoff is

$$V_0^D + \alpha V_0^U - (\alpha - \alpha_1) V_0^U = V_0^D + \alpha_1 V_0^U.$$

Since this is also D_1's payoff without acquisition, D_1 has no incentive to acquire control over U_1.

each shareholder to hold on to his shares. Bagnoli and Lipman (1988) and Holmstrom and Nalebuff (1992) show that the free-rider problem depends crucially on the assumptions that there is a continuum of shareholders rather than finitely many shareholders and on the fact that the acquirer has no private information about the post-acquisition value of the firm.

If the acquisition is followed by the foreclosure of D_2, then U_1's profit, V_1^U, is given by (4), while D_1's profit, V_1^D, is given by (3). Now, D_1's post-acquisition payoff is given by

$$V_1^D + \alpha V_1^U - (\alpha - \alpha_1) V_0^U = V_1^D + \alpha_1 V_0^U - \alpha \underbrace{\left[V_0^U - V_1^U\right]}_{L}.$$

Since $L < 0$, D_1 would only acquire the minimal amount of shares which ensures control, i.e., $\underline{\alpha} - \alpha_1$, such that its post-acquisition stake in U_1 will be $\underline{\alpha}$. Noting that D_1's pre-acquisition payoff is $V_0^D + \alpha_1 V_0^U$, the acquisition of $\underline{\alpha} - \alpha_1$ shares in U_1 is profitable if and only if

$$V_1^D + \alpha_1 V_0^U - \underline{\alpha}\underbrace{\left[V_0^U - V_1^U\right]}_{L} - \left[V_0^D + \alpha_1 V_0^U\right] = \underbrace{\left[V_1^D - V_0^D\right]}_{G} - \underline{\alpha}\underbrace{\left[V_0^U - V_1^U\right]}_{L} \geq 0.$$

This condition, however, is identical to that in Proposition 4, so the existence of a toehold does not affect the equilibrium.

Next, consider the case where U_1 is initially controlled by a single shareholder, whose initial stake is α_C. If D_2 is not foreclosed after the acquisition, then the minimal offer that D_1 needs to make to induce U_1's initial controller to accept is given by (9), except that now, the acquired stake is $\alpha - \alpha_1$ (the difference between the final stake and the toehold) rather than α. Moreover, since we assume that $w = \Delta_1(N, N)$ when D_2 is not foreclosed, the post-acquisition value of U_1 equals its pre-acquisition value, i.e., $V_1^U = V_0^U$, so

$$b^U = (\alpha - \alpha_1) V_0^U,$$

where V_0^U is given by (1). Since D_1's profit absent foreclosure is V_0^D, the post-acquisition payoff of D_1 is:

$$V_0^D + \alpha V_0^U - \underbrace{(\alpha - \alpha_1) V_0^U}_{b^U} = V_0^D + \alpha_1 V_0^U.$$

Since this expression is also equal to D_1's payoff without the acquisition, D_1 has no incentive to acquire control over U_1 if it does not use it to foreclose D_2.

Now suppose that following an acquisition, D_1 uses its control over U_1 to foreclose D_2. Then, the minimal acceptable offer that D_1 needs to make is given by (11), except that now, $\alpha - \alpha_1$ replaces than α. Consequently, D_1's post-acquisition payoff becomes,

$$V_1^D + \alpha V_1^U - \underbrace{\left(\alpha_C L + (\alpha - \alpha_1) V_1^U\right)}_{b^U} = V_1^D - \alpha_C L + \alpha_1 V_1^U,$$

where V_1^U is given by equation (4). Noting that without acquisition, D_1's payoff is $V_0^D + \alpha_1 V_0^U$, and using the definitions of L and G, D_1 will make the offer if and only if his post-acquisition payoff

exceeds his pre-acquisition payoff, i.e.,

$$V_1^D - \alpha_C L + \alpha_1 V_1^U - \left[V_0^D + \alpha_1 V_0^U\right] = G - (\alpha_C + \alpha_1) L \geq 0.$$

■

Intuitively, when U_1's ownership is initially dispersed, D_1 must pay the initial shareholders a price equal to the pre-acquisition value of U_1. As a result, D_1 bears the entire loss in the value of these shares. When D_1 already owns a toehold, it needs to buy fewer shares to gain control over U_1, so it saves on the cost of acquiring shares, but this saving is exactly offset by the decline in the value of its toehold. Hence, D_1 bears a cost equal to $\underline{\alpha}L$, irrespective of how $\underline{\alpha}$ is divided between the toehold, α_1, and the acquired stake, $\underline{\alpha} - \alpha_1$. As a result, the toehold does not affect the equilibrium.

By contrast, when U_1 initially has a controlling shareholder, a toehold in U_1 makes it more costly for D_1 to foreclose D_2, since it lowers the value of D_1's toehold. In other words, D_1 internalizes a larger part of the loss of upstream profit and hence is more reluctant to incur these loses in order to boost its downstream profits. From a competitive standpoint, this implies in turn that a toehold is actually pro-competitive when initially U_1 has a controlling shareholder.

5.2 Acquisition by a controller

So far we have assumed that vertical integration arises when D_1 buys a controlling stake in U_1 or U_1 buys a controlling stake in D_1. However, there are cases in which the controlling shareholders of a firm (rather than the firm itself) buys a controlling stake in a vertically related firm, either directly, or through other firms that are under his control. For example, in 2000, Vivendi, which already held a controlling 49% stake in Canal+ (a major European producer of pay-television channels, with a significant presence in the distribution of films and the licensing of broadcasting rights) acquired Seagram, which owned Universal Studios Inc., as well as Polygram's music activities.[18] A second example is the 2009 offer of International Petroleum Investment Company (IPIC) to acquire a 70% stake in MAN Ferrostaal, which held a controlling 30% stake in Eurotecnica Melamine (the sole supplier and licensor of high pressure technology used in melamine production). The European Commission expressed the concern that after the acquisition, IPIC, which was the controlling shareholder of Agrolinz Melamine International (AMI) (one of the leading melamine producers

[18] See http://ec.europa.eu/competition/mergers/cases/decisions/m2050_en.pdf

world-wide), would foreclose AMI's competitors from Eurotecnica's technology.[19] In this subsection we study how acquisitions by controllers affect the concern for foreclosure.

For the sake of concreteness, suppose that the controlling shareholder of D_1 controls $m \geq 1$ firms; one of these firms is D_1, while the other firms are from other industries. The stakes that the controller holds in the m firms are $\beta_1, \ldots, \beta_m$, where β_1 is the controller's stake in D_1 and $\beta_2, \ldots, \beta_m$ are his stakes in firms $2, \ldots, m$.

D_1's controller can now acquire a controlling stake $\alpha \geq \underline{\alpha}$ in U_1 either directly, through D_1, or through firms $2, \ldots, m$. If he acquires the stake through firm $i = 0, 1, \ldots, m$ ("firm 0" means that the controller acquires a controlling stake in U_1 directly, so naturally, $\beta_0 = 1$) at a price b, and uses his control over U_1 to foreclose D_2, then his payoffs becomes

$$\beta_1 V_1^D + \beta_i \left[\alpha V_1^U - b\right] + \sum_{j=2,\ldots,m} \beta_j V_j,$$

where V_j is the value of firm $j = 2, \ldots, m$. If the controller does not acquires a controlling stake in U_1, his payoff is

$$\beta_1 V_0^D + \sum_{j=2,\ldots,m} \beta_j V_j.$$

The difference between the controller's payoff with and without acquisition is then

$$I = \beta_1 \underbrace{\left[V_1^D - V_0^D\right]}_{G} + \beta_i \left[\alpha V_1^U - b\right]. \tag{18}$$

From (11) we know that if U_1 has initially a controlling shareholder, then $b = \alpha_C L + \alpha V_1^U$. Hence,

$$I = \beta_1 \underbrace{\left[V_1^D - V_0^D\right]}_{G} + \beta_i \left[\alpha V_1^U - \alpha_C L - \alpha V_1^U\right] = \beta_1 \underbrace{\left[V_1^D - V_0^D\right]}_{G} - \beta_i \alpha_C L. \tag{19}$$

Likewise, we know from Lemma 6 that if U_1's ownership is initially dispersed, then $b = \alpha V_0^U$. Hence,

$$I = \beta_1 \underbrace{\left[V_1^D - V_0^D\right]}_{G} + \beta_i \left[\alpha V_1^U - \alpha V_0^U\right] = \beta_1 \underbrace{\left[V_1^D - V_0^D\right]}_{G} + \beta_i \alpha \underbrace{\left[V_1^U - V_0^U\right]}_{-L}. \tag{20}$$

Note immediately that without foreclosure, $G = L = 0$, so acquisition has no effect on the controller's payoff. Moreover, note that I is independent of α in (19), but is decreasing with α in (20). Hence, D_1's controller is indifferent to the precise stake he acquires in U_1 if he acquires it from an initial controller in U_1, but will prefer to acquire the minimal stake $\underline{\alpha}$ that ensures control if U_1's

[19] See http://ec.europa.eu/competition/mergers/cases/decisions/m5406_20090313_20212_en.pdf

ownership is initially dispersed. Moreover, since I is decreasing with β_i and increasing with β_1 in both (19) and (20), D_1's controller will prefer to acquire the controlling stake in U_1 through the firm in which he holds the minimal controlling stake and will have a stronger incentive to acquire control over D_2 when his controlling stake in D_1 is large. These observations imply the following result:

Proposition 8: *Suppose that D_1 has controlling shareholder who also owns controlling shares in firms from other industries and let k be the firm in which his controlling stake, β_k, is the lowest, i.e., $\beta_k \leq \beta_i$ for all $i = 1, \ldots, m$. Then, in equilibrium, D_1 will acquire a controlling stake in U_1 through firm k and will use it to foreclose D_2 if and only if $\beta_1 G \geq \beta_k \widehat{\alpha} L$, where $\widehat{\alpha} = \alpha_C$ when U_1 has initially a single controlling shareholder, and $\widehat{\alpha} = \underline{\alpha}$ when U_1 is initially held by dispersed shareholders. In the latter case, D_1 will acquire the minimal stake that ensures control over U_1, i.e., $\underline{\alpha}$, by making the tender offer restricted.*

Since by definition, $\beta_k \leq \beta_1$, it is clear that the ability of D_1's controller to choose whether to acquire a controlling stake in U_1 through D_1 or through another firm which he controls, expands the range of parameters for which D_2 is foreclosed (unless D_1 happens to be the firm in which the controller has the lowest controlling stake among all firms under his controls). Moreover, so long as $\beta_k < 1$, the controller will not acquire a controlling stake in U_1 directly, but rather through firm k. Intuitively, when the controller has a small stake in firm k, a large fraction of the upstream loss from foreclosing D_2 is borne by the passive shareholders of k. And, when β_1 is large, a large fraction of the associated downstream gain accrues to the controller. Hence, acquiring a controlling stake in U_1 and using it to foreclose D_2 is more attractive when β_k is small and β_1 is large.

5.3 Competition for the acquisition of a controlling stake in U_1

In this section we maintain the assumption that U_1 is the only potential upstream target, but we now consider the possibility that both D_1 and D_2 will compete for acquiring U_1. We begin by considering the case where U_1 has a single controlling shareholder.

Suppose that D_1 and D_2 simultaneously offer U_1's initial controller payments b_1 and b_2 for controlling stakes $\alpha_1 \geq \underline{\alpha}$ and $\alpha_2 \geq \underline{\alpha}$ in U_1, and suppose that the controller accepts D_i's offer. Using $\widehat{V}_1^U$ to denote the post-acquisition payoff of U_1, the payoff of U_1's controller becomes

$$b_i + (\alpha_C - \alpha_i)\,\widehat{V}_1^U = \alpha_C \widehat{V}_1^U + \underbrace{\left(b_i - \alpha_i \widehat{V}_1^U\right)}_{\phi_i},$$

where $\widehat{V}_1^U = V_1^U$ if D_j is foreclosed and $\widehat{V}_1^U = V_0^U$ if D_j is not foreclosed, and ϕ_i is the premium that D_i pays for the acquired stake above and beyond its post-acquisition value. Since $\alpha_C \widehat{V}_1^U$ is constant, it is obvious that U_1's controller will accept the offer with the highest premium ϕ_i, provided that his resulting payoff, $\alpha_C \widehat{V}_1^U + \phi_i$, is at least as high as $\alpha_C V_0^U$, which is his payoff if he rejects both offers. Hence, acceptable offers must be such that

$$\alpha_C \widehat{V}_1^U + \phi_i \geq \alpha_C V_0^U, \qquad \Longrightarrow \qquad \phi_i \geq \alpha_C \left[V_0^U - \widehat{V}_1^U \right]. \tag{21}$$

Using $\widehat{V}_1^D$ to denote the post-acquisition profit of D_i, the associated payoffs of D_i if its offer is accepted is

$$\widehat{V}_1^D + \alpha_i \widehat{V}_1^U - b_i = \widehat{V}_1^D - \phi_i,$$

where $\widehat{V}_1^D = V_1^D$ if D_j is foreclosed and $\widehat{V}_1^D = V_0^D$ if D_j is not foreclosed. The payoff of D_j, denoted $\widehat{V}_j^D$, is either V_0^D if it is not foreclosed and

$$V_F^D = \Pi\left(N-1, N\right) - \left(N-1\right) \Delta_1 \left(N-1, N\right), \tag{22}$$

if it is foreclosed. With these payoffs in place, we now prove the following result:

Proposition 9: *Suppose that D_1 and D_2 compete for the acquisition of a controlling stake in U_1 and assume that U_1 has initially a single controlling shareholder. Then,*

(i) if $G \geq \alpha_C L$, *then in equilibrium, D_1 and D_2 will set $\phi_1 = \phi_2 = V_1^D - V_F^D$ and U_1 will accept one of these offers at random; the acquirer, D_i, will use its control over U_1 to foreclose the downstream rival D_j.*

(ii) *if $G < \alpha_C L$, then in equilibrium, neither downstream firm acquires a sufficiently small controlling stake in U_1 such that after the acquisition the rival will be foreclosed.*

Proof: First, note that $G \geq \alpha_C L$ implies that $G \geq \alpha_i L$ for all $\alpha_i \in [\underline{\alpha}, \alpha_C]$, so once D_i gains control over U_1, it will use it to foreclose D_j (D_i's gain from foreclosure exceeds its share in U_1's loss). Hence, the post-acquisition payoffs are $\widehat{V}_1^D = V_1^D$ and $\widehat{V}_j^D = V_F^D$. The highest premium that D_i will offer is therefore

$$\phi_i = V_1^D - V_F^D,$$

which is the difference between D_i's payoff if it acquires control over U_1 and if D_j acquires it. Noting that $V_F^D < V_0^D$ (foreclosure hurts D_j), and that by assumption, $G \geq \alpha_C L$, and recalling

that $L \equiv V_0^U - V_1^U$ and that $\widehat{V}_1^U$ is either equal to V_1^U (if D_j is foreclosed) or to $V_0^U > V_1^U$ (if D_j is not foreclosed), it follows that

$$\phi_i = V_1^D - V_F^D \geq \underbrace{V_1^D - V_0^D}_{G} \geq \alpha_C L \geq \alpha_C \left[V_0^U - \widehat{V}_1^U\right].$$

Hence, by (21), the offer will be accepted by U_1's controller. Consequently, in equilibrium, $\phi_1 = \phi_2 = V_1^D - V_F^D$, and U_1's controller accepts one of the two offers at random.

Next, suppose that $G < \alpha_C L$. Now, an acquisition will lead to the foreclosure of D_j only if D_i acquires a sufficiently small controlling stake, α, such that $G > \alpha L$, but not if it acquires the entire stake α_C. However if D_i acquires a sufficiently small stake such that $G > \alpha L$, then by (21), it must set $\phi_i \geq \alpha_C \left[V_0^U - V_1^U\right] = \alpha_C L$. But then D_i's payoff is

$$V_1^D - \phi_i \leq V_1^D - \alpha_C L < V_0^D,$$

where the strict inequality follows since $G < \alpha_C L$. Therefore, there does not exist an equilibrium in which D_i alone makes a bid for U_1 and then uses its control over U_1 to foreclosure D_j.

Finally, we need to check that there does not exist an equilibrium in which both D_1 and D_2 offer to acquire controlling stakes in U_1, such that $G \geq \alpha_1 L$ and $G \geq \alpha_2 L$ (if D_i's offer is accepted, then D_j will be foreclosed). Such an equilibrium might exists since each D_i fears that if it will not acquire control, its rival will, and will use its control in U_1 to foreclose it. Now, assume by way of negation that such an equilibrium exists. Since these offers lead to foreclosure, U_1 would accept D_i's offer if $\phi_i \geq \phi_j$ (i offers a higher premium than j) and $\phi_i \geq \alpha_C L$. The resulting payoff of D_i is $V_1^D - \phi_i$. If instead D_j's offer is accepted, then D_i will be foreclosed and its post-acquisition payoff will be V_F^D. Hence, the highest premium that D_i will offer is $\phi_i = V_1^D - V_F^D$. In equilibrium (if it exists) both D_1 and D_2 must offer $V_1^D - V_F^D$, otherwise the firm with the higher offer can cut its offer slightly. Hence, the payoff of both D_1 and D_2 in equilibrium is V_F^D and the payoff of U_1's controller is $\alpha_C V_1^U + V_1^D - V_F^D$.

Now suppose that D_i deviates and offers b_C for the entire stake of U_1's controller, α_C. If the offer is accepted, D_j would not be foreclosed, since by assumption, $G < \alpha_C L$. The deviation is more profitable for U_1's controller's than D_j's offer provided that

$$\underbrace{b_C}_{\text{Accepting } D_i\text{'s offer}} \geq \underbrace{\alpha_C V_1^U + V_1^D - V_F^D}_{\text{Accepting } D_j\text{'s offer}}.$$

If D_i makes the minimal acceptable bid, its payoff under the deviation is

$$V_0^D + \alpha_C V_0^U - \underbrace{\left(\alpha_C V_1^U + V_1^D - V_F^D\right)}_{b_C} = \alpha_C L - G + V_F^D,$$

which exceeds D_i's payoff in the candidate equilibrium since by assumption, $G < \alpha_C L$. ■

Intuitively, competition between D_1 and D_2 induces the two firms to offer more money to U_1's controller because each firm fears that if it does not acquire a controlling stake in U_1, its rival will acquire it and will use it to foreclose it. However, competition between D_1 and D_2 does not alter the range of parameters for which acquisition which leads foreclosure takes place.

We now turn to the case where initially, U_1's ownership is dispersed. The two downstream firms D_1 and D_2 make simultaneous tender offers to U_1's shareholders. Each offer specifies the maximal stake $\overline{\alpha}_i$ that D_i offers to acquire and a price that reflects a value V_i for the entire firm. The shareholders observe the two offers and decide whether to tender to D_1, to D_2, or hold on to their shares. If more than $\overline{\alpha}_i$ shares are tendered to D_i, then the tendered shares are prorated. D_i obtains control over U_1 if it acquires more shares than D_2 and if its final stake is at least $\underline{\alpha}$.

Proposition 10: *Suppose that D_1 and D_2 compete over the acquisition of a controlling stake in U_1, who initially has a dispersed ownership and assume that if $V_1^U < V_i < V_0^U$, then U_1's initial shareholders do not tender their shares to D_i. Then,*

(i) *if $G > \underline{\alpha} L$, then in equilibrium, D_i offers to acquire $\underline{\alpha}$ shares for a price that reflects value $V_i = V_1^U + \frac{V_1^D - V_F^D}{\underline{\alpha}}$ for U_1 and D_j offers to acquire $\alpha_j > 0$ shares at $V_j = V_1^U + \left(V_1^D - V_F^D\right)$. All shareholders tender to D_i who gains control of U_1 and then uses it to foreclose D_j.*

(ii) *if $G < \underline{\alpha} L$, then in equilibrium neither D_1 nor D_2 make a tender offer to U_1's shareholders.*

Proof: Suppose that $G > \underline{\alpha} L$. If D_i gains control over U_1, then U_1's shareholders would tender shares only if the offered price, V_i, exceeds the post-acquisition value of U_1 (otherwise they are better-off holding on to their shares). Hence, D_j would not acquire any shares, while D_i would prefer to acquire the minimal stake that ensures control. The resulting post-acquisition payoffs of D_i and D_j are therefore:

$$\Pi_i = \underline{\alpha}\left(V_1^U - V_i\right) + V_1^D, \qquad \Pi_j = V_F^D.$$

In equilibrium, it must be that $\Pi_i = \Pi_j$, otherwise D_j would outbid D_i slightly, and gain control over U_1. Hence,

$$\underline{\alpha}\left(V_1^U - V_i\right) + V_1^D = V_F^D, \qquad \Rightarrow \qquad V_i = V_1^U + \frac{V_1^D - V_F^D}{\underline{\alpha}}. \tag{23}$$

Note that since $\frac{V_1^D - V_F^D}{\underline{\alpha}} > \frac{V_1^D - V_0^D}{\underline{\alpha}} \equiv \frac{G}{\underline{\alpha}} \geq L$, then $V_i > V_1^U + L = V_0^U$. Hence all shareholders will tender to D_i.

To compute V_j, notice that in equilibrium, shareholders must be indifferent between tendering to D_i and tendering to D_j, otherwise D_i can profitably lower V_i. Since D_i's offer is restricted and therefore prorated, the payoff of each shareholder per share if he tenders to D_i is $\underline{\alpha} V_i + (1 - \underline{\alpha}) V_1^U$. If the shareholder tenders to D_j, his payoff per share is V_j (recall that in equilibrium no shareholder tenders to D_j, so the shareholder's submission will not be prorated). Hence, using (23),

$$V_j = \underbrace{\underline{\alpha}\left(V_1^U + \frac{1}{\underline{\alpha}}\left(V_1^D - V_F^D\right)\right)}_{V_i} + (1 - \underline{\alpha}) V_1^U = V_1^U + V_1^D - V_F^D. \tag{24}$$

We now show that there do not exist equilibria in which neither D_1 nor D_2 gains control over U_1. To this end, assume by way of negation that there exists an equilibrium in which $\alpha_1 < \underline{\alpha}$ and $\alpha_2 < \underline{\alpha}$. Since neither downstream firm controls U_1, neither firm is foreclosed, so the downstream profits of D_1 and D_2 are both equal to V_0^D. Moreover, both firms pay a price of V_0^U for the shares they acquire in U_1 (if any); this price is the minimal price needed to induce U_1's shareholders to tender shares and neither firm would have an incentive to offer a higher price for the shares. Hence, the two firms break even on the shares they acquire in U_1 so their payoffs are both equal to V_0^D. Now, holding α_j fixed, suppose that D_i deviates and acquires a stake $\underline{\alpha}$ in U_1, which ensures that it controls U_1. Since $G > \underline{\alpha} L$, D_i will use its control over U_1 to foreclose D_j. The resulting payoff of D_i is then $V_1^D + \underline{\alpha}\left(V_1^U - V_0^U\right) > V_0^D$, where the inequality follows since $G > \underline{\alpha} L$. The deviation then is profitable and it upsets the putative equilibrium.

Likewise, there do not exist equilibria in which $\alpha_1 = \alpha_2 > \underline{\alpha}$. In these equilibria, D_1 and D_2 have joint control over U_1, so neither firm is foreclosed. As before, each firm pays a price of V_0^U for the shares it acquires, so the payoff of each firm equals its downstream profit V_0^D. Now, suppose that D_i deviates by raising α_i slightly to α_i', while continuing to pay V_0^U for the acquired shares. Following the deviation, D_i obtains controls over U_1, and since $G > \underline{\alpha} L$ it uses its control to foreclose D_j. The resulting payoff of D_i is then $V_1^D + \alpha_i'\left(V_1^U - V_0^U\right)$ implying that the deviation is profitable and it upsets the putative equilibrium.

If $\alpha_1 = \alpha_2 = 1/2$, then again neither firm is foreclosed but now D_i cannot raise α_i without offering a higher price for the shares it buys. If $V_i > V_j$, then D_i can offer to buy $\alpha_i > 1/2$, obtain controls over U_1 and use it control to foreclose D_j. The resulting payoff of D_i is then $V_1^D + \alpha_i\left(V_1^U - V_0^U\right)$ implying that the deviation is profitable and it upsets the putative equilibrium.

If $V_i > V_j$, then D_i can offer to buy $\alpha_i > 1/2$ at a price slightly above V_i. D_i then obtains controls over U_1 and use it control to foreclose D_j. The resulting payoff of D_i is then $V_1^D + \alpha_i \left(V_1^U - V_0^U\right)$ implying that the deviation is profitable and it upsets the putative equilibrium.

Finally, suppose that $G < \underline{\alpha} L$. Then, an acquisition of a controlling stake in U_1 does not lead to foreclose, so neither downstream firm has an incentive to acquire control over U_1. ■

Proposition 10 is very similar to Proposition 4: the condition under which foreclosure emerges remains the same whether or not D_1 competes with D_2 for the acquisition of control over U_1.

6 Conclusions

We consider the effects of partial vertical integration on the foreclosure of downstream competitors. Our analysis shows that firms often have incentives to acquire partial rather than full stakes in vertically related firms and that partial acquisitions may often increase the risk of anticompetitive foreclosure of rivals beyond what is posed by a full vertical integration. The profitability of partial acquisitions depends on the ownership structure of the target firm. We show that an acquisition of an upstream supplier may be unprofitable when the supplier has a single owner, but be profitable when the supplier has a controlling shareholder who holds less than 100% of the shares and provided the controller's initial share is not too high. Partial acquisition is yet more profitable when the shares of the supplier are held by dispersed shareholders.

The profitability of anticompetitive vertical mergers is higher under partial integration due to the potential to expropriate the target's minority shareholders. When minority shareholders are present the acquirer has to pay the shareholders only a partial compensation for the eventual loss of upstream sales due to the foreclosure of the his downstream rivals. We explore some additional implications of this basic intuition and find that it implies that the acquisition of a non-controlling toehold in a supplier often lowers the risk of eventual foreclosure of rivals.

From an antitrust perspective, our analysis suggests that antitrust authorities should view partial acquisitions of control in upstream suppliers as posing a potential bigger anticompetitive risk than similar acquisitions in which all shares are acquired. Antitrust authorities should be particularly wary when the acquisition is carried out through a tender offer to dispersed shareholders. On the other hand, an acquisition of a noncontrolling share in an upstream supplier does not facilitate the eventual acquisition of full control and, at least from this perspective, should not be

viewed as harmful.

The analysis is also relevant to the large literature on corporate government and the protection of minority shareholders. Our paper suggests that firms with a large share of minority shareholders are particularly attractive targets to value-decreasing acquisitions by their downstream buyers, who may then abuse their control to foreclose product-market competitors to the disadvantage of the target's minority shareholders. The paper thus formalizes the wide-spread notion that the foreclosure of rivals can be an important source of private benefits to acquirers in takeovers in which vertically-related firms are involved.

7 Appendix

Following are three examples. The first example motivates the assumptions that we impose in Sections 2 and 3 on the downstream profit functions. The second and third examples show that our basic setup is consistent with varaints of the two main raising your rivals costs models of input foreclosure: In Ordover, Salop and Saloner (1990), and Salinger (1988).

7.1 An example

Suppose that firms D_1 and D_2 are located at the two ends of a unit line and compete by setting prices. Consumers are uniformly distributed on the unit line. The utility of a consumer located at a distance x from firm D_1 if he buys from D_1 is given by

$$U_1(x) = v \log(n_1 + 1) - tx - p_1,$$

where $v \log(n_1 + 1)$ is the "quality" of D_1, which increases with the number of inputs, n_1, it uses, $t > 0$ is the transportation cost per unit of distance, and p_1 is the price that D_1 charges. If the consumer buys from D_2, his utility is

$$U_2(x) = v \log(n_2 + 1) - t(1 - x) - p_2.$$

If the consumer does not buy at all, his utility is 0.

Assuming that the market is fully covered, the location of the indifferent consumer between D_1 and D_2 is

$$x^*(p_1, p_2, n_1, n_2) = \frac{1}{2} + \frac{p_2 - p_1 + v \log\left(\frac{n_1+1}{n_2+1}\right)}{2t}. \tag{25}$$

Assuming in addition that D_1 and D_2 pay a fixed price for the inputs and normalizing their additional costs to 0, the gross profits of D_1 and D_2 are given by

$$\pi_1 = p_1 x^* (p_1, p_2, n_1, n_2), \qquad \pi_2 = p_2 (1 - x^* (p_1, p_2, n_1, n_2)).$$

Solving for the Nash equilibrium prices, we obtain:

$$\begin{aligned} p_1^* (n_1, n_2) &= t + \frac{v}{3} \log \left(\frac{n_1 + 1}{n_2 + 1} \right), \\ p_2^* (n_1, n_2) &= t - \frac{v}{3} \log \left(\frac{n_1 + 1}{n_2 + 1} \right). \end{aligned}$$

To avoid uninteresting complications, we shall assume that $\left| \log \left(\frac{n_1+1}{n_2+1} \right) \right| < \frac{3t}{v}$ for all n_1 and n_2; this assumption ensures that $p_1^* (n_1, n_2)$ and $p_2^* (n_1, n_2)$ are both nonnegative. Substituting $p_1^* (n_1, n_2)$ and $p_2^* (n_1, n_2)$ in (25), and the sunstituting the result in $U_1 (x)$, reveals that in equilibrium, the utility of the indifferent consumer is

$$U_1 (x^* (n_1, n_2)) = \frac{-3t + v \log (n_1 + 1) + v \log (n_2 + 1)}{2}.$$

The market is covered, as we assume, provided that $U_1 (x^* (n_1, n_2)) \geq 0$. This inequality holds if $\frac{3t}{v} < \log (n_1 + 1) + \log (n_2 + 1)$.

Substituting $p_1^* (n_1, n_2)$, $p_2^* (n_1, n_2)$ in the profit functions and using (25), the profit of a downstream firm when it uses k inputs and its rival uses l inputs (e.g., the profit of D_1 when $n_1 = k$ and $n_2 = l$) is

$$\Pi (k, l) = \frac{\left[t + \frac{v}{3} \log \left(\frac{k+1}{l+1} \right) \right]^2}{2}.$$

Our assumption that $\left| \log \left(\frac{n_1+1}{n_2+1} \right) \right| < \frac{3t}{v}$ ensures that $\Pi (k, l)$ is increasing with k and decreasing with l as Assumption 1 states.

Now,

$$\begin{aligned} \Delta_1 (k, l) &\equiv \Pi (k, l) - (k - 1, l) \\ &= \frac{v}{6t} \log \left(\frac{k + 1}{k} \right) \left[2t + \frac{v}{3} \log \left(\frac{k (k + 1)}{(l + 1)^2} \right) \right], \end{aligned}$$

and

$$\begin{aligned} \Delta_2 (k, l) &\equiv \Pi (k, l) - (k, l - 1) \\ &= \frac{v}{6t} \log \left(\frac{l}{l + 1} \right) \left[2t + \frac{v}{3} \log \left(\frac{(k + 1)^2}{l (l + 1)} \right) \right]. \end{aligned}$$

Assumption 3 holds since

$$\begin{aligned}\Delta_{12}(l,k) &\equiv \underbrace{(\Pi(l,k)-(l-1,k))}_{\Delta_1(l,k)} - \underbrace{(\Pi(l,k-1)-(l-1,k-1))}_{\Delta_1(l-1,k)} \\ &= \frac{v^2}{9t}\log\left(\frac{k+1}{k}\right)\log\left(\frac{l}{l+1}\right) < 0.\end{aligned}$$

Assumption 2 holds if

$$= \frac{v}{6t}\log\left(\frac{N+1}{N}\right)\left[2t+\frac{v}{3}\log\left(\frac{N}{N+1}\right)\right] > c,$$

while Assumption A4 holds if

$$\frac{v}{6t}\log\left(\frac{k+1}{k}\right)\left[2t+\frac{v}{3}\log\left(\frac{k(k+1)l^2}{(l+1)^4}\right)\right] > c,$$

for all k and all l. Moreover, Assumption A5 holds if

$$\begin{aligned}&-\frac{v}{6t}\log\left(\frac{N}{N+1}\right)\left[2t+\frac{v}{3}\log\left(\frac{N+1}{N}\right)\right] + N\frac{v^2}{9t}\log\left(\frac{N+1}{N}\right)\log\left(\frac{N}{N+1}\right) \\ =\ & -\frac{v}{3}\log\left(\frac{N}{N+1}\right) - \frac{v^2}{18t}\log\left(\frac{N}{N+1}\right)\log\left(\frac{N+1}{N}\right) + N\frac{v^2}{9t}\log\left(\frac{N+1}{N}\right)\log\left(\frac{N}{N+1}\right) \\ =\ & -\frac{v}{3}\log\left(\frac{N}{N+1}\right) + \frac{v^2}{9t}\left[N-\frac{1}{2}\right]\log\left(\frac{N+1}{N}\right)\log\left(\frac{N}{N+1}\right) \\ =\ & \frac{v}{3}\left[\frac{v\left(N-\frac{1}{2}\right)}{3t}\log\left(\frac{N+1}{N}\right)-1\right]\log\left(\frac{N}{N+1}\right) > 0.\end{aligned}$$

Since $\log\left(\frac{N}{N+1}\right) < 0$, this inequality holds if the square bracketed term is negative, i.e., if

$$\frac{3t}{v} > \left(N-\frac{1}{2}\right)\log\left(\frac{N+1}{N}\right).$$

7.2 A variant of Ordover, Salop and Saloner (1990)

In this example we develop a variant of the Ordover, Salop and Saloner (1990) model (henceforth OSS) which is consistent with our basic setup. OSS consider two upstream suppliers, U_1 and U_2, which produce a homogenous input and sell it to two symmetric downstream firms, D_1 and D_2, which produce substitute products and compete by setting prices. Since U_1 and U_2 engage in Bertrand competition in the upstream market, their profit under non-integration is 0. By definition then, an upstream suppliers cannot lose from vertical integration. Clearly the OSS setting is

extreme. To make it more realistic and ensure that U_1 and U_2 earn a profit before integration, we will modify the OSS setting slightly by assuming that the upstream costs are random.[20]

Specifically, we assume that the per unit cost of each upstream supplier i, c_i, is either high, $\overline{c}$, or low, $\underline{c}$, with equal probabilities, independently across the two suppliers (OSS assume that U_1 and U_2 have the same per unit cost, which is deterministic). Given their cost realizations, U_1 and U_2 set the prices of their respective inputs. Then downstream firms, D_1 and D_2, buy the inputs, convert each unit of input to one unit of the final product, at no additional cost, set their respective prices, and sell to final consumers.

Let w_1 and w_2 be the prices that D_1 and D_2 pay for the input. Since inputs are converted to outputs on a 1:1 basis, w_1 and w_2 are also the marginal costs of D_1 and D_2. The profit of each downstream firm i is then given by

$$\pi_i = (p_i - w_i)\, q_i\,(p_i, p_j)\,,$$

where p_i and p_j are the downstream prices and $q_i\,(p_i, p_j)$ is firm i's quantity. Since the products of D_1 and D_2 are (imperfect) substitutes, $q_i\,(p_i, p_j)$ decreases with p_i and increases with p_j.

The equilibrium price of each downstream firm i is $p_i\,(w_i, w_j)$ and its corresponding profit is $\pi_i\,(w_i, w_j)$.

Lemma 1: *$\pi_i\,(w_i, w_j)$ decreases with w_i and assuming that p_i increases with w_i, $\pi_i\,(w_i, w_j)$ also increases with w_j.*

Proof of Lemma 1: Let $\widehat{w}_i > w_i$ and let $q_i\,(w_i, w_j) \equiv q_i\,(p_i\,(w_i, w_j)\,, p_j\,(w_i, w_j))$. Then by revealed preferences,

$$\begin{aligned}
\pi_i\,(w_i, w_j) &= (p_i\,(w_i, w_j) - w_i)\, q_i\,(w_i, w_j) \\
&\geq (p_i\,(\widehat{w}_i, w_j) - w_i)\, q_i\,(\widehat{w}_i, w_j) \\
&> (p_i\,(\widehat{w}_i, w_j) - \widehat{w}_i)\, q_i\,(\widehat{w}_i, w_j) \\
&= \pi_i\,(\widehat{w}_i, w_j)\,.
\end{aligned}$$

[20] Another possibility is to assume that the inputs are imperfect substitutes. However, this modelling approach would require us to specify how the two inputs are combined into a final product, which would add another layer of complication to the model, which we avoid with our approach.

Moreover,

$$\begin{aligned}\pi_i(w_i, \widehat{w}_j) &= (p_i(w_i, \widehat{w}_j) - w_i)\, q_i(w_i, \widehat{w}_j) \\ &> (p_i(w_i, \widehat{w}_j) - w_i)\, q_i(w_i, w_j) \\ &> (p_i(w_i, w_j) - w_i)\, q_i(w_i, w_j) \\ &= \pi_i(w_i, w_j),\end{aligned}$$

where the first inequality follows because $p_j(w_i, \widehat{w}_j) > p_j(w_i, w_j)$ and because the two final products are substitutes, so $p_j(w_i, \widehat{w}_j) > p_j(w_i, w_j)$ implies that $q_i(w_i, \widehat{w}_j) > q_i(w_i, w_j)$ and the second inequality follows by revealed preference. ■

7.2.1 Nonintegration

Since the input is homogenous, both input prices under nonintegration are equal to $\underline{c}$ if $c_1 = c_2 = \underline{c}$ and $\overline{c}$ if $c_1 = c_2 = \overline{c}$. When $c_i = \underline{c}$ and $c_j = \overline{c}$, U_i can always undercut U_j slightly and sell to both D_1 and D_2, so in equilibrium, only U_i sells the input. We will assume that the difference between $\overline{c}$ and $\underline{c}$ is not too large in the sense that U_i will prefer to set the input price at $\overline{c}$.

Assuming that in case of a tie, D_1 and D_2 buy from the lowest cost supplier (and if costs are the same, they randomize their purchases), it follows that in equilibrium,

$$w_1 = w_2 = \begin{cases} \underline{c} & \text{if } c_1 = c_2 = \underline{c}, \\ \overline{c} & \text{otherwise.} \end{cases}$$

Let $\overline{q} \equiv q_1(\overline{c}, \overline{c}) = q_2(\overline{c}, \overline{c})$ be the equilibrium output levels when $w_1 = w_2 = \overline{c}$, and define $\underline{q}$ similarly. The associated downstream prices are $\overline{p} \equiv p_1(\overline{c}, \overline{c}) = p_2(\overline{c}, \overline{c})$ and $\underline{p} = p_1(\underline{c}, \underline{c}) = p_2(\underline{c}, \underline{c})$. Since the input is converted to output on a 1:1 basis, $\overline{q}$ and $\underline{q}$ are also the demands for the input. The expected profit of each supplier is then:

$$V_0^U = \frac{1}{4} \times 2(\overline{c} - \underline{c})\, \overline{q} = \frac{(\overline{c} - \underline{c})\, \overline{q}}{2}. \tag{26}$$

This expression reflects the fact that a nonintegrated supplier U_i earns a positive profit only when $c_i = \underline{c}$ and $c_j = \overline{c}$; the probability of this event is $\frac{1}{4}$. The supplier then sets a price of $\overline{c}$ and sells $\overline{q}$ units to each downstream firm. The associated expected profits of D_1 and D_2 is

$$V_0^D = \frac{3}{4}\pi_1(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1(\underline{c}, \underline{c}), \tag{27}$$

where

$$\pi_1(\overline{c}, \overline{c}) = \overline{q}(\overline{p} - \overline{c}), \qquad \pi_1(\underline{c}, \underline{c}) = \underline{q}\left(\underline{p} - \underline{c}\right).$$

7.2.2 Integration

When U_1 and D_1 integrate, U_1 supplies D_1 at cost, unless $c_1 = \overline{c}$ and $c_2 = \underline{c}$, in which case U_2 sells the input to D_1 at a price equal to $\overline{c}$. Hence, D_1 buys the input from U_1 at $\underline{c}$ if $c_1 = \underline{c}$ and at $\overline{c}$ if $c_1 = c_2 = \overline{c}$, and buys it from U_2 at $\overline{c}$ if $c_1 = \overline{c}$ and $c_2 = \underline{c}$. Note that in all cases, $w_1 = c_1$.

As in OSS, we assume that when U_1 and D_1 integrate, U_1 commits not to sell to D_2.[21] Hence U_2 becomes the sole supplier to D_2 and sets the input price, w_2, to maximize its profit

$$(w_2 - c_2)\, q_2\,(c_1, w_2)\,.$$

We will assume that this profit is concave in w_2. This assumption holds for example in the linear demand example shown below. The profit maximizing value of w_2 is defined implicitly by the following first-order condition:

$$q_2\,(c_1, w_2) + (w_2 - c_2)\,\frac{\partial q_2\,(c_1, w_2)}{\partial w_2} = 0. \tag{28}$$

The solution for the equation, $w_2\,(c_1, c_2)$, determines D_2's marginal cost. Clearly, $w_2\,(c_1, c_2) > c_2$ for all c_2. Moreover, $w_2\,(\overline{c}, \underline{c}) \geq \overline{c}$ under the following assumption:

Assumption 1: $q_2\,(\overline{c}, \overline{c}) + (\overline{c} - \underline{c})\,\frac{\partial q_2(\overline{c},\overline{c})}{\partial w_2} \geq 0.$

Since $\frac{\partial q_2(\overline{c},\overline{c})}{\partial w_2}$ is bounded from above and $q_2\,(\overline{c}, \overline{c}) > 0$, Assumption 1 holds when $\overline{c} - \underline{c}$ is sufficiently small.

The expected profit of D_1 under integration is

$$V_1^D = \frac{1}{4}\pi_1\,(\underline{c}, w_2\,(\underline{c}, \underline{c})) + \frac{1}{4}\pi_1\,(\underline{c}, w_2\,(\underline{c}, \overline{c})) + \frac{1}{4}\pi_1\,(\overline{c}, w_2\,(\overline{c}, \underline{c})) + \frac{1}{4}\pi_1\,(\overline{c}, w_2\,(\overline{c}, \overline{c}))\,. \tag{29}$$

Notice that since $\pi_i\,(w_i, \widehat{w}_j) > \pi_i\,(w_i, w_j)$ for $\widehat{w}_j > w_j$ and since $w_2\,(\overline{c}, \overline{c}) > w_2\,(\overline{c}, \underline{c}) \geq \overline{c}$ and $w_2\,(\underline{c}, \underline{c}) > \underline{c}$,

$$\begin{aligned}
V_1^D &= \frac{1}{4}\pi_1\,(\underline{c}, w_2\,(\underline{c}, \underline{c})) + \frac{1}{4} \times \pi_1\,(\underline{c}, w_2\,(\underline{c}, \overline{c})) + \frac{1}{4}\pi_1\,(\overline{c}, w_2\,(\overline{c}, \underline{c})) + \frac{1}{4}\pi_1\,(\overline{c}, w_2\,(\overline{c}, \overline{c})) \\
&> \frac{1}{4}\pi_1\,(\underline{c}, \underline{c}) + \frac{1}{4}\pi_1\,(\underline{c}, \overline{c}) + \frac{1}{4}\pi_1\,(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1\,(\overline{c}, \overline{c}) \\
&> \frac{1}{4}\pi_1\,(\underline{c}, \underline{c}) + \frac{1}{4}\pi_1\,(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1\,(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1\,(\overline{c}, \overline{c}) \\
&= V_0^D.
\end{aligned}$$

[21] There is a debate about U_1's ability to make this commiment: see Hart and Tirole (1990) and Reiffen (1992), and see Ordover, Salop, and Saloner (1992) for a response. Several papers have proposed models that are immune to this criticism, inluding Ma (1997), Chen (2001), Choi and Yi (2001), and Church and Gandal (2000). We will follow OSS in assuming that U_1 can commit not to sell to D_2 because our purpose here is to show that a (variant of) the OSS model predicts that there are cases in which $G > L$ and there are cases in which the opposite holds.

That is, vertical integration and the foreclosure of D_2 boost the profit of D_1. Since U_1 commits not to sell to D_2 and since it transfers the input to D_1 at cost, its profit is $V_1^U = 0$. Given that its pre-merger profit is $V_0^U > 0$, it follows that integration and the foreclosure of D_2 involve a transfer of profits from U_1 to D_1.

Vertical integration is profitable if the downstream gain exceeds the upstream loss:

$$\underbrace{V_1^D - V_0^D}_{G} > \underbrace{V_0^U - V_1^U}_{L} = V_0^U, \tag{30}$$

where G is the downstream benefit from vertical integration and the foreclosure of D_2 and L is the associated upstream loss. The next example shows that $G > L$ for a broad range of parameters.

7.2.3 Example

Assume that $q_i = A - p_i + \gamma p_j$, where $\gamma \in [0, 1]$ is the degree of product differentiation. The profit of each downstream firm i is $\pi_i = q_i (p_i - w_i)$. The Nash equilibrium when both firms choose their prices simultaneously is

$$p_1(w_1, w_2) = \frac{(2+\gamma) A + 2w_1 + \gamma w_2}{4 - \gamma^2}, \qquad p_2(w_1, w_2) = \frac{(2+\gamma) A + 2w_2 + \gamma w_1}{4 - \gamma^2}.$$

The resulting quantities are

$$q_1(w_1, w_2) = \frac{(2+\gamma) A - \left(2 - \gamma^2\right) w_1 + \gamma w_2}{4 - \gamma^2},$$

and

$$q_2(w_1, w_2) = \frac{(2+\gamma) A - \left(2 - \gamma^2\right) w_2 + \gamma w_1}{4 - \gamma^2}.$$

The equilibrium profits are $\pi_1(w_1, w_2) = q_1(w_1, w_2)^2$ and $\pi_2(w_1, w_2) = q_2(w_1, w_2)^2$. Notice that $\pi_i(w_i, w_j)$ decreases with w_i and increases with w_j, as Lemma 1 above states.

Given these expressions, the expected pre-merger profits of D and U_1 are:

$$\begin{aligned} V_0^D &= \frac{3}{4}\pi_1(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1(\underline{c}, \underline{c}) \\ &= \frac{3(A - (1-\gamma)\overline{c})^2 + (A - (1-\gamma)\underline{c})^2}{4(2 - \gamma^2)}, \end{aligned} \tag{31}$$

and

$$\begin{aligned} V_0^U &= \frac{(\overline{c} - \underline{c})}{2} \times \underbrace{\frac{(2+\gamma) A - \left(2 - \gamma^2\right) \overline{c} + \gamma \overline{c}}{4 - \gamma^2}}_{\overline{q}} \\ &= \frac{(\overline{c} - \underline{c})(A - (1-\gamma)\overline{c})}{2(2-\gamma)}. \end{aligned} \tag{32}$$

To calculate the price at which U_2 sells to D_2 after U_1 and D_1 integrate, recall that after integration, $w_1 = c_1$. Substituting $q_2(c_1, w_2)$ into (28) and solving for w_2 yields

$$w_2(c_1, c_2) = \frac{(2+\gamma)A + \left(2-\gamma^2\right)c_2 + \gamma c_1}{2\left(2-\gamma^2\right)}.$$

Hence, the profit of D_1, given c_1 and c_2, is

$$\pi_1(c_1, w_2(c_1, c_2)) = q_1(c_1, w_2(c_1, c_2))^2 = \left(\frac{\beta A - c_1\left(8 - 9\gamma^2 + 2\gamma^4\right) + \gamma c_2\left(2-\gamma^2\right)}{2\left(2-\gamma^2\right)\left(4-\gamma^2\right)}\right)^2,$$

where

$$\beta \equiv 8 + 6\gamma - 3\gamma^2 - 2\gamma^3.$$

Substituting into (29) and rearranging,

$$\begin{aligned} V_1^D &= \frac{1}{4}\pi_1(\underline{c}, w_2(\underline{c}, \underline{c})) + \frac{1}{4}\pi_1(\underline{c}, w_2(\underline{c}, \overline{c})) + \frac{1}{4}\pi_1(\overline{c}, w_2(\overline{c}, \underline{c})) + \frac{1}{4}\pi_1(\overline{c}, w_2(\overline{c}, \overline{c})) \\ &= \frac{2\beta^2 A^2 + \phi\left(\underline{c}^2 + \overline{c}^2\right) - \gamma\left(32 - 52\gamma^2 + 26\gamma^4 - 4\gamma^6\right)\underline{c}\overline{c} - 2(1-\gamma)A\beta^2(\underline{c} + \overline{c})}{8(2+\gamma)^2\left(4 - 2\gamma - 2\gamma^2 + \gamma^3\right)^2}, \end{aligned} \tag{33}$$

where

$$\phi \equiv 64 - 16\gamma - 140\gamma^2 + 26\gamma^3 + 109\gamma^4 - 13\gamma^5 - 35\gamma^6 + 2\gamma^7 + 4\gamma^8.$$

To simplify the computations, we will use the normalizations $A = 1$ and $\underline{c} = 0$. To ensure that $w_2(\overline{c}, \underline{c}) \geq \overline{c}$, we will also

$$\overline{c} \leq \frac{2+\gamma}{4 - \gamma - 2\gamma^2}. \tag{34}$$

Substituting from (31), (32), and (33) into (30) and using the normalizations, we get

$$\underbrace{V_1^D - V_0^D}_{G} - \underbrace{V_0^U}_{L} = \frac{2\beta^2 + \phi\overline{c}^2 - 2(1-\gamma)\beta^2\overline{c}}{8(2+\gamma)^2\left(4 - 2\gamma - 2\gamma^2 + \gamma^3\right)^2} - \frac{4 - 2(1-2\gamma)\overline{c} - \left(1-\gamma^2\right)\overline{c}^2}{4(2-\gamma)^2}. \tag{35}$$

This expression depends only on the degree of product differentiation, γ, and on $\overline{c}$. Figure 1 shows that the combinations of γ and $\overline{c}$ for which (35) holds. The relevant range of parameters which satisfy (34) are those below the $\frac{2+\gamma}{4-\gamma-2\gamma^2}$ curve. The figure shows that the downstream benefit from vertical integration and the foreclosure of D_2, G, exceeds the associated upstream loss, L, when γ is sufficiently large, i.e., the downstream products are sufficiently close substitutes. When γ is low, L exceeds G (note in particular that when $\gamma = 0$, D_1 and D_2 do not compete with each other, so $G = 0$, implying that $L > G$; by continuity this is also true when γ is positive but small).

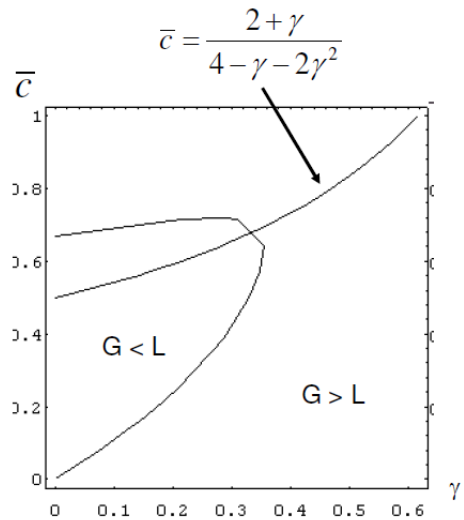

Figure 1: The profitability of vertical integration in a variant of the OSS model

7.3 A variant of Salinger (1988)

This example shows that our basic setup is also consistent with Salinger (1988). In his model, there are $N \geq 2$ symmetric upstream suppliers $U_1, ..., U_N$, which is produce a homogenous input at a cost c per unit. The upstream firms compete by setting quantities and the input price, w, clears the input market. For simplicity, we will assume here that there are only two downstream firms, D_1 and D_2, which convert the input to a final product on a 1:1 basis at no additional cost. The two downstream firms also compete by setting quantities. The demand for the final good is given by $p = A - Q$, where Q is the sum of the quantities of D_1 and D_2.

7.3.1 Nonintegration

Since D_1 and D_2 convert the input to a final product on a 1:1 basis at no additional cost, their marginal costs are equal to the input price w. Noting that D_1 and D_2 engage in Cournot competition, the output of each firm is $\frac{A-w}{3}$. Hence, the total demand for the input is $Q = \frac{2(A-w)}{3}$, so the inverse demand for the input is $w = A - \frac{3Q}{2}$, where $Q = q_1 + \ldots + q_N$ is the total output of the upstream suppliers.

The profit of each upstream supplier i is given by $q_i (w - c)$. Each upstream supplier i chooses q_i to maximize his profit. The resulting Nash equilibrium output of each upstream firm is

$$q^* = \frac{2(A-c)}{3(N+1)},$$

and the equilibrium price of the input is

$$w^* = A - \frac{3Nq^*}{2} = \frac{A + Nc}{N+1}.$$

The equilibrium profit of each upstream firm then is

$$V_0^U = q^* (w^* - c) = \frac{2}{3} \left(\frac{A - c}{N+1} \right)^2, \tag{36}$$

and the equilibrium profit of each downstream firm is

$$V_0^D = \left(\frac{A - w^*}{3} \right)^2 = \left(\frac{N (A - c)}{3 (N+1)} \right)^2. \tag{37}$$

7.3.2 Integration

As Salinger argues, when U_1 and D_1 integrate, U_1 finds it optimal to withdraw from the input market and supply only D_1, who buys the input at a cost c. Hence, $V_1^U = 0$, implying that the upstream loss from vertical integration is $L = V_0^U$.

Now, D_2 buys the input at w, while D_1 buys it at c. In a Nash equilibrium in the downstream market, the output of D_1 is $\frac{A-2c+w}{3}$ and the output of D_2 is $\frac{A-2w+c}{3}$. Since only D_2 buys the input in the upstream market (D_1 is supplied by U_1 at marginal cost), the inverse demand for the input is $w = \frac{A+c-3Q}{2}$.

The profit of each nonintegrated upstream supplier i is given by $q_i (w - c)$. Each upstream supplier i chooses q_i to maximize his profit. The resulting Nash equilibrium output of each upstream firm is

$$q^{**} = \frac{A - c}{3N},$$

and the equilibrium price of the input is

$$w^{**} = \frac{A + c - 3 (N-1) q^*}{2} = \frac{A + (2N-1) c}{2N}.$$

Consequently, the equilibrium profit of D_1 is

$$V_1^D = \left(\frac{A - 2c + w^{**}}{3} \right)^2 = \left(\frac{(2N+1)(A-c)}{6N} \right)^2. \tag{38}$$

Substituting from (37), (36), and (38) into (30) and using the normalizations, we get

$$\begin{aligned} V_1^D - V_0^D - V_0^U &= \left(\frac{(2N+1)(A-c)}{6N} \right)^2 - \left(\frac{N (A-c)}{3 (N+1)} \right)^2 - \frac{2}{3} \left(\frac{A-c}{N+1} \right)^2 \\ &= \left(\frac{A-c}{6N (N+1)} \right)^2 \left(1 + 6N - 11N^2 + 12N^3 \right). \end{aligned} \tag{39}$$

This expression positive for all N so vertical integration is always profitable in the Salinger model.

8 References

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