

Applying Goal-Based Investing Principles to the Retirement Problem

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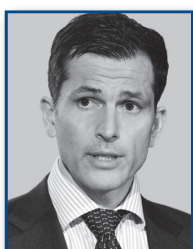
Abstract

In most developed countries, pension systems are being threatened by rising demographic imbalances as well as lower growth in productivity. With the need to supplement public and private retirement benefits via voluntary contributions, individuals are becoming increasingly responsible for their own retirement savings and investment decisions. This global trend poses substantial challenges to individuals, who typically lack the expertise required to make such complex financial decisions. Unfortunately, currently available products such as target date funds or annuities and variable annuities are ill-suited to investors' needs, either because of their lack of focus on securing minimum levels of replacement income in retirement or because of their lack of flexibility and upside potential. In this paper, we propose to apply the principles of goal-based investing to the design of a new generation of retirement goal-based investing strategies, which can be regarded as *risk-controlled* target date funds that strike a balance between safety and performance with respect to the objective of generating replacement income. To provide the investment community with a concrete illustration of these concepts, EDHEC-Risk Institute has teamed up with the Operations Research and Financial Engineering (ORFE) department at Princeton University to launch the EDHEC-Princeton Retirement Goal- Based Investing Index series. The live performance of these indices is published on the EDHEC-Risk and Princeton ORFE websites.

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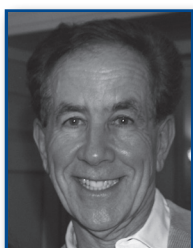
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A major global pension crisis is threatening the two main pillars of pension systems, due to a combination of increasing demographic imbalances and decreasing economic productivity growth. In parallel, defined-benefit arrangements, which used to be dominant among occupational pension schemes, are progressively being closed and replaced by defined-contribution arrangements for new workers. As a result, individuals are increasingly responsible for their own savings and investment decisions.

In most developed countries, pension arrangements are organised on the basis of a three-pillar system. The first pillar, which is key for social coherence, is made of public social security benefits and aims at providing a universal core of pension coverage to address basic consumption needs in retirement through funded public pension systems or unfunded pay-as-you-go systems. Most countries that have opted for a funded system, as is the case in the United Kingdom, are faced with a systemic deficit that is getting worse. The situation is unfortunately no better in countries like France that have adopted an unfunded pay-as-you-go system, the sustainability of which is deeply threatened by rising life expectancy and the impending retirement of baby boomers, as well as low population and productivity growth. The second pillar of pension systems, made of public or private occupational pensions that are expected to provide additional replacement income for retirees, is also weakening. In particular, private pension funds have been strongly impacted by the shift in accounting standards towards the valuation of pension liabilities at market rates, instead of fixed discount rates, which has resulted in increased volatility for pension liabilities. This new constraint has been

reinforced in parallel by stricter solvency requirements following the 2000-2003 pension fund crisis. The evolution of accounting and prudential regulations have subsequently led a large number of corporations to close their defined-benefit pension schemes to new members and increasingly to further their accrual of benefits to reduce the impact of pension liability risk on their balance sheets and income statements. Overall, a massive shift from defined-benefit pension to defined-contribution pension schemes is taking place across the world, implying a transfer of retirement risks from corporations to individuals. As a result of these evolutions, public and private pension schemes deliver replacement income levels that are significantly lower than labor income. According to the *OECD report Pensions at a Glance 2017*, pension replacement rates range from 42.4% to 52.9% in the US and fall to 11.9% for high-earnings individuals in South Africa.

With the need to supplement public and private retirement benefits via voluntary contributions, individuals are becoming increasingly responsible for their own retirement savings and investment decisions within individual retirement accounts, which form the third pillar of pension systems. This global trend poses substantial challenges, not only to individuals, who typically lack the time and expertise required to make such complex financial decisions, but also to policy makers and regulators. In the context of such a massive shift of retirement risk onto individuals, the investment management industry is facing an ever greater responsibility in terms of the need to provide suitable retirement solutions. Unfortunately, available retirement products distributed by asset managers or insurance companies hardly provide a satisfactory solution to

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investors' and households' replacement income needs in retirement.

Insurance companies, asset managers and investment banks offer a variety of so-called retirement products such as annuities and target date funds, but they hardly provide a satisfactory answer to the need for retirement investment solutions. Annuities lack flexibility and have no upside potential, and target date funds have no focus on securing minimum levels of replacement income.

The most natural way to frame an investor's retirement goal is in terms of how much lifetime guaranteed replacement income they can afford at retirement. Given that the biggest risk in retirement is the risk of outliving one's retirement assets, securing replacement income in retirement can be achieved with annuities distributed by insurance companies. Annuities, as well as variable annuities (annuity products that offer participation to the upside of equity markets) suffer from a number of fatal flaws, namely, their cost-inefficiency due to prohibitive costs of capital for insurers offering formal guarantees, their unavailability early on in the accumulation phase, as well as a severe lack of transparency and lack of flexibility, which leaves investors with no exit strategy other than high cost surrender charges.

These elements undoubtedly explain a large part of the "annuity puzzle", which refers to the fact that individual do not invest in annuities unless such an investment is mandatory or strongly incentivised. A good case can actually be made that annuitisation is a decision that is best taken close to retirement, if ever, and that annuities should be used for hedging against late life longevity risk, and not for

providing replacement income in early retirement. Turning to asset management products, life-cycle funds (also known as target date funds), which are often used as the default option in retirement plans, may seem attractive alternatives to annuities due to the fact that these are positioned as one-stop solutions to provide long-term investors with a diversified investment and an allocation strategy that favours wealth accumulation in early years and gradually switches towards safety as retirement date approaches.

Target date funds, however, generally focus on reducing uncertainty over capital value near the retirement date, regardless of the beneficiaries' objectives in terms of replacement income in retirement. The so-called "safe" bond portfolio used in these strategies is actually unsafe when it comes to securing a replacement income because it is not explicitly designed to deliver a stable income during the decumulation period. As a result, they offer no protection to investors with respect to unexpected changes in retirement risk factors. Besides, academic research has shown that the deterministic strategy is suboptimal and that the true optimal strategy should depend on market conditions in addition to the investment horizon.¹

Other products are also proposed by investment banks, such as capital guarantee products, which are considered as a possible default investment option in the legislative proposal for a regulation on a pan-European personal pension product by the European Commission on 29 June 2017. It can be argued that such guaranteed products are not suited to the needs of future retirees because even though capital is protected, the replacement income that it delivers is not known in advance, which does not facilitate retirement planning. Besides, the

1 - See Kim and Omberg (1996), Munk, Sørensen, and Vinther (2004) and Sangvinatsos and Wachter (2005) for examples of optimal strategies exhibiting dependence with respect to market conditions. See Cocco, Gomes, and Maenhout (2005) and Cairns, Blake, and Dowd (2006) for the calculation of utility costs with respect to deterministic policies.

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presence of the formal guarantee implies a strong opportunity cost in terms of upside potential, especially when excessively high distribution costs are factored in.

In retirement investing, the goal is to generate replacement income. The EDHEC-Princeton Goal Price Index series measures the cost of one dollar of replacement income for a fixed period of time in retirement and thus allows investors to find out how much income can be financed with current savings. A truly safe "goal-hedging portfolio" should track the performance of the Goal Price Index. Bonds and cash are not good substitutes for this suitably-designed retirement bond.

The goal-based investing (GBI) paradigm puts investors' goals at the heart of the design of the investment strategy (see Chhabra (2005); Wang et al. (2011); Deguest et al. (2015)). The first step is the identification of a safe "goal-hedging portfolio"(GHP), which effectively and reliably secures an investor's essential goal, regardless of assumptions on parameter values such as risk premia on risky assets. In other words, the GHP should secure the purchasing power of retirement savings in terms of replacement income, an objective that is clearly different from securing the nominal value of retirement savings.

To help investors find how much replacement income can be financed with a given capital, EDHEC-Risk Institute and the Operations Research and Financial Engineering (ORFE) department of Princeton University have partnered to launch the EDHEC-Princeton Retirement Goal Price Index series. Each Goal Price Index is associated with a fixed retirement date, and it gives the price of one dollar per year starting at the retirement

date for a fixed period roughly equal to the average life expectancy in retirement (say 15 or 20 years). Following no-arbitrage pricing principles, each index is simply valued by discounting the one-dollar cash flows at the zero-coupon rates of appropriate maturities. To address the concern over inflation, there exists a version of the indices with a cost-of-living adjustment (COLA). For example, a 2% COLA means that income grows by 2% per year in order to make up for expected inflation. The Goal Price Index series can be used to measure the purchasing power of a given capital in terms of replacement income in a straightforward way. For instance, if the index value is 10, it means that a \$100,000 contribution can finance $\$100,000/10 = \$10,000$ per year for the specified period.

A Goal Price Index can be regarded as the price of a "retirement bond", which starts paying off at the retirement date and pays constant or cost-of-living adjusted cash flows for a fixed period in retirement (e.g. for the first 20 years of retirement). This cash flow schedule is different from the pattern of standard sovereign and corporate bonds, which provide unequal cash flows by delaying capital amortisation (see Exhibit 1). In the absence of these retirement bonds that could be issued by sovereign states or highly rated corporations, the GHP can be synthesised by standard cash flow-matching or duration-matching techniques. It is important to note that assets traditionally regarded as safe, such as short-term or long-term sovereign bonds, are actually highly risky when it comes to securing a stream of income unless they are combined in such a way as to match the duration of the required replacement income cash flows. While they have low standalone volatility (especially for cash), the duration mismatch implies the presence of unrewarded interest rate risk and a large

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tracking error with respect to replacement income cash flows.

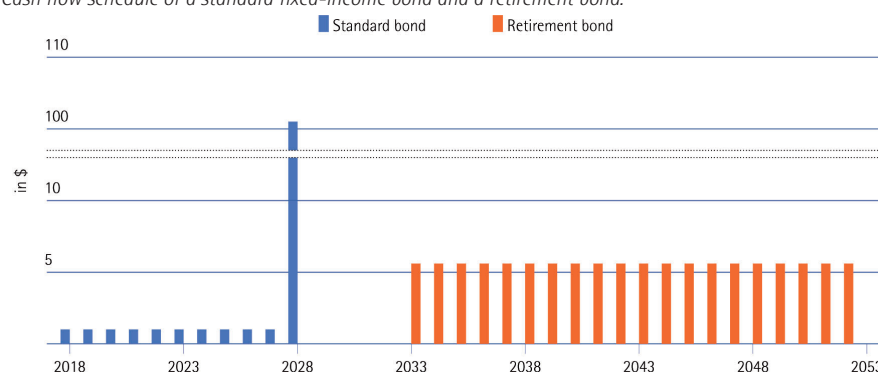
To maximise the probability of reaching a non-affordable target level of replacement income while protecting an essential minimum level, an investor should engage in a dynamic portfolio strategy invested in a performance-seeking portfolio and the suitably-designed goal-hedging portfolio. Such strategies are known as *goal-based investing strategies* since they make explicit use of the information about investor's goals for defining the allocation to the performance and hedging portfolios at each point in time.

A target level of replacement income that the investor would like to reach but is unable to secure given current resources is said to be an *aspirational goal*. On the other hand, an *essential goal* is an affordable level of income that the investor would like to secure with the highest confidence level. In most cases, current savings are insufficient to finance the target income level that allow the desired standard of living to be financed, so the investor needs to have access to upside potential via some performance-seeking portfolio (PSP). Mathematically, the retirement investing problem can be laid

out as follows: maximise the probability of reaching a target level of replacement income in retirement, while securing a minimum level. This problem can be solved by standard probabilistic techniques, but the theoretically optimal approach would not be implementable because it would require unreasonably high levels of leverage as well as continuous trading.

Fortunately, it can be shown that the optimal payoff can be approximated with a simple dynamic (GBI) strategy in which the dollar allocation to the PSP is given by a multiple of the risk budget, defined as the distance between current savings and a floor equal to the present value of the essential goal. This form of strategy is reminiscent of the dynamic core-satellite investment approach of Amenc, Malaise, and Martellini (2004), with the GHP as the core and the PSP as the satellite. It allows the tracking error with respect to the replacement income portfolio to be managed in a non-symmetric way, by capturing part of the upside of the PSP while limiting funding ratio downside risk to a fixed level. From an implementation standpoint, it has the advantage over the probability-maximising strategy that it is only based on observable parameters.

Exhibit 1: Cash flow schedule of a standard fixed-income bond and a retirement bond.



The standard bond has a face value of \$100, a coupon rate of 1% and an annual coupon frequency. It matures in 2028, and the last payment consists of the principal plus the last coupon. The deferred bond starts paying off in 2033 and makes 20 annual payments of \$5.6. The sum of cash flows is the same for both securities.

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In order to achieve the highest success probability, the GBI strategy embeds a stop-gain mechanism, by which all assets are transferred to the GHP on the first date the aspirational goal is hit, that is if and when current wealth becomes sufficiently high to purchase the target level of replacement income cash flows.

To provide a more meaningful retirement investment solution to individuals, a new target date fund strategy can be designed, which uses the GHP as a safe building block and includes a risk control mechanism in order to protect the purchasing power of invested contributions on an annual basis. The EDHEC-Princeton Retirement Goal-Based Investing Index series represents the performance of these strategies that can be regarded as improved forms of "risk-controlled life-cycle funds".

The previously defined GBI strategy allows a well-defined essential goal to be secured, while opening access to the upside performance of the PSP, in the context of a transparent and liquid investment vehicle. This represents an improvement over annuities, which are irreversible and have no upside potential, and over target date funds, which use a mislabelled "safe" building block and neither secure nor explicitly intend to secure any minimum level of income.

In order to take care of periodic, say annual, contributions, the minimum funding ratio level must be secured on an annual basis. Assuming that new contributions are made at the end of each calendar year, we introduce a class of retirement GBI strategies in which the floor is reset at the beginning of every year to protect 80% of the purchasing power of accumulated capital in terms of replacement income

over the following 12-month period. In addition, we take the multiplier to be a decreasing function of time as opposed to being a constant, so that the percentage allocation to the PSP matches the equity allocation of a deterministic target date fund at the beginning of the year, and reflects the desire to benefit from mean reversion in the equity risk premium. For instance, if the target allocation at the 20-year horizon is 80%, the multiplier to be applied within the 40th year to retirement is $80/[100 - 80] = 4$.

With this investment policy, which is rebalanced on a monthly basis, the GBI strategy represents an improvement over standard forms of target date funds, which do not enjoy the benefits of risk management. In the retirement GBI strategy, the safe component is truly safe because it is defined as a GHP determined by the investor's goal and horizon, and the allocation strategy is designed to reliably secure a well-defined essential goal, namely to cap the annual loss of purchasing power in terms of replacement income.

The EDHEC-Princeton Retirement Goal-Based Investing Index series complements the EDHEC-Princeton Retirement Goal Price Index series by representing the performance of dynamic GBI strategies invested in the hedging and performance building blocks. It will also be published on the EDHEC-Risk Institute and Princeton ORFE websites.

GBI strategies deliver attractive probabilities of reaching aspirational goals set by investors, while securing their essential goals. As such they stand in sharp contrast with traditional target date funds, which can experience potentially unbounded losses in adverse market conditions. Our hope

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and ambition is that the publication of the EDHEC-Princeton Retirement Goal-Based Investing Index series will provide incentives for investment management firms to develop and launch welfare-improving forms of retirement investment solutions.

While asset management products are often marketed on the basis of their past track records, historical scenarios are not of direct relevance for investment *solutions*, even though they can be used to perform stress test analysis. The proper evaluation criterion for a retirement investment solution is indeed its ability to secure an essential goal and its potential to reach aspirational goals over multiple scenarios. Monte-Carlo analysis is well suited for this purpose because it can be used to simulate a wide range of possible market environments, and allows for an analysis of the opportunity costs of various essential goals in terms of the probability to reach aspirational goals. Exhibit 2 shows some statistics on the simulated distribution of the *funding ratio*, defined as the ratio of the currently affordable income level to the initially affordable one. A ratio of 100% means that the purchasing power of wealth in terms of replacement income

has not changed since inception, while a ratio greater than 100% means that the portfolio has outperformed the Goal Price Index, thus leading to an improvement in purchasing power. Exhibit 2 shows that the GBI strategy has similar upside potential as a traditional target date fund, as can be seen from the comparable, albeit slightly lower, probabilities of reaching aspirational goals. Chances of success are further improved if a PSP with a higher Sharpe ratio is available in place of the standard cap-weighted index, thus suggesting that improved benchmarks based on so-called smart factor indices (see Amenc, Goltz, and Lodh (2012); Amenc et al. (2014)) are ideally suited as ingredients in retirement solutions.

The main difference between the standard form of a target date fund and its goal-based investing risk-controlled version is in the annual losses in terms of purchasing power, that is relative to the retirement Goal Price Index. For the GBI strategy, these losses are limited at 20% by construction, although the presence of gap risk inherent in dynamic portfolio strategies can on occasion cause mild violations of these targets, violations that do not materialise in this analysis with monthly rebalancing. In contrast, the target date fund can experience losses as large

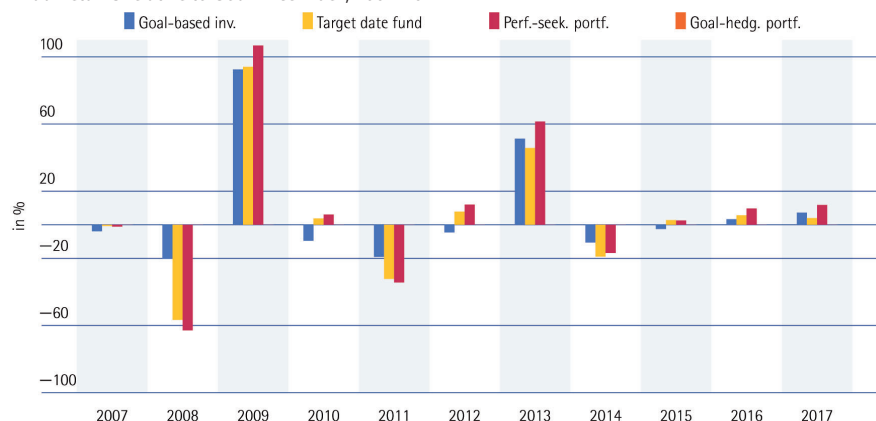
Exhibit 2: Simulation of funding ratio with a target date fund and a goal-based investing strategy.

	Target date fund	GBI strategy	GBI strategy - Improved PSP
Expected funding ratio (%)	225.1	209.9	322.2
Success probability (%)			
130%	89.7	86.2	96.2
150%	81.1	75.4	92.2
200%	56.1	50.2	77.9
Volatility of annual changes (%)	10.9	12.1	13.0
Probability of annual loss > 20% (%)	15.6	0.0	0.0
Worst annual loss (%)	35.6	18.6	18.4

10,000 paths for the target date fund and the goal-based investing strategy are simulated using the stochastic model and the parameter values described in Appendix B. Interest rate parameters are estimated on 1 January 2018. The improved performance-seeking portfolio is simulated by raising the base case Sharpe ratio by 50% (i.e. from 0.39 to 0.59), so that the expected return in excess of the risk-free rate grows from 6.4% to 9.6% per year. The investor starts to accumulate in January 2018 and plans to retire twenty years later, in 2038.

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Exhibit 3: Annual returns relative to Goal Price Index; 2007-2017.



The plot displays annual returns relative to the goal-hedging portfolio, that is annual changes in the level of affordable income. They are zero for the goal-hedging portfolio. The reference investor plans to retire in January 2027 and expects constant (not inflation-adjusted) annual replacement income.

as 35.6% and has a 15.6% probability of experiencing at least one loss larger than 20%. More importantly, the 0% failure rate for the GBI strategy is robust with respect to parametric assumptions (again in the limits of gap risk) while the probability for its deterministic counterpart to maintain losses in purchasing power below 20% on an annual basis is highly parameter-dependent. In robustness checks, we find that raising equity volatility from its base case value of 16.2% to 24.3% implies that the probability jumps from 15.6% to 51.4%.

The lack of robustness of target date fund strategies can also be seen in stress tests conducted over historical periods with strong adverse market conditions. Exhibit 3 shows the annual relative returns of the PSP as well as target date fund and retirement GBI strategies between 2007 and 2017. In 2008 and 2011, the severe underperformance of the PSP with respect to the GHP caused losses in the level of affordable income for both strategies, but such losses are much larger for the target date fund, reaching respectively 56.7% and 32.3%. In these years, the built-in risk control mechanism of the GBI strategy maintains the loss roughly at the target level of 20%.

Taken together, these results suggest that GBI principles can be used to design improved forms of retirement investment strategies that retain some of the desirable features of existing target date funds and annuities, which are, respectively, the ability to generate upside potential in a liquid investment vehicle and the ability to secure minimum levels of replacement income, while avoiding their respective drawbacks. By using the proper GHP and a risk-controlled investment approach, retirement GBI strategies secure a fixed fraction of the purchasing power of each dollar invested without sacrificing upside potential. The publication of the EDHEC-Princeton Retirement Goal-Based Investing Index series on the EDHEC-Risk Institute and Princeton ORFE websites can be regarded as an attempt to provide the investment industry with an incentive to launch new forms of retirement investment solutions better aligned with investors' objectives.

1. Introduction



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Triggered by the introduction of ever stricter accounting and prudential pension fund regulations, a massive shift from defined-benefit (DB) to defined-contribution (DC) pension schemes is taking place across the world. As a result of this trend, individuals have become increasingly responsible for making the right saving and investment decisions that will provide them with sufficient replacement income in retirement. This is a substantial challenge for individuals who not only suffer from behavioural limitations, but also typically lack the expertise needed to make educated investment decisions and must choose between a wealth of investment products presented as “solutions” to their needs. It can be argued, however, that currently available products manufactured and distributed by asset managers or insurance companies fall short of providing satisfactory solutions to future retirees.

On the asset management side, *target date funds* may seem to be an attractive option because they are inherently long-horizon strategies that let young investors benefit from the performance of equity markets, and progressively secure accumulated savings by shifting to assets with lower volatility and lower potential downside risk, such as cash and sovereign bonds. This substitution takes place according to a pre-defined schedule, in which the split between “risky” and “safe” assets is given as a function of an investor’s horizon. One key limitation of this approach is that it aims to reduce uncertainty for investors over their terminal *wealth* as they approach their maturity date, while the objective in retirement investing is instead to generate replacement *income*. Replacement income goals are not equivalent to wealth goals because the price to pay upon retirement to receive one dollar of income every year in retirement is not known in advance. Should

this price double by the time an individual retires, he/she would need to possess twice as much wealth to afford the same level of replacement income in retirement. It therefore makes intuitive sense that a meaningful retirement investment solution should have a focus on securing minimum levels of replacement income, and nothing in the design of target date funds guarantees that they meet this requirement. The deterministic allocation strategy implemented in target date funds also poses a conceptual problem because it ignores changes in market conditions, in particular in volatilities and risk premia of underlying risky asset classes.

Products with a capital guarantee have also been positioned as retirement investment products. The legislative proposal for a regulation on a pan-European personal pension product (PEPP) issued by the European Commission on 29 June 2017 gave some sort of official support to them by suggesting that the default investment option that PEPP providers should offer to PEPP savers should include such a capital guarantee. While it seems intuitively desirable that the default option should aim to preserve capital over time, one key concern is that the introduction of minimum return or capital guarantees would have a number of negative consequences. The most important of these consequences would be an exceedingly large opportunity cost for beneficiaries, given the presence of strict prudential regulations, which make such guarantees prohibitively expensive. In addition to the direct opportunity cost deriving from the introduction of a formal insurance guarantee, as well as the costs implied by the typical distribution channels for such guaranteed products, one may also be concerned by the indirect opportunity costs implied by the use of low-yielding fixed-income instruments in the hedging

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component of the guaranteed products. Moreover, the typical use of single-class liquid underlying instruments such as stock indices for guaranteed products (as opposed to well-diversified multi-asset portfolios) may also contribute to a lack of diversification.

In contrast to asset management products, annuities explicitly seek to deliver a fixed level of replacement income for an individual's life (in which case they are known as "lifetime annuities") or for a pre-determined period (in which case they are known as "period certain annuities"). Some of these annuity products promoted by insurance companies include an adjustment for the cost of living or for realised inflation, to protect the purchasing power of income in terms of goods and services. In theory, they are the perfect asset for investors seeking to secure a stream of replacement income in retirement, but the observed demand for annuities remains low, a fact referred to as the "annuity puzzle". Multiple reasons can explain the low level of annuitisation, including the existence of "pre-annuitised" wealth (Social Security and DB pension plan benefits), adverse selection ruling out groups with higher mortality, and multiple frictions such as minimum investment, irreversibility, etc. Other reasons include the perceived cost-inefficiency of annuities, which are not sold at an actuarially fair price, the fact that they do not contribute to bequest objectives, and the fact that annuitised wealth cannot be recovered in the form of capital even if the beneficiary experiences a severe health problem that would generate large upfront expenses. Besides, an annuity purchase involves giving up on risk premia, which is not an option for most individuals who need upside potential to finance their target consumption needs in retirement.² All in all, a good case can be made that

annuitisation is a decision that is best taken late in the life cycle, if ever.

This paper provides an attempt to contribute to the debate over the desirable characteristics of retirement investment solutions by showing that goal-based investing (GBI) principles can be applied to the design of retirement investment strategies that combine features of target date funds and annuities while circumventing their main shortcomings. The GBI paradigm seeks to compare different investment strategies based on their ability to reach investors' goals, as opposed to using risk and performance metrics such as annual return, volatility, Sharpe ratio or information ratio, which are mostly irrelevant with respect to investors' goals. Because of its explicit focus on goals, the GBI approach aims at providing genuine *solutions* to investors' problems. Formally, Deguest et al. (2015) propose to define GBI strategies as strategies that secure goals qualified as "essential" while offering a high probability of reaching "aspirational" goals. The first step in the framework is to sort goals as "affordable" and "non-affordable", where affordability means that the goal can be secured with the investor's resources. Among the former class, essential goals are those that should be protected at all cost in all market conditions, and non-essential or non-affordable goals form the class of aspirational goals. The second step is the construction of two types of portfolios, namely the goal-hedging portfolio (GHP) designed to hedge against unexpected changes in risk factors that impact the present value of investors' goals, and a performance-seeking portfolio (PSP) intended to most efficiently harvest risk premia across and within asset classes so as to generate the upside potential required for the investor to reach otherwise non-affordable aspirational goals with

² - Variable annuities, which are annuity products that offer participation in the upside of equity markets, suffer from similar flaws, namely, unavailability early on in the accumulation phase, cost-inefficiency due to prohibitive costs of capital for insurers offering formal guarantees, as well as a lack of transparency and lack of flexibility, which leaves investors with no exit strategy, unless at the cost of high surrender charges.

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high probabilities. The third step is the design of an allocation to the hedging and performance building blocks that reliably secures essential goals while maximising the exposure to the outperformance of the performance-seeking component. Overall, this process constitutes disciplined use of the three forms of risk management identified by Amenc et al. (2010), namely *diversification*, *hedging* and *insurance*.

Mass production (in terms of investment products) has happened a long time ago within investment management through the introduction of mutual funds and, more recently, exchange-traded funds. The new frontier for this industry is now to address the mass customisation challenge, which consists in manufacturing and distributing investment solutions at low unit costs, to provide a large number of individuals with meaningful solutions to their problems, and in particular the retirement financing problem. That mass customisation is the key challenge that this industry is facing has long been recognised, but it is only recently that the actual capacity to provide such dedicated investment solutions to individuals has been developed. This point was made very explicitly in Merton (2003): "It is, of course, not new to say that optimal investment policy should not be 'one size fits all'. In current practice, however, there is much more uniformity in advice than is necessary with existing financial thinking and technology. That is, investment managers and advisors have a much richer set of tools available to them than they traditionally use for clients. (...) I see this issue as a tough engineering problem, not one of new science. We know how to approach it in principle (...) but actually doing it is the challenge." In this paper, we leverage on the fact that funding consumption in retirement is a common concern for most individual investors, hence the need to

introduce scalable *mass-customised* forms of retirement solutions, thus alleviating the need to design and implement a dedicated strategy for each individual investor.

Specifically, the goal-based strategies that we introduce provide an answer to the scalability concern by offering the same level of protection to all dollars invested in accumulation, regardless of their arrival date. This allows us to adequately deal with the case of different investors entering the strategy at different points in time, as well as the regular contributions made by a given investor during the accumulation phase. These strategies can be described as a simple and pragmatic risk-managed improvement over deterministic target date funds. Consistent with the above discussion, the building blocks of the strategies are a PSP and a retirement GHP, which replicates the present value of one dollar of replacement income for a fixed period, say 20 years, in retirement. The time-varying allocation to these blocks is taken to be an explicit function of the risk budget, defined as the distance between current wealth and the present value of the essential goal. The aim is to cap the annual loss of the strategy relative to the GHP to a predefined threshold (e.g. 20%). This is equivalent to securing at least 80% of an investor's purchasing power in terms of replacement income. The selection of these building blocks components and allocation policy has a natural justification, since it is rooted in the analysis of strategies that maximise the probability of reaching a target level of replacement income in retirement while securing a minimum level. The GBI solutions introduced in this paper can be regarded as implementable versions of these theoretically optimal portfolio strategies, and as a risk-managed improved form of life-cycle funds.

1. Introduction

To provide the investment community with a concrete and vivid illustration of these concepts, EDHEC-Risk Institute has teamed up with the Operations Research and Financial Engineering (ORFE) department Princeton University to launch the EDHEC-Princeton Retirement Goal-Based Investing Index series. This initiative has been supported by Merrill Lynch in the context of the Risk Allocation Framework for Goals-Driven Investing Strategies research chair at EDHEC-Risk Institute and the indices will be published on the EDHEC-Risk and Princeton ORFE websites.

The rest of the paper is organised as follows. Section 2 provides a big picture analysis of the structural crisis of pension systems in developed countries. In Section 3, we define the goals in retirement planning and make a key distinction between essential goals, which must be secured with a 100% confidence level, and aspirational goals, which cannot be secured but should be reached with a high probability. Section 4 describes the strategy that maximises the probability of reaching an aspirational level of income that is not affordable at the initial date, while protecting a lower level. In Section 5, we introduce an implementable version of this optimal policy with an investment rule that is only based on observable quantities. In Section 6, we propose using GBI principles to design a novel retirement solution that can be regarded as a simple improvement over existing target date funds. Section 7 shows the benefits of the risk-controlled approach over the deterministic glide path approach to target date funds using Monte-Carlo simulations and historical backtests. Section 8 concludes.

1. Introduction

2. The Crisis of Pension Systems



2. The Crisis of Pension Systems

Pension systems around the world are being threatened by rising demographic imbalances as well as lower growth in economic productivity. This section provides an overview of the current situation. Unless otherwise stated, figures are borrowed from OECD report *Pensions at a Glance 2017*. In most developed countries, pension arrangements are organised on the basis of a three-pillar system.

The first pillar, which is key for social coherence, is made of public social security benefits and aims at providing a universal core of pension coverage to address basic consumption needs in retirement. Pillar I systems can be classified either as *unfunded* pay-as-you-go systems, in which current retirement benefits are covered by current contributions from workers, or as *funded* systems, in which current contributions are invested to finance future benefits. The second pillar is made of occupational pension schemes set up by public or private employers. Finally, individual retirement accounts funded by voluntary contributions, form the third pillar of pension systems.

Pillar I: Demographic Imbalances

The sustainability of both funded and pay-as-you-go systems is compromised by demographic imbalances, reflected in the number of individuals aged 65 and over per 100 individuals aged between 20 and 64, a range that corresponds to working ages. In the OECD, this number rose from 13.9 in 1950 to 27.9 in 2015, and is expected to grow to 58.6 by 2075. At the same time, the level of public expenditure on pensions relative to Gross Domestic Product (GDP) is expected to rise, from 8.9% on average in OECD countries between 2013 and 2015 to 10.9% in

2060. As a consequence, unfunded public pension liabilities, obtained by discounting future benefits, are sizable, even though their large sensitivity to discount rates and mortality assumptions makes them difficult to evaluate accurately. For instance, it has been estimated by Citi GPS (2016) that liabilities amount to approximately 350% of GDP in France, 330% in the UK and 110% in the USA.³ In many OECD countries, they represent a bigger fraction of GDP than public debt.

Pillar II: Underfunding Problems and Inadequacy Risk

The second pillar consists of public or private occupational pension plans, which can be DB or DC plans, or involve a mixture of both features, such as in the case of collective DC schemes in the Netherlands for example. The size of assets in private pension plans varies widely across countries, reflecting the mandatory or non-mandatory nature of occupational plans and their relative importance with respect to public systems. In 2016, they amounted to only 9.8% of GDP in France, where Pillar II is under-developed, versus 134.9% in the US, and they reached a maximum across OECD countries of 209.0% in Denmark. But the total size of the private pension plan asset pool is large, at \$38.1 trillion for OECD countries, of which US assets represent 65.9%. Regarding the split between DB and DC plans, the former category still contains a large fraction of invested assets in some countries (e.g. 82% in 2016 in the UK or 40% in the US in 2016 according to Willis Tower Watson (2017)), but there is a clear trend from DB to DC, as can be seen from the evolution in the number of participants. The number of individuals enrolled in DC plans rose faster in the US between 2000 and 2013, and the number

3 - Estimation is based on 1996 data for the US, 2006 data for France and 2010 data for the UK.

2. The Crisis of Pension Systems

of participants in DB plans went down in many countries, including the UK, the Netherlands and Ireland (OECD 2016b). In terms of invested assets, Willis Tower Watson (2017) report an increase in the share of DC plans from 2006 to 2016 for Australia, Canada, Japan, the Netherlands and the US.

Several reasons contribute to explaining this shift. DB plans have been particularly impacted by changes in accounting standards, which recommend that pension liabilities be valued at a market rate, as opposed to a fixed discount rate. This practice gives a better sense of the size of commitments, but it creates volatility in the actuarial value of liabilities, defined as the sum of discounted cash flows, which is reflected in the sponsor companies' income statements and/or balance sheets. In addition, the historically low interest rates that have prevailed since the 2008 downturn inflate this value and depress funding ratios. At the same time, DB plans have also been subject to increasingly strict prudential regulations that force them to comply with minimum funding constraints intended to protect beneficiaries. In the US, these rules are expressed in the Employee Retirement Income Security Act of 1974, which states that DB plans must be fully funded. But in several countries, accumulated assets are insufficient to cover liabilities, and the deficit is sometimes severe. In 2016, the average funding ratio was 60% in Iceland, close to 70% in the US, 90% in the UK and slightly less than 100% in Canada. As a general rule, DB plans leave employers to bear the risk of adverse market events like the financial crises of 2000-2002 and 2008-2009, and the risk of unexpected increases in longevity.

4 - These numbers are net replacement rates, which account for the differential tax treatment of wages and pension benefits.

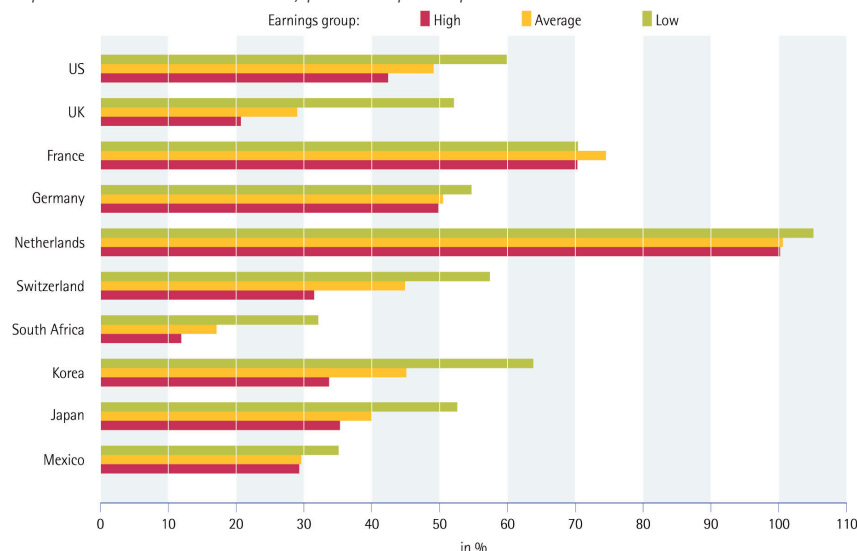
With the rise of DC plans, these risks are increasingly transferred to individuals who, in addition, face *inadequacy* risk, that is the risk of insufficient replacement income given that benefits are not predictable. As noted by the OECD (2016b), this risk is exacerbated by the fact that contributions tend to be lower in DC than in DB arrangements. Typical contribution rates are greater than 20% of wages in public or private DB schemes (e.g. 21.3% in France and 20.9% in the Netherlands in 2014; see OECD (2015)), and are substantially lower in DC schemes (e.g. 9.5% in Australia). Inadequacy risk materialises in the pension replacement rate, defined as the ratio of benefits from mandatory public and private arrangements to labour income. This rate is generally decreasing in the income level, reflecting the redistributive nature of many systems, and it ranges from 42.4% to 59.9% in the US, from 20.7% to 52.1% in the UK, and from 32.4.7% to 70.0% in Ireland (see examples in Figure 1).⁴ Denmark has the highest rates, from 76.2% to 110.3%, and South Africa is among the lowest, with rates lower than 35%. These numbers shows that individuals are likely to experience a severe decrease in their income when retiring unless they engage in voluntary savings plans.

Pillar III: Inadequacy Risk Again and Inappropriate Investment Decisions

The third pillar of the pension system is made of voluntary savings accounts, which are typically invested in asset management products like target date funds or in insurance products like annuities. The third pillar shares many of the risks of the second pillar. In particular, contributions may be too low to build a big enough nest egg and generate adequate replacement income in retirement. Taking into account

2. The Crisis of Pension Systems

Figure 1: Net replacement rates from mandatory public and private pension schemes.



Statistics are reproduced from Table 4.11 in the OECD report *Pensions at a Glance 2017* and are as of 2016. The net replacement rate is the ratio of pension benefits in retirement to labour earnings, for an individual earning 0.5 times the average worker earnings or less ("low earners"), between 0.5 and 1.5 times the average ("average earners") and more than 1.5 times the average ("high earners"). Detailed methodology, with assumptions on inflation, earnings growth rate, rate of return on assets in funded pension schemes, loading factor for annuities, discount rates, longevity and tax rates, is presented in p. 98-99 of the OECD report. In the US, UK, Germany, South Africa and Japan, a wide fraction of the population is covered by voluntary private pension arrangements, so the total replacement rate, which aggregates mandatory and voluntary sources, is higher than the one shown in this diagram.

voluntary plans improves replacement rates, sometimes significantly. In the US, the average worker can expect 49.1% of labour income after retiring without including voluntary savings, and 87.1% including all sources. The difference is also spectacular for South Africa, with a replacement rate growing from 17.1% to 52.0%. However, percentages remain low in many countries (e.g. 62.2% for an individual earning average salary in the UK, 65.4% in Germany, 44.9% in Switzerland and 42.6% in Australia). By making assumptions on retirement income needs, longevity and returns on savings, it is possible to calculate the gap between what savings can effectively finance and the replacement income needs. The consulting firm Mercer thus estimated that the gap was \$8 trillion in 2015 in the UK and would grow to \$33 trillion by 2050 (World Economic Forum 2017).

The amount of contributions is one of the two factors that determine replacement income, the other being the returns on investment. A key challenge is that most individuals do not have the financial education needed to make adequate choices when it comes to deciding how much to save and how to invest. The *OECD Pensions Outlook 2016* discusses these issues at length, and emphasises that basic financial concepts such as interest compounding are not well understood by a large audience, while individuals have a tendency to delay important decisions when benefits are far ahead in the future, and to adhere to default options when available, without actively comparing to other products. Annuitisation, which is in principle the safest approach to secure a lifetime income, is often declined for psychological reasons but also because of frictions on the annuity market. Making annuity purchases compulsory forces individuals to get past their reluctance,

2. The Crisis of Pension Systems

but raises other problems. When forced annuitisation was abolished in April 2014 in the UK, the Financial Conduct Authority pointed that competition between providers was limited and that about 60% of individuals did not take advantage of the opportunity to shop around, ending up with poor returns on their investment.

In its 2012 *Roadmap for the Good Design of Defined Contribution Pension Plans* and the *Pensions Outlook 2016*, the OECD also issues a number of policy messages regarding the design of DC pension plans and annuity products, and calls for better financial education, so as to improve the adequacy of retirement income. The recommendations include, among others (OECD (2016b), p. 33-4):

- "Ensure the design of DC pension plans is internally coherent between the accumulation and pay-out phases and with the overall pension system";
- "Encourage people to enrol, to contribute and contribute for long periods";
- "Promote low-cost retirement savings instruments";
- "Promote the supply of annuities and cost-efficient competition in the annuity market".

Some of these prescriptions apply specifically to annuities, while the others refer to DC plans and re-affirm the principles stated in the *OECD Core Principles of Private Pension Regulation* (OECD 2016a). They emphasise that the objective is to generate retirement income, and that investment strategies should be aligned with this objective:

- "Risk management concepts, such as diversification and asset-liability matching, should be appropriately employed in order to achieve the best outcome for the plan members and

beneficiaries" (Guidelines 4.1 in OECD (2016a), p. 33);

- "A sound investment risk management process that supports the achievement of the investment objectives should be established" (Guidelines 4.9, p. 35).

It is striking that current forms of target date funds, often used as the default option in DC plans and individual retirement accounts, do not comply with all these principles, which should apply equally to voluntary savings plans of Pillar III and to mandatory arrangements of Pillar II.

2. The Crisis of Pension Systems

3. Goals in Retirement Investing



3. Goals in Retirement Investing

The main concern for future retirees is to generate replacement income in retirement. In general, the level of income that can be achieved in decumulation is not known until retirement for two main reasons. First, the capital reached at the retirement date is uncertain because it depends on the performance of investments made by the individual during accumulation, and second, the income to which each dollar of savings entitles him/her is unknown until the capital is turned into income, by purchasing an annuity or a cash flow-matching bond portfolio. This section introduces a formal definition for replacement income goals and for the notion of affordability, which serves to qualify the attainability of a goal.

3.1 Definition of Goals

This paper focuses on the accumulation phase, during which the individual saves money to finance expenses in retirement, when he/she no longer receives labour income. By convention, the beginning of the investment period is referred to as date 0 and the retirement date is denoted by date T , where T is an integer number of years. We assume that the broad investor's objective, or goal, is defined as a level of (annual) replacement income for a fixed period of time after retirement date. Consumption needs in late retirement can be accommodated via the purchase of deferred annuities upon retirement in order to gain protection against tail longevity risk. The fixed period, over which retirement income is expected, should roughly coincide with the life expectancy of a recently retired individual. Assuming a retirement age of 65, the life expectancy in the US is 19.4 years according to the November 2017 issue of the National Vital Statistics Reports.⁵ In view of this figure, we assume a 20-year decumulation

period in the remainder of this paper, as well as in the construction of the EDHEC-Princeton Goal-Based Investing Indices.

Given the length of the horizon typically involved in a retirement planning problem, inflation risk is a concern because the purchasing power of a given level of annual income at a remote point in the future is likely lower than what it is today. For this reason, it is often desirable to express the goal in terms of constant dollars, as opposed to current dollars. There are two ways to provide protection of future cash flows in terms of purchasing power. The first approach consists of indexing replacement income cash flows to realised inflation, and the second is to index them to a fixed expected inflation level (e.g. 1% or 2% per year. The latter case is by far the most frequent in the annuity industry, where it is referred to as a cost-of-living adjustment (COLA).

3.2 Qualifying Affordability

Broadly speaking, a goal is said to be affordable if there exists an investment strategy that makes it attainable with 100% probability given available resources. To find whether a goal is affordable or not, we need to know how much capital is needed to secure it, or conversely how much income can be financed with the current capital. In other words, we need a method to translate wealth into income, or income into wealth.

Let ri represent a replacement income level. The principle of absence of arbitrage opportunities says that the minimum contribution to invest in order to secure a stream of cash flows equal to ri for τ years is the sum of the discounted cash flows. Obviously, the answer is proportional to ri , so we can focus with no loss of generality

5 – See Table 3 in National Vital Statistics Reports (2017). This number is the life expectancy for a generic US individual assuming that no other characteristic is observed that is known to impact life expectancy. Examples of such characteristics are gender and social group.

3. Goals in Retirement Investing

on the case where ri is \$1, and multiply the capital by ri to find how much is needed for an arbitrary goal ri . Discount rates vary over time, so the capital needed to secure \$1 of replacement income over a given period depends on the date t at which it is evaluated. Mathematically, the minimum capital needed to secure a replacement income of \$1 starting at the retirement date and lasting τ years is

$$\beta_t = \sum_{s=T}^{T+\tau-1} D(t, s),$$

where $D(t, s)$ is the discount factor at date t for cash flows occurring at date s . β_t is the present value of \$1 of income.

When income is fixed in current dollars, $D(t, s)$ is the price of a pure discount bond with a unit face value that matures at date s . If it is fixed in constant dollars, $D(t, s)$ is the price of an inflation-indexed pure discount bond if income is indexed on realised inflation, or the price of the nominal bond adjusted for the growth rate if income is tied to a constant COLA. The discount factor $D(t, s)$ can be obtained from a zero-coupon curve, which is itself constructed from coupon-paying bond prices through standard bootstrapping techniques. Figures 2 and 3 show examples with curves downloaded from the Federal Reserve website on 2 January 2018.⁶ For an investor retiring in January 2038, the prices of receiving \$1 per year for 20 years estimated on 2 January 2018 are respectively $\beta = 8.68$ for constant cash flows, and $\beta = 16.2$ for cash flows indexed on realised inflation.⁷

Knowing the price of \$1 of income, the *maximum affordable income* with a capital W is W/β_t and conversely, the minimum capital needed to finance an income stream ri is $ri \times \beta_t$. Hence, the

quantity β_t allows us to translate back and forth the amount of savings and the level of replacement income, since the level of affordable income measures the purchasing power of a given capital in terms of replacement income. Therefore, β_t plays an important role in goal-based reporting for individual investors, where the level of income that can be financed with current savings is a more relevant piece of information than the nominal value of savings. In the remainder of this paper, we will argue that it is also useful in implementing risk management principles for goal-based investing. Given its importance in the retirement problem, we call β_t it a *Goal Price Index*, and EDHEC-Risk Institute and Princeton ORFE will publish its value on a regular basis on their respective websites.

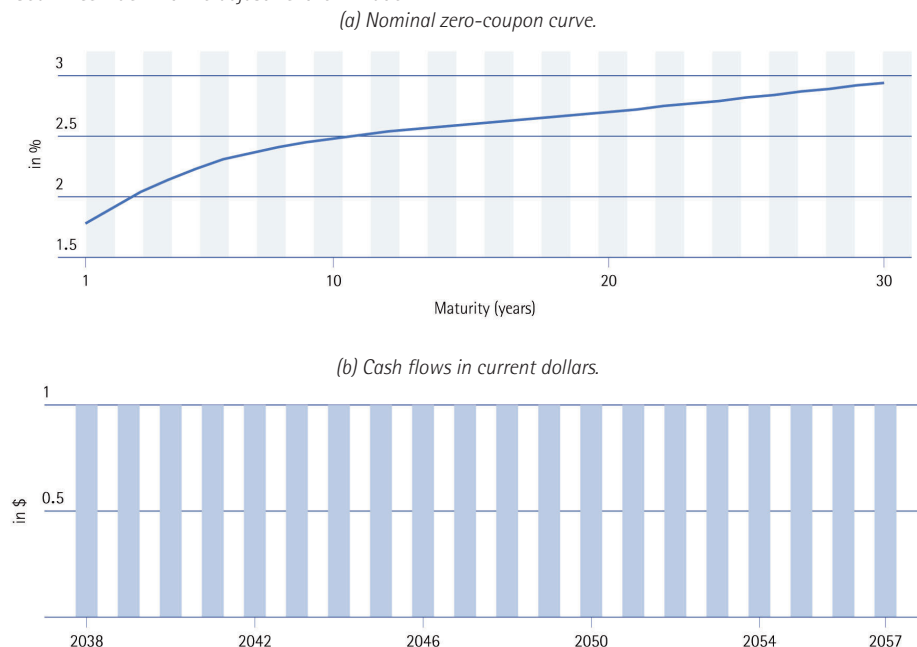
The value of the Goal Price Index is determined by three subjective characteristics: the retirement date (T), the decumulation period (τ) and the indexation mode (no indexation, COLA or realised inflation). Once these variables are set, it is a function of the discount rates, which are objective parameters. It is therefore exposed to the same risk factors as the nominal or the real yield curve.

It should be noted that the affordability calculation only takes into account the current savings of an individual, as opposed to including contributions expected to be made in the future which will eventually increase affordable income levels. Indeed, future contributions are hypothetical in nature, so they cannot be regarded as granted. The distinction between the actual purchasing power of current savings and the "virtual" purchasing power also including expected future contributions is analogous to the distinction between "accumulated

6 - Data is available at <https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html> and <http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html>, and is constructed based on the methodology of Gürkaynak, Sack, and Wright (2007, 2010).
7 - We assume continuously compounded rates in the calculation of the Goal Price Index.

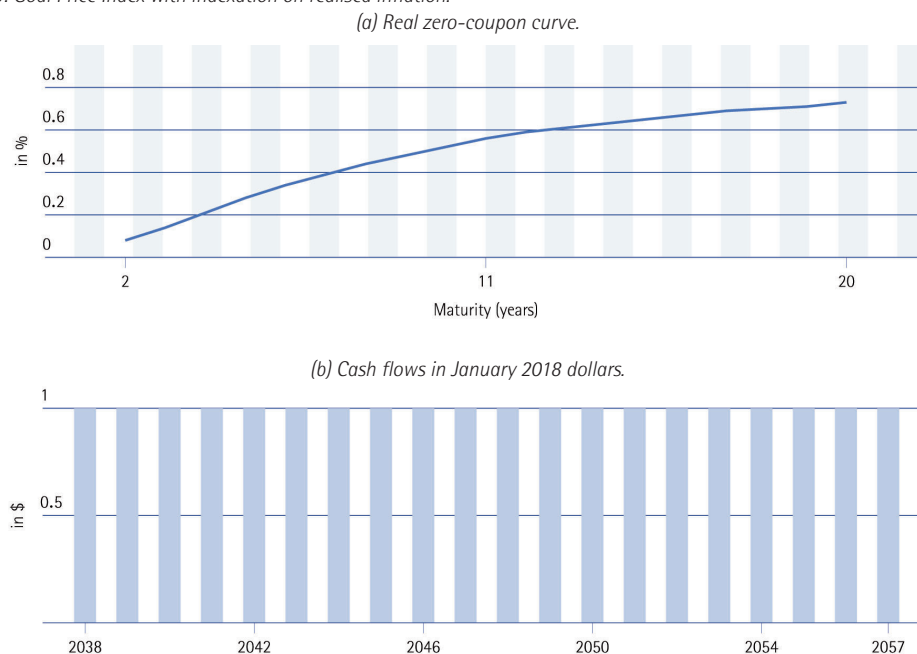
3. Goals in Retirement Investing

Figure 2: Goal Price Index with no adjustment for inflation.



Nominal zero-coupon rates of maturities 1, 2, ..., 30 years on 2 January 2018 are obtained from the Federal Reserve website. Cash flows are fixed in current dollars.

Figure 3: Goal Price Index with indexation on realised inflation.



Real zero-coupon rates of maturities 2, 3, ..., 20 years on 2 January 2018 are obtained from the Federal Reserve website. Cash flows are fixed in constant dollars (2018 dollars)

3. Goals in Retirement Investing

benefit obligation" and "projected benefit obligation" in pension fund management.

3.3 Securing a Goal

To secure a replacement income of \$1 from date T to date $T + \tau - 1$, an individual needs to invest in a fixed-income security that pays \$1 every year for τ years starting from the retirement year. In a series of recent articles, Muralidhar (December 2015), Muralidhar, Ohashi, and Shin (2016) and Merton and Muralidhar (2017) argue that sovereign states could issue such bonds, which are called "RB" (Retirement Bonds), "BFFS" (Bonds for Financial Security), "FBS" (Forward-Starting Bonds) or "SeLFIES" (Standard of Living indexed, Forward-starting, Income-only Securities), and make them available to investors in accumulation. Before they eventually become available, these retirement bonds can be replicated as a basket of unit pure discount bonds paying \$1 (in current or constant dollars) per year from date T to date $T + \tau - 1$. In practice, the bond ladder is often synthesised in the accumulation phase by duration-matching (or duration-convexity matching) techniques instead of cash flow-matching. A bond portfolio that tracks the present value of replacement income is a *goal-hedging portfolio* (GHP), and it can be regarded as the equivalent of the liability-hedging portfolio of institutional investors in an individual money management context.

Figure 4 illustrates the difference between the cash flows of a standard fixed-income bond (e.g. a sovereign bond) and a "retirement bond". The standard bond starts paying coupons to the holder right after it is delivered, and principal is only redeemed at maturity. This results in unequal cash flows that do not satisfy the need for stable income. In contrast, the

retirement bond spreads interest payment and capital amortisation over time in the same way as fixed-rate housing mortgages, so that the holder receives constant cash flows.

Assets that are traditionally regarded as "safe" because of their low volatility from an absolute return perspective are actually not safe relative to the Goal Price Index. Figures 5 and 6 illustrate the difference between absolute risk and relative risk, which is well known in asset-liability management. Cash delivers consistently low but rather stable returns from one month to the next. Bonds with long maturities exhibit high exposure to interest rate changes, which translates into higher levels of short-term volatility, and the retirement bond, or GHP, appears to be the most volatile instrument, since it has even longer duration.

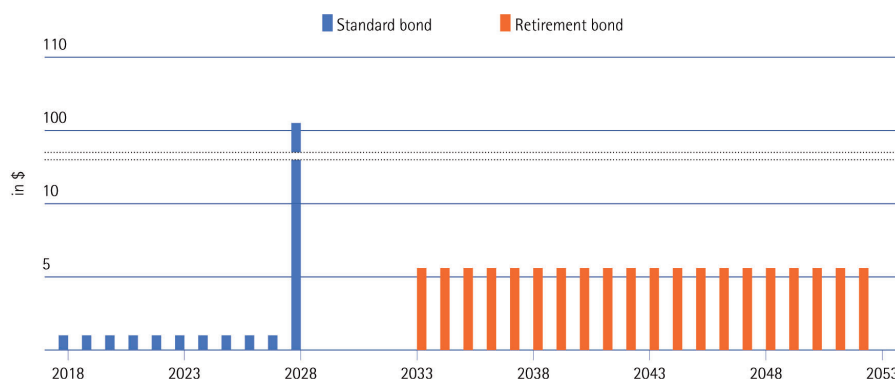
The picture is very different, however, if one focuses on the evolution of affordable income or *funding ratio*. The *funding ratio* in the retirement saving problem can be defined as follows. Consider an investor starting in January 2007 with an initial wealth W_0 and planning to retire in January 2017. The initially affordable income is W_0/β_0 , where β_0 is the Goal Price Index in 2017. At date t , his/her wealth becomes W_t and the index is β_t , so the new affordable income is W_t/β_t . The funding ratio measures the fraction of the initially affordable income that the investor has been able to preserve, or the multiple that he/she has reached. It is the ratio of the affordable income of date t to the affordable income of date 0:

$$R_t = \frac{W_t}{\beta_t} \times \frac{\beta_0}{W_0}.$$

By construction, the GHP implies no relative risk and a constant funding

3. Goals in Retirement Investing

Figure 4: Cash flow schedule of a standard fixed-income bond and a retirement bond.



The standard bond has a face value of \$100, a coupon rate of 1% and an annual coupon frequency. It matures in 2028, and the last payment consists of the principal plus the last coupon. The retirement bond starts paying off in 2033 and makes 20 annual payments of \$5.6. The sum of cash flows is the same for both securities.

ratio. As appears from Figure 6, the cash account and the bond index, on the other hand, leave the investor with a substantial amount of short-term volatility in the funding ratio. The key insight from this analysis is that the GHP is the true safe asset with respect to investor's goals, which are defined in terms of replacement income in retirement.

3.4 Aspirational and Essential Goals

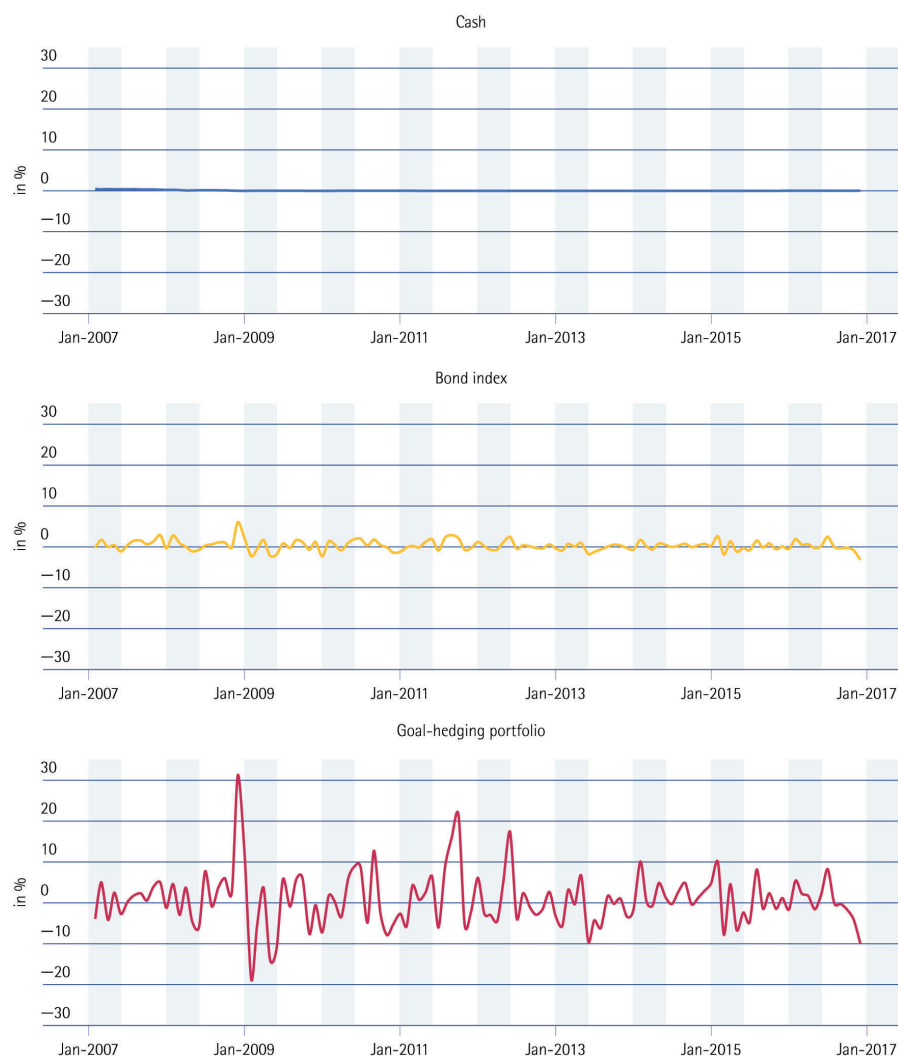
The Goal Price Index allows an investor to answer the following question: how much replacement income can be financed given my current retirement savings? The other question that the investor should ask is: how much replacement income do I need? It is his/her responsibility to estimate the latter quantity, by taking into account how much he/she will receive from Social Security (Pillar I) and employer-sponsored plans (Pillar II), and how much he/she expects to need in the future in order to achieve a target standard of living. In the US, the Social Security website (<https://www.ssa.gov/>) provides individuals with online calculators and guides to estimate benefits and expenses. If expenses exceed attainable replacement income, individuals are left with a gap

to be financed via voluntary savings in individual retirement accounts.

Should the currently affordable income be greater than the desired income, the investor would be able to secure the income objective by investing in the GHP, but more often than not the target level of replacement income is unaffordable. Following the terminology of Deguest et al. (2015), we call this unaffordable target level of replacement income an *aspirational goal*. By the very definition of non-affordability, no strategy can guarantee that this goal will be reached with a 100% probability, at least if this probability is required to be robust to model and parameter assumptions. In the best case, the individual will have access to a strategy that reaches the goal with a "high" probability. Such a strategy cannot be based on the GHP alone, since investing savings in the GHP implies a constant level of affordable income and cannot generate the upside potential required to eventually reach the aspirational goal with a positive probability. One therefore also has to invest in performance-seeking assets in order to increase the chances to reach the goal. Maximising the odds of achieving an aspirational goal is the

3. Goals in Retirement Investing

Figure 5: Absolute risk: Monthly returns to asset classes.

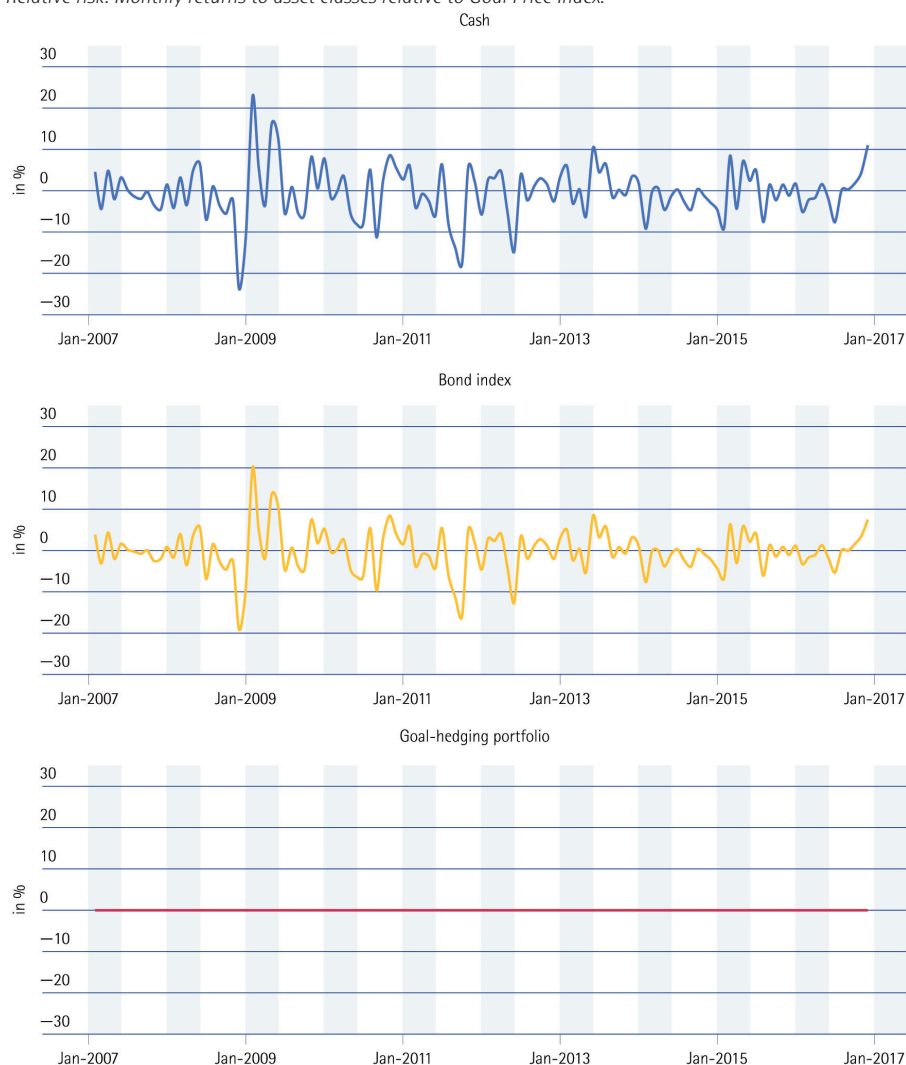


The cash account earns the continuously compounded interest rate on three-month Treasury bills (from the Federal Reserve), and the bond index is the Barclays US Treasury index. The goal-hedging portfolio (GHP) replicates the performance of the Goal Price Index for an individual with a 20-year decumulation period retiring in January 2017. Returns are monthly arithmetic returns.

main objective of the GBI strategies that we introduce in this paper, subject to the constraint that the strategy also needs to secure an essential goal, which must be met with a robust 100% confidence level (implying that it is affordable at the initial date) and which is typically defined as a minimum level of replacement income to secure over a short or a long horizon.

3. Goals in Retirement Investing

Figure 6: Relative risk: Monthly returns to asset classes relative to Goal Price Index.



The cash account earns the continuously compounded interest rate on three-month Treasury bills (from the Federal Reserve), and the bond index is the Barclays US Treasury index. The goal-hedging portfolio (GHP) replicates the performance of the Goal Price Index for an individual with a 20-year decumulation period retiring in January 2017. The figure displays the monthly relative returns, that is the returns of the funding ratios.

4. Maximising the Probability of Reaching a Target Level of Replacement Income



4. Maximising the Probability of Reaching a Target Level of Replacement Income

An investor endowed with an aspirational goal and an essential goal is faced with the following portfolio choice problem: find a strategy that delivers the highest *ex-ante* probability of reaching the target while securing the essential goal in a reliable way. By "reliable", it is understood that the protection is offered in all scenarios and does not hinge on specific parameter assumptions. This problem has a straightforward mathematical expression: maximise the probability of reaching the target level of replacement income (aspirational goal) while securing the minimum level of replacement income (essential goal). This section describes the mathematical solution to this optimisation program (with the technical details relegated to Appendix A), and the translation of the theoretically optimal strategy into a version that is more compatible with implementation constraints.

8 - See also Munk and Sørensen (2010) and Deguest, Martellini, and Milhau (2015) for examples of optimal strategies with the short position in the contribution-hedging portfolio under another optimisation criterion (expected utility maximisation).

4.1 The Portfolio Choice Problem

The investor starts accumulating money at date 0, with an initial wealth W_0 that can be invested in various financial assets until retirement date T . The value of retirement savings at a given point in time, W_t , depends on the investment decisions made up to date t and on how well the portfolio has performed so far. It also depends on contributions made prior to date t , but in order to simplify the discussion, we assume in this section that the portfolio is self-financing. Martellini and Milhau (2016) provide a more general analysis that allows for contributions down the road to retirement. In the setting of their paper, one has to reason in terms of "total wealth", equal to the sum of financial wealth plus the present value of future contributions, which gives rise to a short position in a "contribution-hedging

portfolio" in the optimal strategy to make up for the implicit long position implied by the contributions.⁸ This feature is the only difference between the probability-maximising strategies with and without intermediate contributions.

The investor is endowed with an aspirational goal expressed as a replacement income level ri_{asp} . The Goal Price Index determined by the decumulation period, the retirement date and the indexation mode is denoted by β_t . Among the various available assets is a GHP, a retirement bond assumed to perfectly replicate the performance of the Goal Price Index. In fact, for the purpose of solving for the optimal portfolio, it is convenient to assume that the market is *complete*, so that all relevant sources of risk can be hedged with existing assets or with a (possibly dynamic) trading strategy in these assets.

Over a period as long as a typical accumulation phase, the restriction to buy-and-hold strategies would be unrealistic, so investors are allowed to dynamically rebalance their portfolio. For the purpose of computing an optimal investment policy, we take this assumption one step further by allowing for *continuous* rebalancing, which is a standard assumption in Mertonian portfolio choice models (Merton 1969).

The replacement income objective can be translated into a wealth objective with a random level. Indeed, the minimum wealth needed at date t to secure the objective is $ri_{asp}\beta_t$, so the objective of being able to secure ri_{asp} is equivalent to the objective of reaching the wealth level $ri_{asp}\beta_t$ and because β_t is time-varying, this is a random target wealth. Similarly, the long-term essential goal translates

4. Maximising the Probability of Reaching a Target Level of Replacement Income

into a stochastic floor that wealth should respect with a 100% probability upon retirement. This floor is equal to $ri_{ess}\beta_t$, where ri_{ess} is the essential income level. With these notations, the problem can be stated as follows: maximise the probability of reaching the target wealth $ri_{asp}\beta_t$ at date T while respecting the floor $ri_{ess}\beta_t$.

4.2 The Optimal Payoff

Browne (1999) and Föllmer and Leukert (1999) solve a related optimisation problem. The solution is derived Appendix A, where we extend their results to a setting with both a random level of target wealth and stochastic investment opportunities (i.e. with conditional risk premia, volatilities and correlations that can randomly vary over time). The solution method is the “duality technique” approach that was introduced by Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989) to solve expected utility maximisation problems, but can also be used for other types of objectives. The first step is to find an investor-specific optimal payoff, consistent with the horizon and the budget constraints, and the second step is the derivation of the strategy that replicates the payoff.

The optimal payoff is shown to be of the “binary” type, with two possible outcomes: either it is equal to $ri_{ess}\beta_t$ or to $ri_{asp}\beta_t$, so the individual gets either the essential level of income, or the aspirational level. The separation between these two cases depends on the performance of the *growth-optimal portfolio*, which is the portfolio that maximises the expected logarithmic return on wealth at horizon T . This portfolio consists of two building blocks: the locally risk-free asset – a cash account in which funds are rolled over at the money market rate – and the

maximum Sharpe ratio (MSR) portfolio of the risky assets. Since our model allows for stochastic expected returns and volatilities, the composition of the MSR portfolio is time-varying, and so are its conditional Sharpe ratio and volatility. The allocation rule in the growth-optimal strategy expressed in terms of fractions of wealth can be stated as follows:

At date t , invest

$$\begin{cases} \lambda_{MSR,t}/\sigma_{MSR,t} & \text{in the MSR portfolio} \\ 1 - \lambda_{MSR,t}/\sigma_{MSR,t} & \text{in the cash account} \end{cases}.$$

If W_T^{go} denotes the final wealth generated by the growth-optimal portfolio starting from capital W_0 , then the probability-maximising payoff is

$$W^* = \begin{cases} ri_{asp}\beta_T & \text{if } \frac{W_T^{go}}{\beta_T} \geq h \frac{W_0}{\beta_0}, \\ ri_{ess}\beta_T & \text{otherwise} \end{cases}, \quad (4.1)$$

h being a positive constant. The value of h must be calculated numerically from the budget constraint, which says that the expected discounted value of W^* must equal the initial wealth.

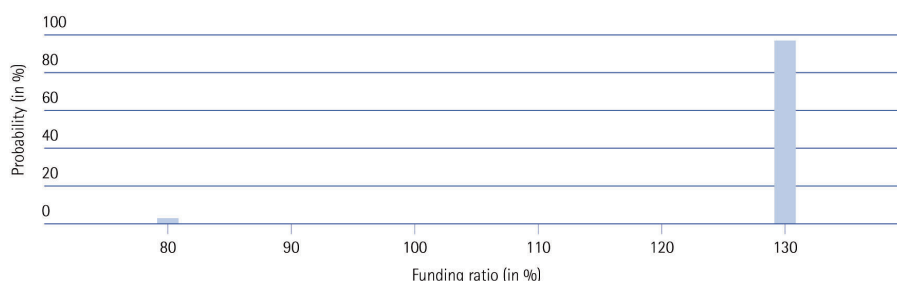
In words, Equation (4.1) says that the aspirational goal is reached in those states of the world where the growth-optimal portfolio performs sufficiently well with respect to the Goal Price Index, in the sense that the ratio of the gross portfolio performance over the gross index return is at least equal to h . It is shown in Appendix A that for a given Goal Price Index, h is decreasing in the ratio

$$\frac{W_0 - ri_{ess}\beta_0}{[ri_{asp} - ri_{ess}]\beta_0}.$$

This property makes intuitive sense: a lower initial wealth or a more ambitious aspirational goal imply a lower h , hence

4. Maximising the Probability of Reaching a Target Level of Replacement Income

Figure 7: Distribution of probability-maximising funding ratio at retirement date.



The probability-maximising strategy is simulated for an investor retiring in January 2038 conditional on parameter values of 2 January 2018. The essential goal is to secure 80% of the initially affordable replacement income in all states of the world, and the aspirational goal is to reach a replacement income equal to 130% of the initially affordable level. Parameter values for the stochastic processes are given in the text.

a narrower success region and a lower success probability.

The distribution of the optimal terminal funding ratio is shown in Figure 7. The investment universe consists of a stock index, a bond index, the GHP and a cash account. The stock index aims to represent a broad US stock index (the S&P 500 index), with a volatility of 16.2% and a Sharpe ratio of 0.395, and the bond index is representative of a Government bond index (the BofA ML AAA US Treasury/Agency Master), with a volatility of 6.4% and a Sharpe ratio of 0.234.⁹ We set the correlation between the stock and the short-term rate to the neutral value of zero, and the bond-rate correlation is estimated as the correlation between the monthly returns to the BofA ML AAA US Treasury/Agency Master and the monthly changes in the secondary market rate on 3-month Treasury bills, which turns out to be -0.588 for the period from April 1978 to May 2017. The short-term interest rate is simulated with the Vasicek model (Vasicek 1977), with parameters estimated by a mixture of historical estimation and calibration to the yield curve of 2 January 2018. Appendix B describes the procedure in detail.

Because the essential goal is affordable, it can be defined as a fraction of the initially

affordable income, which is set to 80% for illustration purposes in what follows. Similarly, the aspirational goal is defined as a multiple, say equal to 130%, of the initial level of income. As a result, the funding ratio at retirement date under the probability-maximising strategy can take on the values of 80% and 130%, a bimodal distribution that is displayed on Figure 8. With the assumed parameter values, the probability of reaching the aspirational goal is 97.0%.

4.3 Optimal Strategy in a “Black-Scholes” Framework

Once the probability-maximising payoff has been derived, finding a strategy that replicates the payoff boils down to a standard dynamic option hedging/replication problem. The seminal work of Black and Scholes (1973) and Merton (1974) has shown that under certain conditions, holding a European call option is equivalent to executing a dynamic trading strategy in the underlying asset and the cash account, in which the number of shares of the underlying asset to be held is equal to the option delta. This equivalence relies on the assumptions that continuous trading is possible and that the volatility of the underlying is constant. In our model, the payoff to replicate is of the binary type,

9 - These parameter values have been obtained from Merrill Lynch's Capital Market Assumptions for 2017.

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and the underlying of the binary option is the value of the growth-optimal portfolio expressed in a numeraire proportional to the Goal Price Index. In order to derive an explicit expression for the dynamic replication strategy, we need to assume that the volatility of the growth-optimal portfolio relative to the Goal Price Index is constant. It is shown in Appendix A that this condition is satisfied if the following conditions are met: the conditional Sharpe ratio of the MSR portfolio is a constant λ_{MSR} , and the conditional volatility and the conditional Sharpe ratio of the GHP are constant σ_β and λ_β .

Under these assumptions, it is possible to calculate the binary option price in closed form and to derive the hedging strategy, which is the probability-maximising policy. As shown in Appendix A, the strategy can be seen as a time-varying allocation to two building blocks, namely the growth-optimal portfolio and the GHP. Because the former block is itself a combination of the MSR performance-seeking portfolio (PSP) and the cash account, one can equivalently describe the strategy as an investment in the MSR portfolio, the GHP and the cash account. The weights allocated to these blocks depend on the distance between the current level of affordable income and the minimum and target levels. In details, the investment rule can be expressed as follows:

At date t , invest

$$\begin{cases} \varphi_t \lambda_{MSR} / \sigma_{MSR,t} & \text{in the MSR portfolio} \\ 1 - \varphi_t & \text{in the GHP} \\ \varphi_t [1 - \lambda_{MSR} / \sigma_{MSR,t}] & \text{in the cash account} \end{cases}.$$

The quantity φ_t is given by

$$\varphi_t = \frac{r_{i_{asp}} - r_{i_{ess}}}{\eta_t r_{i_{aff},t}} n \left[\mathcal{N}^{-1} \left(\frac{r_{i_{aff},t} - r_{i_{ess}}}{r_{i_{asp}} - r_{i_{ess}}} \right) \right], \quad (4.2)$$

where n and \mathcal{N} are respectively the probability density function and the cumulative distribution function of the Gaussian distribution, \mathcal{N}^{-1} is the reciprocal function of \mathcal{N} , $r_{i_{aff},t} = W_t / \beta_t$ denotes the level of income that can be financed with current wealth, and η_t is the standard deviation of $\log [W_t^{\theta_0} / \beta_t]$ conditional on information set of date t , that is

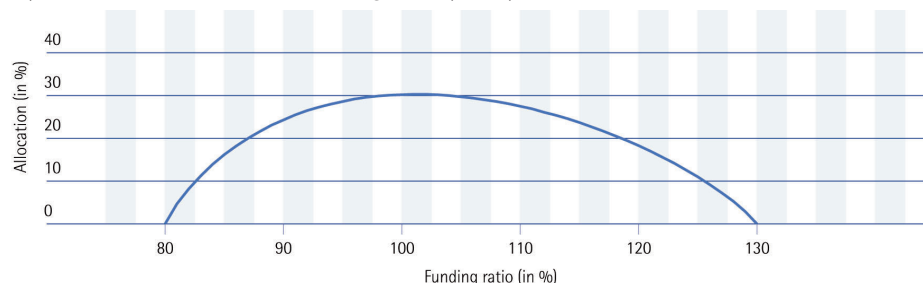
$$\eta_t = \sqrt{\lambda_{MSR}^2 + \sigma_\beta^2 - 2\lambda_\beta \sigma_\beta} \sqrt{T - t}.$$

It should be emphasised again that the above analytical expression for the optimal strategy is derived in a specific setting, with constant Sharpe ratios for the MSR portfolio and the GHP and a constant volatility for the GHP. We obtain an interesting insight: the allocation to the MSR portfolio depends on the current affordable income level, and it approaches zero as the quantity $\mathcal{N}^{-1} \left(\frac{r_{i_{aff},t} - r_{i_{ess}}}{r_{i_{asp}} - r_{i_{ess}}} \right)$ goes to minus or plus infinity, that is when $r_{i_{aff},t}$ approaches the bounds $r_{i_{ess}}$ or $r_{i_{asp}}$. In fact, because final wealth is always equal to $r_{i_{ess}} \beta_T$ or to $r_{i_{asp}} \beta_T$, it follows from the absence of arbitrage opportunities that wealth at any date is between $r_{i_{ess}} \beta_t$ and $r_{i_{asp}} \beta_t$, so that affordable income can never fall short of the essential level or exceed the aspirational level of replacement income.

The property that the allocation to the performance building block shrinks to zero as current wealth approaches a floor is shared with portfolio insurance strategies (see Black and Perold (1992) for constant proportion portfolio insurance and Teplá (2001) for option-based portfolio insurance). It allows investors to avoid breaching the floor by slowing down on risk-taking when the margin for error vanishes. We also obtain that risk taking vanishes when wealth approaches

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Figure 8: Optimal fraction of wealth allocated to the growth-optimal portfolio.



This figure shows the fraction of wealth allocated to the growth-optimal portfolio by the probability-maximising strategy, as a function of the current funding ratio. The optimal allocation is calculated with Equation (4.2), and the following parameter values: the time-to-retirement is 10 years, the essential and the aspirational levels of income are respectively 80% and 130% of the initially affordable level, the maximum Sharpe ratio is 0.6, the Sharpe ratio of the GHP is 0.3, and the annual volatility of the GHP is 7%. The funding ratio is the ratio of the currently level of affordable income to the initial level.

a level equal to the minimum wealth required to secure the aspirational goal, but this responds to a totally different concern. Indeed, if $ri_{aff,t}$ could exceed the level ri_{asp} at some point, there would be a positive probability for the final $ri_{aff,T}$ to do so as well (still by absence of arbitrage opportunities), but this would entail no welfare gain for an investor with a focus on reaching ri_{asp} rather than exceeding that level. As a consequence, it is optimal for the investor to slow down on risk taking near ri_{asp} and secure the aspirational goal as soon as it is reached because risk taking is no longer needed then.

Figure 8 plots the percentage allocation to the growth-optimal portfolio (i.e. the quantity ϕ_t from Equation (4.2)), as a function of the current funding ratio. By construction of the probability-maximising strategy, the funding ratio is constrained to lie between the essential and the aspirational levels. The picture clearly shows the non-monotonic nature of the allocation, which is a hump-shaped function of the funding ratio and is zero at the essential and at the aspirational levels.

5. A Simple Class of Implementable Retirement Goal-Based Investing Strategies



5. A Simple Class of Implementable Retirement Goal-Based Investing Strategies

While being optimal in theory, probability-maximising strategies would be hard to implement in practice for several reasons. First, they involve continuous-time trading and possibly large levels of leverage, which are not realistically feasible. Besides, the payoff is all the more difficult to replicate with discrete trading because it is discontinuous. Moreover, their implementation requires the use of the volatility of the underlying asset as an input, which is an unobservable parameter. This volatility is a function of the volatility of the GHP and the volatility and the Sharpe ratio of the MSR portfolio, the latter parameter being particularly difficult to estimate.

Additionally, the digital nature of the payoff may make it unacceptable for real-world investors. Since the aspirational goal is either reached in full or completely missed, the expected shortfall (the distance with respect to the target when it is missed) is sizable, being equal to the distance between the aspirational and the essential levels. Moreover, the discontinuity in the payoff implies that a small change in the value of the underlying asset around the barrier causes a sudden switch between failure and success. It is possible that investors anxious about replacement income in retirement will perceive such strategies as gambles, and that they will prefer a more widespread distribution of replacement income, even if it comes at the expense of a decrease in the success probability. This concern could be mitigated by penalising deviations from the goal in the objective function, but adding such constraints would add to the complexity of the optimisation problem without alleviating the concern over implementation.¹⁰ In this section, we introduce a simple class of strategies that share most of the desirable properties

of the optimal probability-maximising strategies, while being consistent with realistic implementation constraints.

5.1 Key Properties of Goal-Based Investing Strategies

The probability-maximising strategy enjoys several properties that make intuitive sense. First, it makes use of two building blocks – the GHP and the growth-optimal strategies – that have well-defined roles. The GHP has a focus on hedging, which allows the investors to secure replacement income by keeping up with the performance of the Goal Price Index. The growth-optimal strategy, by definition, maximises the expected growth rate on the portfolio, to generate the upside performance needed to move on to higher funding ratio levels. This portfolio is entirely independent from subjective characteristics and is driven by the risk and return characteristics of risky assets. A clear separation of roles between building blocks is typical of fund separation theorems in portfolio theory, and is a principle of liability-driven investing strategies in institutional money management. The second noteworthy property of optimal strategies is that the allocation to the safe and risky building blocks is a function of market conditions through the current level of affordable income, and that it shrinks to zero as the funding ratio approaches the essential or the aspirational level.

We propose designing strategies that retain these properties while respecting real-world implementation constraints. Like the optimal one, they combine the GHP and a PSP, but since leverage is costly or even forbidden in practice, the PSP is fully invested in risky assets. The ideal PSP would have the highest Sharpe

¹⁰ – Cvitanic (2000) and Föllmer and Leukert (2000) minimise a loss function of the shortfall.

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ratio, but this criterion is not operational because expected returns on assets are notoriously difficult to estimate.¹¹ Fortunately, a variety of alternative methods are available to construct well-diversified portfolios without relying on imprecise expected return estimates. They include variance minimisation, equal weighting, risk parity, and diversification techniques in terms of risk factors, subject of the introduction of suitable weight constraints.¹²

The allocation rule to the two building blocks must also be modified to avoid the use of unobservable risk and return parameters. To ensure that the dollar allocation to the PSP vanishes as wealth approaches the floor, the simplest specification is a linear function of the risk budget, defined as the distance between wealth and the essential goal taken as a floor. As a result, the fraction of wealth invested in the PSP is

$$\psi_t = m \left[1 - \frac{r_{iess}}{r_{iaff,t}} \right], \quad (5.1)$$

where m is a constant multiplier in the simplest case (we introduce a strategy with a time-varying multiplier in the next section). The remainder of wealth goes to the GHP. In order to preclude leverage or short positions, the right-hand side of Equation (5.1) is floored at 0 and capped at 100%.

The strategy defined in Equation (5.1) is somewhat reminiscent of constant proportion portfolio insurance (CPPI), where the dollar allocation is a multiple of the distance between wealth and a floor. Several important differences exist, however, including the fact that the floor here is proportional to a stochastic benchmark (the Goal Price Index), rather than being the discounted value of a fixed

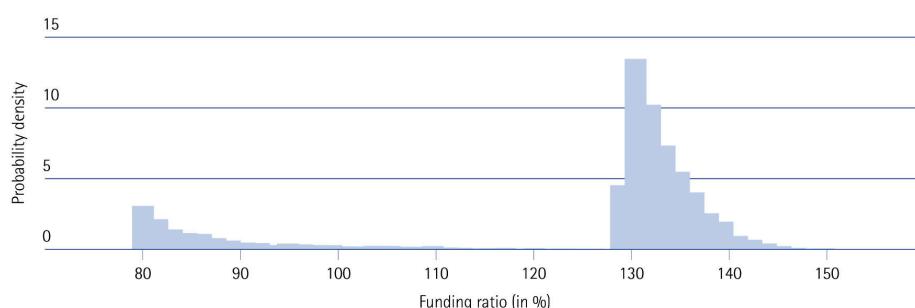
minimum of wealth. This is because the strategy takes care of relative, as opposed to absolute, risk, by protecting a fixed level of replacement income. In this sense, it is similar to a dynamic core-satellite strategy, which is the extension of CPPI to a situation where the concern is over relative risk with respect to a benchmark (see Amenc, Malaise, and Martellini (2004)). More generally, it belongs to the class of GBI strategies introduced by Deguest et al. (2015), where the non-symmetric management of risk allows an essential goal to be secured while taking advantage of the PSP performance (at least in part) to reach an aspirational goal. The role of the multiplier, which is taken to be greater than 1, is to increase the exposure to the PSP despite the need to secure the essential goal.

The GBI strategy provides a way to reliably secure an essential level of income while benefitting from the upside potential of the PSP relative to the GHP, which leads to increasing the level of affordable income with respect to the starting point in favourable market conditions. The main risk involved in the approach is gap risk. When the strategy is rebalanced in discrete time, unexpected violations of the floor can occur within a rebalancing period, in case of relative poor performance of the PSP with respect to the GHP. The consequence is sterilisation because after a violation of the floor, the portfolio gets entirely invested in the GHP and is "sterilised" in the sense that there is no longer any potential to outperform the GHP. This concern is alleviated in strategies where the floor is reset on an annual basis, as discussed in the next section.

11 - This is documented in a vast literature on portfolio choice under parameter uncertainty. See Merton (1980) for a statistical argument that sample mean is inaccurate at usual sample sizes, and Kan and Zhou (2007) for a numerical study of the effects of estimation errors.
12 - See Martellini and Milhau (2018) for a review of these approaches.

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Figure 9: Distribution of the funding ratio at retirement date under a goal-based investing strategy.



The GBI strategy is simulated for an investor retiring in January 2038 conditional on parameter values of 1 January 2018, with a multiplier of 3. The essential goal is to secure 80% of the initially affordable replacement income in all states of the world. A stop-gain decision is made on the first date the funding ratio hits the aspirational level of 130%. Parameter values for the stochastic processes are given in the text.

5.2 Distribution of the Funding Ratio

The probability-maximising strategy lets the allocation to the performance building block decrease to zero as the distance to this goal gets smaller. As explained previously, the rationale for cutting risk taking then is that the aspirational goal should be gradually secured as it gets within reach. We can adapt this rule in the case of the implementable GBI strategy by setting the allocation to zero as soon as the level of affordable income hits the target. As a result, the portfolio is entirely invested in the GHP after the aspirational goal has been reached, and remains in this state until retirement. This “stop-gain” mechanism amounts to securing the aspirational goal as soon as it is reached, to avoid the risk of a downturn in the PSP value. Once the stop-gain mechanism is activated, the funding ratio remains constant, so the aspirational funding level plays the role of a cap in the strategy. In principle (that is, up to upside gap risk, as we shall see below), the funding ratio cannot exceed the aspirational level because there is a priori no interest expressed by the investor to exceed this level.

We now simulate the value of the GBI strategy, to compare its success

probabilities with those of the optimal investment rule. To this end, we assume a monthly rebalancing frequency, and the value of the funding ratio is monitored on a monthly basis in order to trigger the stop-gain mechanism as soon as it reaches the aspirational goal again assumed to be at 130%. The essential level is still taken to be 80%, and parameter values for expected returns, volatilities and correlations are those given in Section 4.2. Interest rate parameters relate to the yield curve at the beginning of January 2018 and are described in Appendix B.

Figure 9 shows the distribution of 10,000 outcomes for the funding ratio. It is clearly bi-modal even though not in a strict sense, and shows two clusters, at 80% and 130%, a shape that is reminiscent of the distribution under the probability-maximising strategy. Funding levels located between 80% and 130% are attained with a positive probability with the implementable strategy, so that it is possible to miss the aspirational goal without being stuck at the essential level. If the funding ratio was monitored continuously, it would never exceed 130% upon retirement, because the stopgain decision would be made on the exact moment when it is equal to 130%.

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With monthly monitoring, however, the decision is made after the fact, when it is already above the barrier. This discrete trading generates “upside gap risk”, implying that the final funding level can eventually exceed the target.

5.3 Numerical Comparison of Success Probabilities

In Table 1, we compare the success probabilities delivered by implementable GBI strategies to the theoretical optimal probabilities, and we analyse the impact of the multiplier and the level of the essential goal. These results allow us to confirm that while being suboptimal, the implementable GBI strategies still offer attractive probabilities, and also allow us to calibrate the multiplier and the essential goal level. For the probability-maximising strategy, we simulate the theoretical payoff using the expression given in Equation (4.1), which assumes that this strategy is implemented with continuous rebalancing. For the more realistic GBI strategies, we simulate the full path of values, starting at date 0 and finishing at retirement date. In accordance with the decision to activate a stop-gain mechanism if and when the aspirational goal is reached, the probability of being successful in reaching a given aspirational level is estimated as the probability for the funding ratio to hit this level at least once during the accumulation phase.¹³

As a general rule, for a given class of strategies, the probability of reaching a given aspirational goal is decreasing in the value of the essential goal, as expected given that requiring a higher protection level implies that less money is available to invest in performance-seeking assets. Among GBI strategies, the decrease is sharper for low multiplier

values: the probability of reaching the 130% funding ratio at least once before retirement decreases from 74.6% for an essential goal at 70% to 34.0% for a goal at 90% when $m = 1$, while it moves from 83.1% down to 65.3% when $m = 3$. This amounts to a decrease by 54.4% when $m = 1$ and 21.4% for $m = 3$. Therefore, a larger multiplier partially offsets the negative effect of selecting a higher essential goal, and therefore decreases the associated opportunity cost. This is clearly apparent from Equation (5.1), which shows that the allocation to the PSP in the GBI strategy is decreasing in the essential income level and increasing in the multiplier. To get “the best of both worlds”, one may be tempted to set a high essential level and compensate with a large multiplier, but this will in principle increase the likelihood of gap risk, although Table 1 shows no violation of the floor.¹⁴

The approach consisting in increasing the multiplier also has its limits when it comes to aspirational goals because the allocation to the PSP is capped to 100%. As a result, with a very large multiplier, the GBI strategy is most often invested in the PSP, except when the risk budget is small. In fact, for an unboundedly large multiplier value, the GBI strategy eventually becomes a “stop-loss/stop-gain” strategy, which is entirely invested in the PSP until it hits the floor or the cap and then gets fully invested in the GHP. The motivation behind the stop-loss approach is to avoid bearing the opportunity cost in terms of performance that a GBI strategy generates with respect to an investment in the PSP. Its main shortcoming is that the portfolio can become irrevocably sterilised in the GHP following poor PSP performance even before the aspirational goal is attained, while a more prudent GBI strategy would have avoided sterilisation

13 - In Appendix A, we argue that given that there is positive probability for the optimal strategy to miss the aspirational goal, the assumption of no arbitrage opportunities implies that the level of affordable income with the optimal strategy is strictly less than the aspirational level until retirement. Hence, no opportunities to secure the goal are lost by assuming that no stop-gain is used for this strategy, and the comparison of success probabilities between the optimal and the GBI strategies is fair.

14 - The absence of violations even with high multiplier values is due to our assumption of monthly rebalancing. With quarterly trading, a high multiplier would be more harmful.

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Table 1: Probabilities of securing a long-term essential goal and reaching aspirational goals by optimal strategies and goal-based investing strategies (in %).

	Probability-maximising	Goal-based investing				
Multiplier		1	2	3	4	5
Essential 70%						
Essential Goal	100.0	100.0	100.0	100.0	100.0	100.0
Aspirational Goal (%)						
110	99.8	92.3	93.2	92.0	90.7	89.9
120	99.2	83.4	88.0	87.0	86.1	85.0
130	98.5	74.6	83.4	83.1	82.5	81.5
140	97.5	66.1	79.1	80.1	79.5	78.8
150	96.5	57.7	74.9	77.1	77.1	76.6
Essential 80%						
Essential Goal	100.0	100.0	100.0	100.0	100.0	100.0
Aspirational Goal (%)						
110	99.5	88.3	90.3	89.1	86.4	84.2
120	98.5	74.6	83.4	82.3	79.9	77.8
130	97.0	61.8	76.9	77.3	75.4	72.9
140	95.4	50.9	71.3	73.3	71.8	69.5
150	93.9	41.6	66.5	69.3	68.9	67.0
Essential 90%						
Essential Goal	100.0	100.0	100.0	100.0	100.0	100.0
Aspirational Goal (%)						
110	98.5	74.9	83.4	82.4	79.2	75.5
120	95.5	51.3	71.5	73.0	69.5	64.8
130	92.5	34.0	62.2	65.3	63.2	59.1
140	89.8	23.3	55.1	59.6	58.2	54.9
150	87.4	16.0	49.1	55.7	54.7	51.6

Strategies are simulated for an investor who starts to accumulate money on 1 January 2018 and plans to retire in January 2038. The essential goal is expressed as a percentage of the initially affordable income level to protect at the retirement date, and the aspirational goal is a percentage, greater than 100%, of the same level to reach at least once during the accumulation period. For each pair of goals, we simulate the binary-like option payoff that maximises the probability of reaching the aspirational goal subject to the constraint of securing the essential goal. For each essential goal, we simulate five goal-based investing (GBI) strategies that secure this goal and deliver a (hopefully) high probability of reaching aspirational levels of funding. The five GBI strategies differ through their multiplier, which ranges from 1 to 5.

in the first place and would have benefitted from the recovery of the PSP. Strategies with a large multiplier are exposed to the same risk, so the probability of reaching an aspirational goal ends up being a hump-shaped function of the multiplier, which suggests that suitable choices of multiplier value can improve the design of GBI retirement solutions.

6. Introducing Risk-Managed Target Date Funds



6. Introducing Risk-Managed Target Date Funds

So far, we have focused on strategies that protect a long-term essential goal, expressed as a percentage of the initial level of affordable income. Although they are useful from a pedagogical perspective, such strategies, however, do not offer any protection to subsequent contributions made down the way to retirement. In retirement planning, regular contributions represent a total amount that is usually much larger than the initial capital invested, except for investors in the late transition phase, who have already accumulated a substantial capital. In the same vein, strategies with a long-term essential goal also lack scalability from the perspective of new investors, since their guarantee applies to a fixed period, implying that individuals who enter during the fund's life do not benefit from the same protection.

To avoid setting new investors or new contributions at risk, Martellini and Milhau (2016) introduce a class of strategies that secure a given fraction of their purchasing power in terms of replacement income at the retirement horizon. In this section, we consider a different type of essential goal expressed as a maximum annual loss in purchasing power, and we show that this short-term essential goal can be secured with a class of goal-based investing strategies that extend traditional forms of target date funds. Having target date funds as natural benchmarks, or anchor points, for retirement GBI strategies shall presumably facilitate the adoption of the latter solutions by asset managers and distributors.

6.1 Problems with Existing Forms of Target Date Funds

In target date funds, the target mix of stocks, bonds and cash evolves in time

until a date called the target date or target maturity date of the fund, with a deterministic decrease in equity allocation according to a predetermined glide path. Figure 10 shows an example, with an equity allocation that starts at 100% when the investor enters the job market at age 25. During the first 20 years, the allocation decreases by 100 basis points per year, from 100% at the 40-year horizon to 80% at the 20-year horizon, and during the last 20 years, the annual shift is 200 basis points, so the allocation just before retirement is 40%.

Since target date funds were first launched in the early 1990s, they have become one of the fastest growing segments in the mutual fund industry. Embedding the life-cycle allocation decisions within a one-stop decision is obviously a valuable attempt at helping unsophisticated investors implement long-term investment portfolio policies. The problem, however, is that standard forms of target date funds suffer from two shortcomings, which can be described as the use of inappropriate building blocks and the use of a simplistic allocation strategy.

Use of Inappropriate Building Blocks

Fund separation theorems in portfolio theory suggest that the building blocks for an asset allocation exercise are not asset classes, such as stocks and bonds, but instead suitably designed portfolios known as the PSP and the liability-hedging portfolio. While this approach is well-understood in the context of institutional money management, their transposition into individual money management is not yet widely developed. The so-called "safe" bond portfolio used in most target date funds is actually unsafe given the investor's horizon and preferences. For

6. Introducing Risk-Managed Target Date Funds

instance, neither a generic sovereign bond index nor a money market account matches the duration of the investor's dedicated retirement bond or GHP, which is the true safe asset in a retirement context. In fact, bond indices will inevitably have too short a duration, especially for investors who are far from retirement. A cash account will have an even shorter duration, thus generating extremely high levels of unrewarded interest rate risk exposure. As a consequence of this duration mismatch, the value of the bond index or the cash account expressed in the Goal Price Index numeraire exhibits large short-term variability, as shown in Figure 6. Similarly, inflation risk is usually not hedged, although long-term investors are often concerned with the purchasing power of their savings.

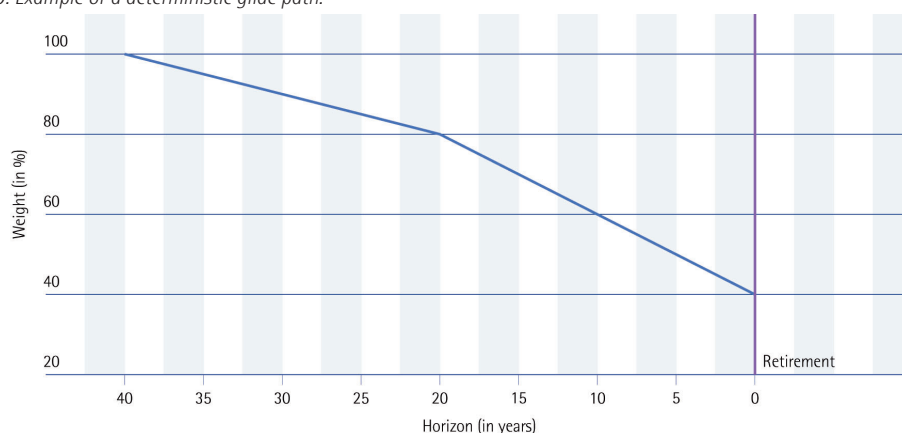
The PSP used in most target date fund strategies is also typically inefficient in that it does not allow for the most efficient harvesting of risk premia in equity markets. Indeed, the equity portfolio in these target date funds is typically benchmarked against cap-weighted indices, which have been shown to deliver a poor risk-adjusted return (Haugen and Baker 1991; Grinold 1992) because of an excessive concentration in relatively

a few large stocks as well as a sub-optimal exposure to rewarded risk factors. Recently introduced smart factor indices have been shown to provide a better access to the sources of profitability in the stock market (Amenc, Goltz, and Lodh 2012; Amenc et al. 2014), and they can be fruitfully combined to take advantage of their respective premia (Amenc et al. 2017; Martellini and Milhau 2018).

Use of a Simplistic Allocation Strategy

Another criticism relates to the allocation policy implemented by target date funds, which is purely deterministic (as in Figure 10) and does not allow for revisions of the asset allocation as a function of changes in market conditions. Common sense suggests that the allocation to equities should be lower when the equity risk premium is lower and when equity volatility is higher, everything else equal. This intuitive approach is formally justified by dynamic portfolio theory, which shows in an expected utility maximisation framework that the optimal allocation to stocks is a function of conditional equity volatility and equity risk premium, and that a higher premium and/or a lower volatility commands a greater exposure to equities. Academic research has found that the use of a deterministic allocation

Figure 10: Example of a deterministic glide path.



The glide path is the allocation to equities in a deterministic target date fund, expressed as a function of the time-to-horizon.

6. Introducing Risk-Managed Target Date Funds

strategy often incurs large welfare costs with respect to the optimal policy.¹⁵

A motivation behind a deterministic glide path is to progressively secure accumulated savings by shifting to low-risk assets, but it is unclear what exact essential goal this allocation method secures. Indeed, it is mathematically impossible to calculate what minimum wealth or the minimum level of affordable income would be implied by such a strategy, even if the proper GHP was used as a safe building block. Lower bounds on the final wealth value or the final value of affordable income can be estimated by running Monte-Carlo simulations of the portfolio returns, but these estimates are necessarily model- and parameter-dependent, and, as such, lack robustness. For instance, even an ambitious minimum wealth or income level at the retirement date may seem attainable with the greatest confidence level provided optimistic assumptions on expected returns are made.

6.2 Target Date Funds with a Focus on Replacement Income

To improve upon existing target date funds, we introduce a new form of risk-controlled strategies that have a focus on replacement income in retirement. From a conceptual standpoint, these strategies are of the *goal-based investing* type in that the choice of their building blocks and their allocation policy are driven by the identification of specific *goals* that are relevant to individuals. Because the objective in retirement investing is to generate replacement income, it is natural to expect that GBI strategies should protect some fraction of the purchasing power of savings in terms of income as an essential goal, while allowing the

remainder to be at risk in exchange for the potential to increase the purchasing power.

As explained in the introduction to this section, a long-term essential goal (i.e. a minimum level of replacement income to be guaranteed at the retirement horizon) is of limited interest given that it has to be affordable with initial savings and is therefore in general very low compared to what the investor can expect provided he/she makes regular contributions until he/she retires. Setting a goal of that type implies that some fraction (e.g. 70%, 80% or 90%) of the purchasing power of initial savings is secured, while the remainder is set at risk to get a chance to reach a higher income level. No guarantee can be made, however, that the same fixed percentage of the forthcoming contributions will be protected. As a result, such a goal is relevant only for investors who have large initial savings compared to expected future savings, and who express no concern over short-term decreases in funding ratios. For investors with a longer accumulation phase, and/or for investors concerned with short-term protection, it is necessary to replace the long-term essential goal with a series of shorter-term goals.

A uniform treatment of all dollars invested, regardless of their schedule, can be achieved by setting a short-term essential goal which consists in protecting the purchasing power of contributions on an annual basis. Assuming for simplicity that contributions are made once a year, say at the end of December, the goal is then to protect a given fraction of the purchasing power of each invested dollar (e.g. 80%) over a calendar year. This type of essential goal allows for strategies that are relatively scalable with respect to the

15 - See Cocco, Gomes, and Maenhout (2005), Cairns, Blake, and Dowd (2006) and Martellini and Milhau (2010).

6. Introducing Risk-Managed Target Date Funds

entry point as long as contributions come in at the beginning of the calendar year.

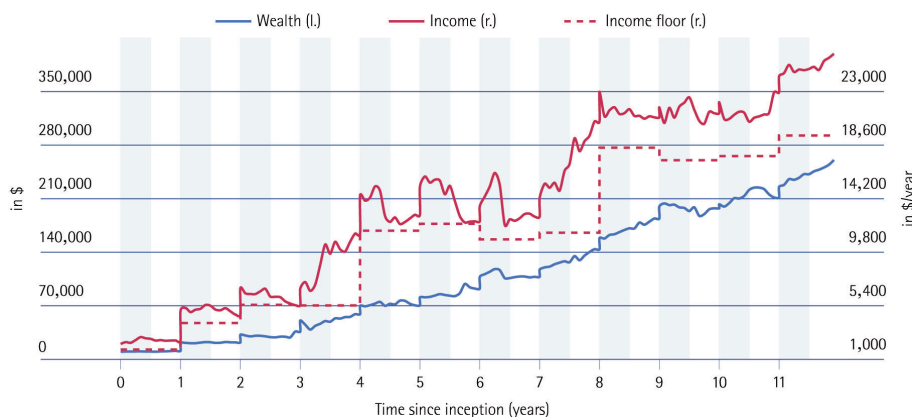
6.3 Introducing Risk-Controlled Target Date Funds

The first improvement consists in replacing the "safe" component of target date funds with a proper GHP that replicates the performance of the Goal Price Index. The GHP is associated with the same three subjective characteristics as its reference index, namely the retirement date, the duration of the decumulation phase and the indexation mode. It is the true safe asset with respect to the objective, unlike a generic bond index, as was shown in Figures 5 and 6. Like a regular target date fund, the risk-controlled strategy uses a PSP in addition to the GHP. An optional improvement would be to also improve the PSP with respect to a standard target date fund by considering an equity portfolio with a better risk-return profile than a broad cap-weighted index (e.g. a portfolio of smart factor indices) or even by adopting a well-diversified multi-asset portfolio.¹⁶

The second aspect in which the risk-controlled retirement GBI strategy departs from a standard target date fund is the investment policy, which must offer upside potential while securing the essential goal. To this end, we introduce a floor that the strategy should respect at all times, given by the price of the retirement bond that pays for example 80% of the replacement income that was affordable at the beginning of the year. This floor is reset every year to be equal to 80% of current savings, including the annual contribution. This mechanism is depicted in Figure 11. In this example, we consider an investor who makes an annual contribution of \$10,000 every year. Cash additions take place at dates 0, 1, 2, 3, etc., which respectively represent the beginnings of years 1, 2, 3, 4, etc. The "income floor" is the minimum level of affordable income to protect at all times, and it is constant within each year, and jumps to 80% of the newly affordable income level at the beginning of each year. As the picture shows, the jump can be either up or down. In this example, date 0 was taken to be January 2006, and a large

16 - See Martellini and Milhau (2015, 2018) for a discussion of competing methodologies that can be used to efficiently harvest risk premia across asset classes.

Figure 11: Annual reset of floor in a goal-based investing strategy with periodic contributions.



In this example, the individual starts out with \$10,000 savings, follows a goal-based investing strategy and makes an additional contribution of \$10,000 at the beginning of every year. "Income" is the level of replacement income that can be financed with current wealth. "Income floor" is the minimum level of income that the strategy should protect at all times, and is defined as 80% of the income level that was affordable at the beginning of the current year. To construct this figure, it was assumed that date 0 is 2 January 2006, that the retirement date is 1 January 2026 and that the decumulation period is 15 years. Each tick on the horizontal axis represents the beginning of a year, when a contribution is made.

6. Introducing Risk-Managed Target Date Funds

increase in the floor value takes place in January 2010, after good PSP (equity) performance in 2009. The floor decreases two years later, in 2012, after a bad year for equity markets in 2011. The data is described in Section 7.2, where we also analyse the relation between the strategy performance and the PSP performance in more detail. The point of Figure 11 is just to illustrate the reset mechanism.

Protection of the floor can be achieved by the means of a dynamic insurance strategy, in which the dollar allocation to the PSP is taken to be a multiple of the risk budget, or margin for error, defined as the distance between current wealth and floor levels. To mathematically write down the investment policy, let us introduce some notation: W_t is the current wealth level, δ is the percentage of purchasing power to be secured on a yearly basis, m is the multiplier, F_t is the floor that wealth should respect, and β_n as before, is the Goal Price Index determined by investor's characteristics. Let also W_n and β_n denote wealth and the price index at date $n - 1$, understood as the beginning of year n . (Date 0 is the beginning of year 1, date 1 is the beginning of year 2, etc.) Then, the percentage allocation to the PSP at date t is

$$w_t = m \left[1 - \frac{F_t}{W_t} \right], \quad (6.1)$$

with the floor

$$F_t = \delta \frac{W_n}{\beta_n} \beta_t.$$

The value of the right-hand side in Equation (6.1) is capped at 100% to avoid leverage, and the remainder of the wealth is invested in the GHP. δ is set to 80%, which means that the maximum tolerable relative loss within a year is 20%.

In fact, the multiplier in Equation (6.1) is not taken to be constant, but given as a time-varying quantity the value of which is revised every year at the same time the floor is reset. This degree of freedom is chosen to match the pre-determined allocation to the PSP to that of a standard target date fund, and thus to anchor the design of the GBI strategy with respect to standard forms of retirement savings products. In details, if TDF_{n-1} is the percentage allocation to the PSP in the target date fund at the beginning of year n (as given by Figure 10), then the multiplier for this year is

$$m_n = \frac{TDF_{n-1}}{1 - \delta_{ess}}.$$

The glide path of the multiplier can be derived from Figure 10. For instance, when the horizon is 40 years, the allocation to the PSP on the target date fund is 100%, so the multiplier is $100/[100 - 80] = 5$. Just before retirement, the target weight is 40%, which implies a multiplier $40/[100 - 80] = 2$.

The GBI strategy is comparable to a standard target date fund in that it has the same allocation to the performance and the safe building blocks at the beginning of each year, but the difference is that the glide path applies to the multiplier as opposed to the weights themselves. As a result, the allocation to the two components can vary within each year to secure the essential goal. As already mentioned, the GBI strategy also uses the proper GHP as a safe asset, as opposed to using some bond index with an uncontrolled duration.

7. Comparing Risk-Managed and Deterministic Target Date Funds



7. Comparing Risk-Managed and Deterministic Target Date Funds

This section is not a competitive horse race between the GBI strategy and its reference target date fund strategy to test which one exhibits the strongest performance. Indeed, while asset management *products* are often promoted on the grounds of a track record, historical performance should not be the choice criterion for investment *solutions*, and expected performance is not a relevant criterion either. Instead, a solution should be evaluated against its adequacy with respect to an investor's goals, namely its ability to secure essential goals in a reliable way and its potential to reach aspirational goals. Monte-Carlo simulations are well suited for this purpose because they cover a wide range of possible scenarios, while a historical backtest shows the behaviour of the strategy in just one scenario, in which an aspirational goal may or may not have been achieved. On the other hand, historical scenarios can serve as "stress-tests" to check the ability of a strategy to secure essential goals even in particularly adverse market conditions. For these reasons, this section presents an *ex-ante* comparison of strategies based on stochastic simulations, and an *ex-post* comparison using historical data.

17 – In robustness checks, we have found that some gap risk arises when the GBI strategy is rebalanced quarterly, as opposed to monthly. Over the 20-year period, however, this risk is limited in probability (0.2%) and in severity (worst annual loss at 23.4%).

target date fund is invested in the stock index and the bond index with the glide path of Figure 10, and the GBI strategy aims to cap the annual relative loss with respect to the Goal Price Index at 20%, or in other words to secure 80% of the purchasing power in terms of replacement income of each dollar invested within a given year. We simulate 10,000 scenarios, and strategies are rebalanced every month.

Table 2 displays several statistics on the distribution of funding ratios. It appears that risk-managed target date GBI retirement solutions are comparable to conventional target date funds in terms of long-term expected funding ratio and probabilities of reaching aspirational levels of funding. On the other hand, standard forms of target date funds are unable to reliably secure minimum levels of funding ratios on an annual basis, especially for investors far from retirement: the failure probability is 15.6% over 20 years and it reaches 40.5% over 30 years. The GBI strategy may in principle be subject to gap risk, but no violations of the floor occur in the simulations.¹⁷ The control of short-term risk is especially useful when retirement is far ahead in the future, because the glide path in Figure 10 prescribes a particularly aggressive allocation, with more than 80% invested in equities. As a result, the target date fund exposes young investors to the risk of a severe under-performance of the PSP with respect to the GHP in early years.

Interestingly, realistic improvements to the Sharpe ratio of the PSP simulated here by raising the Sharpe ratio by 50% (that is, from 0.39 to 0.59), which can be obtained in practice by shifting from a cap-weighted index to a well diversified

7.1 Ex-Ante Comparison

The purpose of the *ex-ante* comparison is to assess the probabilities of reaching various goals, and more generally to simulate the distribution of the future replacement income that can be financed with one or another strategy. The distribution is simulated by generating many (here, 10,000) scenarios for the returns of the strategies and the Goal Price Index. For simplicity and parsimony, we adopt the same stochastic model that was used in Section 5.3, which is described in detail in Appendix B. The

7. Comparing Risk-Managed and Deterministic Target Date Funds

Table 2: Simulation of a funding ratio with a target date fund and a goal-based investing strategy.

	Target date fund	GBI strategy	GBI strategy - Improved PSP
10 years			
Expected funding ratio (%)	139.6	135.9	162.3
Success probability (%)			
130%	67.8	61.7	79.3
150%	43.9	40.2	60.3
200%	9.1	10.7	24.4
Volatility of annual changes (%)	8.8	9.7	10.5
Probability of annual loss > 20% (%)	1.9	0.0	0.0
Worst annual loss (%)	32.5	17.6	17.4
20 years			
Expected funding ratio (%)	225.1	209.9	322.2
Success probability (%)			
130%	89.7	86.2	96.2
150%	81.1	75.4	92.2
200%	56.1	50.2	77.9
Volatility of annual changes (%)	10.9	12.1	13.0
Probability of annual loss > 20% (%)	15.6	0.0	0.0
Worst annual loss (%)	35.6	18.6	18.4
30 years			
Expected funding ratio (%)	371.0	337.6	687.3
Success probability (%)			
130%	95.0	93.7	99.1
150%	90.9	88.4	98.3
200%	78.8	73.4	93.9
Volatility of annual changes (%)	12.5	13.6	14.6
Probability of annual loss > 20% (%)	40.5	0.0	0.0
Worst annual loss (%)	44.6	19.6	19.5

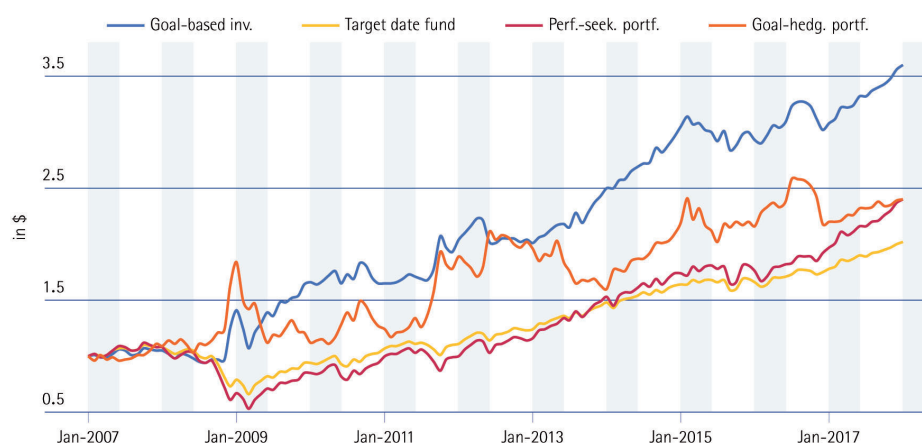
10,000 paths for the target date fund and the goal-based investing strategy are simulated using the stochastic model and the parameter values described in Appendix B. Interest rate parameters are estimated on 1 January 2018. The improved performance-seeking portfolio is simulated by raising the base case Sharpe ratio by 50% (i.e. from 0.39 to 0.59), so that the expected return in excess of the risk-free rate grows from 6.4% to 9.6% per year. The investor starts to accumulate in January 2018 and plans to retire 10, 20 or 30 years later, in 2028, 2038 or 2048.

portfolio of smart factor equity indices, would lead to an extremely significant increase in the probability for investors to achieve their target levels of replacement income. For example a 200% increase in purchasing power can be obtained with 77.9% probability over a 20-year period, while there is only a 50.2% chance to reach this objective if a poorly diversified cap-weighted equity portfolio is used.

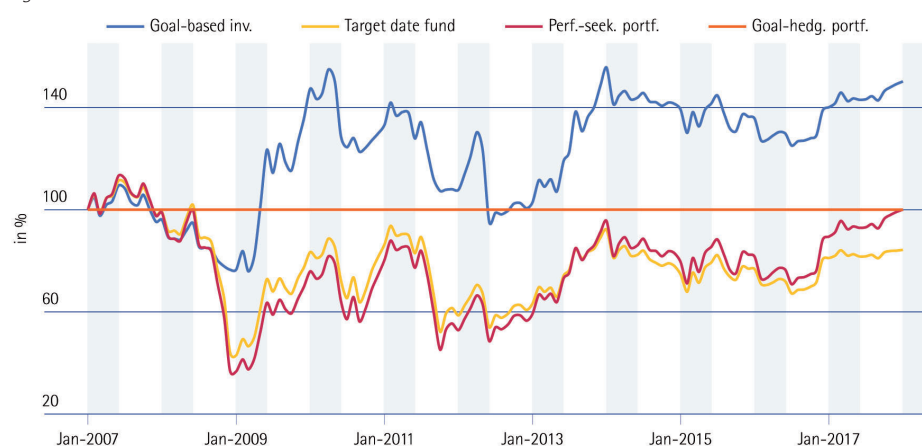
7. Comparing Risk-Managed and Deterministic Target Date Funds

Figure 12: Backtest of strategies; 2007-2018.

(a) Value of \$1 invested.



(b) Funding ratio.



The top plot shows the value of \$1 invested on 1 January 2007 in the goal-based investing strategy, the performance-seeking portfolio, the goal-hedging portfolio and the target date fund. The bottom plot shows the funding ratio, which is equal to the value of each strategy divided by the value of the goal-hedging portfolio: this ratio measures the evolution of the level of affordable income since January 2007. The reference investor plans to retire in January 2027 and expects constant annual replacement income.

7.2 Ex-Post Comparison

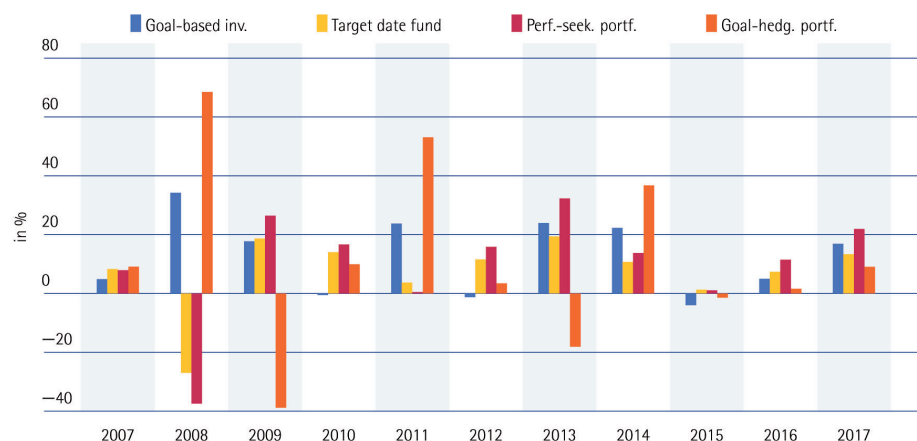
Unlike the *ex-ante* analysis, which aggregates multiple scenarios, the *ex-post* analysis focuses on a single historical path for the returns of the strategies and the Goal Price Index. The backtest procedure has inherent limitations: it reflects the *ex-post* application of a methodology to past data, but does not represent the returns on actual portfolios. Besides, a GBI strategy may have a high probability of reaching aspirational goals, but it does not guarantee that these will be attained in each scenario, so it may

fail in a particular one. On the other hand, it should protect the essential goal in each scenario, up to gap risk. This is not the case for target date funds, which have no built-in mechanism that can satisfy any essential goal. To showcase the benefits of the GBI approach for the control of short-term risk, we perform a "stress-test" by considering an accumulation phase that started in January 2007, some 20 months before the Lehman collapse and the subsequent financial crisis.

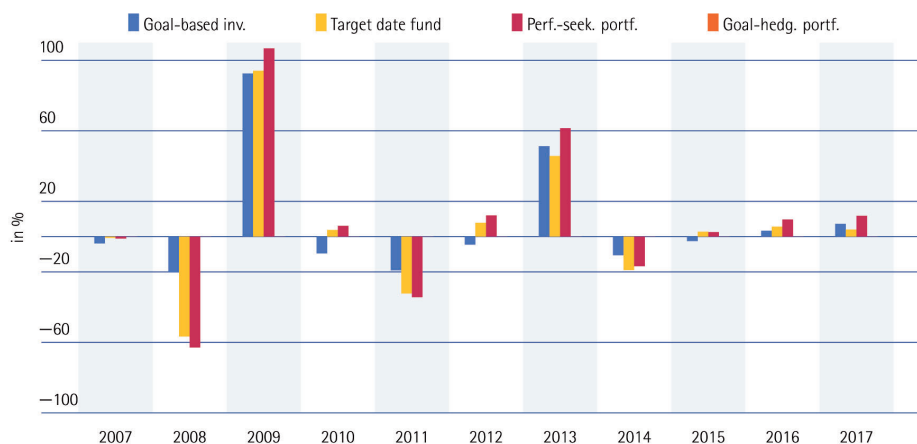
7. Comparing Risk-Managed and Deterministic Target Date Funds

Figure 13: Annual returns; 2007-2017.

(a) Annual returns.



(b) Annual returns relative to Goal Price Index.



The top plot shows the annual returns on the goal-based investing strategy, the target date fund, the performance-seeking portfolio and the goal-hedging portfolio. The bottom plot displays annual returns relative to the goal-hedging portfolio, that is annual changes in the level of affordable income. They are zero for the goal-hedging portfolio. The reference investor plans to retire in January 2027 and expects constant annual replacement income.

18 - The ERI Scientific Beta US cap-weighted index is not a publicly available series, but an alternative choice for the PSP is the market factor of Ken French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The realised performances of the GBI strategy and the target date fund, and consequently their relative performances with respect to the GHP, are very close to the ones shown in Figure 12.

19 - Availability period depends on the maturity: it starts in June 1961 for rates of maturities from 1 to 7 year(s), in August 1971 for maturities of 8 to 10 years, in November 1971 for 11 to 15 years, in July 1981 for 16 to 20 years and in November 1985 for 21 to 30 years.

The individual starts on 1 January 2007 and plans to retire 20 years later, in January 2027. The PSP is represented by the ERI Scientific Beta US cap-weighted index, which is a cap-weighted index of the 500 largest US stocks.¹⁸ As already mentioned, while a broad cap-weighted index is a standard default choice for US equity investors, a better risk-return profile could be achieved by switching to equity factor or smart beta indices. The Goal Price Index is valued using the same nominal zero-coupon rates that were already used in Section 3.2,

and that are available from the Federal Reserve website at daily frequency since December 1985.¹⁹ Dynamic strategies (i.e. the GBI one and the target date fund) are rebalanced every month.

Figure 12 shows the value of \$1 invested in January 2007 in the PSP, the GHP, the GBI strategy or the target date fund, and the corresponding funding ratio: recall that the funding ratio measures the evolution in the level of affordable income since January 2007. By definition, it is constant at 100%

7. Comparing Risk-Managed and Deterministic Target Date Funds

for the GHP. The PSP and the GHP have similar cumulative performance over the period (+140% for both), but exhibit very different sequences of intermediate returns. This is illustrated in Figure 13, where returns are plotted year by year. In 2008, the PSP experiences a strongly negative return of –37.5%, while the GHP grows by 68.5%, in response to the sizeable decrease in interest rates during the liquidity crisis triggered by the Lehman collapse. In 2009, the situation reverses, with a recovery of stock prices and an increase of long-term interest rates, which generate a positive return of 26.5% for the PSP and a fall by 38.8% for the GHP. 2013 is another rally year for stocks, but a bad year for long-term bonds, while 2011 is a “lost year” for the PSP but a very good one for the GHP. These return spreads can also be measured by calculating the relative return of each strategy with respect to the Goal Price Index, or, equivalently, with respect to the GHP, since relative returns measure the annual change in the level of affordable income. In goal-based reporting, these relative returns are a more relevant piece of information than absolute returns because they show how an investor's portfolio evolves relative to the cost of his/her objectives.

Market conditions like those prevailing in 2008 and 2011 are problematic for the GBI strategy, which feeds off the outperformance, if any, of the PSP with respect to the GHP. In these two years, the annual risk budget is entirely consumed, with annual relative returns of –20.4% and –19.2%, respectively. In 2008, the loss is actually slightly greater than the targeted 20%, as a manifestation of some gap risk that arises with discrete rebalancing. On the other hand, the target date fund exhibits

a much more severe loss, with a relative return of –56.7%, while the loss reaches 32.3% in 2011. These results confirm that investing in standard target date funds involves potentially large losses of purchasing power in situations where the PSP severely underperforms the Goal Price Index.

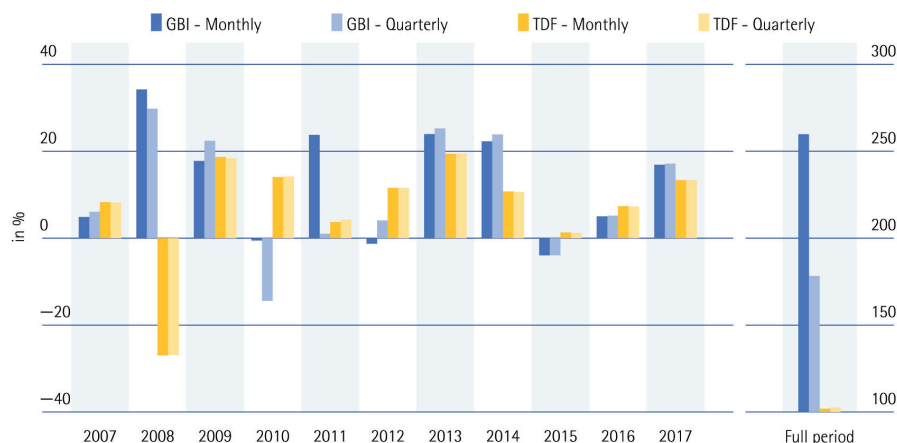
Impact of Rebalancing Frequency

The previous backtest assumes monthly rebalancing for dynamic strategies. We now present the results of a second backtest assuming quarterly rebalancing, and we report the annual returns to the GBI strategy and the target date fund in Figure 14. The target date fund shows very little sensitivity to the trading frequency, since it has about the same cumulative performance whether it is rebalanced every month or every quarter. On the other hand, switching to quarterly rebalancing involves a sizeable violation of the essential goal by the GBI strategy in 2011: the relative loss reaches 34.0%, while it was maintained under the 20% cap, at 19.2%, with monthly trading. In 2008, the deviation from the 20% objective is also larger, with a loss of 23.0%, while monthly rebalancing allowed the violation to stay at a very low level of 0.4% beyond the cap. Differences in annual returns accumulate year after year and result in a large difference in January 2018, both in absolute and in relative terms. In implementation, an asset manager may decide to go for quarterly revisions as a base case, and to increase this frequency in bear markets. Alternatively, he/she could follow a move-based strategy, under which trading takes place if and when effective weights deviate from target weights by too large an amount.

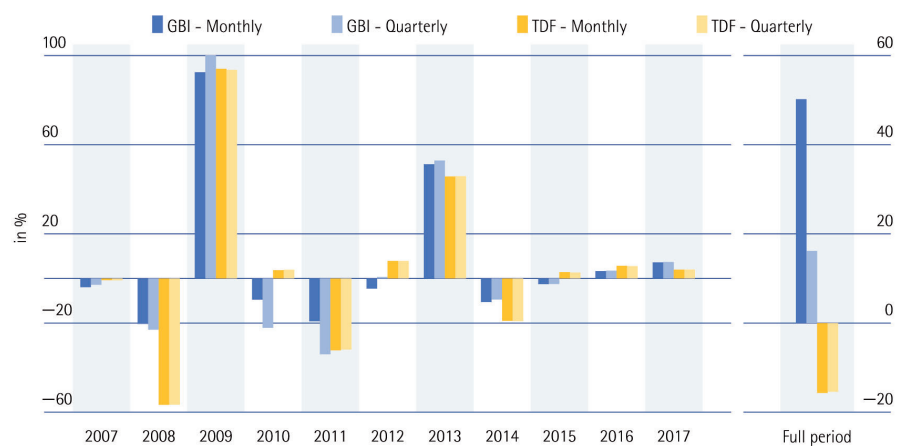
7. Comparing Risk-Managed and Deterministic Target Date Funds

Figure 14: Impact of rebalancing frequency; 2007-2018.

(a) Returns.



(b) Returns relative to Goal Price Index.



The top plot shows the returns on the goal-based investing strategy and the target date fund with monthly and quarterly rebalancing. The bottom plot shows the returns relative to the Goal Price Index. The reference investor plans to retire in January 2027 and expects constant annual replacement income.

7. Comparing Risk-Managed and Deterministic Target Date Funds

8. Conclusion



8. Conclusion

The general shift from DB towards DC pension plans, as well as the growing importance of individually managed retirement accounts, raise the need for improved forms of retirement investment solutions, that is investment funds focusing on the production of replacement income in retirement, and striking a reasonable balance between performance and safety. While these principles are explicitly stated by the OECD in its 2016 *Core Principles of Private Pension Regulation*, there are reasons to believe that existing products provide at best incomplete answers to these concerns. The deterministic glide path from equities to bonds and cash followed by target date funds cannot be justified since it does not allow investors to react to changes in market conditions. Besides, the bond portfolio that is typically used as a “safe” building block is in general not suited to the needs of a particular individual because its duration does not match that of the stream of replacement income cash flows he/she requires. As a result, such products offer no focus on replacement income. On the other hand, annuities sold by insurers do secure a stream of lifetime income in retirement, but their inherent irreversibility and their cost inefficiency, whether perceived or real, are major obstacles to their widespread adoption.

In this paper, we propose to apply the principles of goal-based investing (GBI) to the design of a new generation of retirement GBI strategies, which can be regarded as *risk-controlled target date funds* that strike a balance between safety and performance with respect to the objective of generating replacement income. To this end, we first introduce the *Goal Price Index*, defined as the price of \$1 of replacement income

for a fixed period in retirement, possibly including indexation to inflation. This index is also a useful tool from a reporting perspective because it allows investors to estimate the level of income that can be financed with their retirement savings. We then introduce dynamic strategies engineered to secure a given fraction of the level of income that can be financed with current retirement savings, while giving access to the upside potential of the performance-seeking portfolio (PSP). These strategies can be regarded as an evolution of deterministic target date funds and a form of *risk-controlled target date funds*. They differ from traditional life-cycle funds through the choice of the safe building block, taken to be the goal-hedging portfolio (GHP) as opposed to a fixed-income portfolio with unmatched duration, and through the investment strategy, which explicitly aims to protect a fixed fraction of the purchasing power of invested contributions in terms of income. Specifically, the *essential goal* for these strategies is to secure say 80% of affordable income on an annual basis, an objective that deterministic target date funds are unable to reach, except by chance. The risk budget, taken to be 20% in this example, is chosen in order to open access to the upside potential of PSP. This gives the individual the ability to reach *aspirational goals*, defined as replacement income levels that are not affordable *ex-ante* given current levels of retirement savings.

This approach is firmly grounded on simple and robust asset pricing academic principles, which suggest that a good risky asset should be a well-rewarded portfolio that efficiently harvests risk premia across and within asset classes, and that a good safe

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asset should replicate replacement income cash flows. Finally, the allocation to the two building blocks is chosen to extend portfolio insurance principles to the management of relative risk and performance. In other words, this new generation of retirement GBI risk-controlled target date funds make use of the three known forms of risk management, namely diversification, hedging and insurance, and as such offer a significant improvement with respect to standard forms of target date funds with no risk management mechanism. This approach to GBI solutions for households and individuals can be applied to other meaningful goals, such as securing financing for children's education for example, as explained by Deguest et al. (2015).

foster interest in the investment industry for the launch of new forms of retirement investment solutions that are better aligned with investors' objectives.

In order to provide the investment community with a concrete illustration of the concepts presented in this paper, EDHEC-Risk Institute has teamed up with Princeton University's ORFE department to launch the EDHEC-Princeton Retirement Goal-Based Investing Index series. The first types of indices are the *Retirement Goal Price Indices*, which represent the price of \$1 of replacement income per year, for various retirement dates. The indices in the second group represent the performance of the improved risk-controlled target date fund strategies invested in the GHP and a PSP, and are named *Retirement Goal-Based Investing Indices*. The value of the two index series will be published on the EDHEC-Risk Institute and Princeton ORFE websites, together with the allocation to the two building blocks and the probabilities of reaching aspirational goals. It is our hope and ambition that the publication of the so-called EDHEC-Princeton Retirement Goal-Based Investing Index series can

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Appendices



Appendices

A. Derivation of the Probability-Maximising Strategy

In this appendix, we provide the detailed mathematical derivation of the strategy that maximises the probability of reaching an aspirational goal while respecting a long-term essential goal.

A.1 The Optimisation Problem

We cast the problem in a standard continuous-time framework where the time span is the time interval $[0, T]$. Uncertainty is represented by a standard probability space $(\Omega, \mathcal{A}, \mathbb{P})$, where Ω is the set of outcomes, \mathcal{A} is a sigma-algebra and \mathbb{P} is the “physical” probability measure. The probability space supports a vector Brownian motion \mathbf{z} of dimension d , where d is the number of independent sources of risk to take into account. It is equipped with the filtration $(\mathcal{F}_t)_{t \in [0, T]}$ generated by the Brownian motion. The expectation operator conditional on information available at date t is denoted with \mathbb{E}_t .

The investor has access to N locally risky securities whose prices follow diffusion processes:

$$\frac{dS_{it}}{S_{it}} = \mu_{it} dt + \sigma'_{it} d\mathbf{z}_t, \quad i = 1, \dots, N.$$

The volatility vectors σ_{it} contain the exposures of the securities to the d sources of risk. They are collected in the $d \times N$ volatility matrix

$$\sigma_t = \begin{bmatrix} \sigma_{1t} & \cdots & \sigma_{Nt} \end{bmatrix}.$$

The market completeness assumption implies that this matrix is square (hence, $d = N$) and non-singular. Hence, there exists a unique price of risk vector, given by

$$\lambda_t = \sigma_t [\sigma'_t \sigma_t]^{-1} \begin{bmatrix} \mu_{1t} - r_t \\ \vdots \\ \mu_{Nt} - r_t \end{bmatrix},$$

where r_t is the short-term interest rate. The unique stochastic discount factor is the stochastic process $(M_t)_{t \in [0, T]}$ defined as

$$M_t = \exp \left[- \int_0^t \left[r_s + \frac{\|\lambda_s\|^2}{2} \right] ds - \int_0^t \lambda'_s d\mathbf{z}_s \right].$$

The quantity $\|\lambda_s\|$ is the Euclidian norm of the vector λ_s . Prices and portfolio values multiplied by M_t follow martingales under \mathbb{P} .

In addition to the N risky assets, the investor can trade in a locally risk-free asset, or cash account, that earns the continuously compounded short-term rate. Let w_{it} be the weight allocated to security i at date t , and \mathbf{w}_t be the $N \times 1$ vector of weights. The fraction of wealth allocated to the cash account is one minus the sum of the w_{it} -s. Trading takes place in continuous time, so the intertemporal budget constraint reads

$$dW_t = r_t W_t dt + \sum_{i=1}^N W_t w_{it} \left[\frac{dS_{it}}{S_{it}} - r_t dt \right]. \quad (\text{A.1})$$

The Goal Price Index that corresponds to the individual's characteristics is β_t , and it follows the stochastic process

$$\frac{d\beta_t}{\beta_t} = \mu_{\beta t} dt + \sigma'_{\beta t} d\mathbf{z}_t.$$

Because the market is complete, there exists a cash flow matching strategy for the retirement bond, so the process $M\beta$ follows a martingale.

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At date 0, the essential goal is affordable, but the aspirational one is not, so we have

$$ri_{ess}\beta_0 \leq W_0 < ri_{asp}\beta_0.$$

Let $\delta_{ess} = ri_{ess}\beta_0/W_0$ and $\delta_{asp} = ri_{asp}\beta_0/W_0$.

The investor's problem is to maximise the probability of reaching the aspirational goal while respecting the essential goal. The essential goal implies a floor $F_T = ri_{ess}\beta_T$ on terminal wealth, and the aspirational goal is reached if wealth reaches the target $G_T = ri_{asp}\beta_T$. Mathematically, the optimisation problem reads

$$\max_w \mathbb{P}[W_T \geq G_T],$$

$$\text{subject to } W_T \geq F_T \text{ and (A.1).} \quad (\text{A.2})$$

A.2 The Optimal Payoff

Our derivation of the optimal payoff follows the lines of Föllmer and Leukert (1999). We first define a change of probability measure, from \mathbb{P} to the probability that makes prices expressed in the Goal Price Index numeraire be martingales. This probability measure \mathbb{Q}^β is defined by its Radon-Nikodym density:

$$\frac{d\mathbb{Q}^\beta}{d\mathbb{P}} = \frac{M_T\beta_T}{\beta_0}.$$

We define the event $E_0 = \{hM_T\beta_T \leq \beta_0\}$, where h is chosen in such a way that

$$\mathbb{Q}^\beta(E_0) = \frac{W_0 - ri_{ess}\beta_0}{[ri_{asp} - ri_{ess}]\beta_0}, \quad (\text{A.3})$$

a condition equivalent to

$$\mathbb{E}[M_T X^*] = W_0, \quad (\text{A.4})$$

where

$$X^* = ri_{ess}\beta_T + [ri_{asp} - ri_{ess}]\beta_T \mathbb{1}_{E_0}.$$

Equation (A.3) can be rewritten as

$$\mathbb{Q}^\beta\left(\frac{\beta_0}{M_T\beta_T} \geq h\right) = \frac{W_0 - ri_{ess}\beta_0}{[ri_{asp} - ri_{ess}]\beta_0},$$

so h is a quantile of the cumulative distribution function of $\beta_0/[M_T\beta_T]$. Hence, h is decreasing in the ratio

$$[W_0 - ri_{ess}\beta_0] / [[ri_{asp} - ri_{ess}]\beta_0]$$

X^* is our candidate optimal payoff. Together with the assumption of market completeness, Equation (A.4) shows that it is an attainable payoff. It clearly respects the floor. The success region, in which it reaches the aspirational goal, is E_0 . Thus, the success probability under \mathbb{Q}^β is

$$\mathbb{Q}^\beta(X^* \geq G_T) = \mathbb{Q}^\beta(E_0) = \frac{W_0 - ri_{ess}\beta_0}{[ri_{asp} - ri_{ess}]\beta_0}.$$

Consider now any strategy with a terminal value W_T satisfying $W_T \geq F_T$ almost surely, and let $E = \{W_T \geq G_T\}$ be its success region. We have

$$\begin{aligned} \mathbb{Q}^\beta(E) &= \frac{1}{\beta_0} \mathbb{E}[M_T\beta_T \mathbb{1}_E] \\ &= \frac{1}{[ri_{asp} - ri_{ess}]\beta_0} \mathbb{E}[M_T[G_T - F_T] \mathbb{1}_E]. \end{aligned}$$

But, on E , $G_T \leq W_T$. Hence,

$$\mathbb{Q}^\beta(E) \leq \frac{1}{[ri_{asp} - ri_{ess}]\beta_0} \mathbb{E}[(W_T - F_T) \mathbb{1}_E].$$

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Because $W_T - F_T \geq 0$ and $\mathbb{1}_E \leq 1$, it follows that

$$\begin{aligned} \mathbb{Q}^\beta(E) &\leq \frac{1}{[ri_{asp} - ri_{ess}] \beta_0} \mathbb{E} [W_T - F_T] \\ &= \frac{W_0 - ri_{ess} \beta_0}{[ri_{asp} - ri_{ess}] \beta_0} = \mathbb{Q}^\beta(E_0). \end{aligned}$$

Then, Neyman-Pearson lemma implies that $\mathbb{P}(E) \leq \mathbb{P}(E_0)$, so X^* is the probability-maximising payoff subject to the constraint of reaching the essential goal.

With the growth-optimal strategy, the vector of weights in the risky assets is (see Long (1990))

$$\mathbf{w}_t^{go} = \boldsymbol{\sigma}_t^{-1} \boldsymbol{\lambda}_t,$$

so the final wealth achieved is

$$W_T^{go} = \frac{W_0}{M_T}.$$

Hence, the success region for the optimal payoff can be rewritten as

$$E_0 = \left\{ \frac{W_T^{go}}{\beta_T} \geq h \frac{W_0}{\beta_0} \right\}.$$

In no case can the aspirational goal be reached with 100% probability under the probability measure of interest, which is \mathbb{P} . Indeed, \mathbb{P} and \mathbb{Q}^β are equivalent probability measures, so they have the same sets of zero measure. If the probability of E_0 under \mathbb{P} was one, it would be one under \mathbb{Q}^β too. But Equation (A.3) implies that $\mathbb{Q}^\beta(E_0)$ is strictly less than one as long as ri_{asp} is strictly greater than W_0/β_0 , the currently affordable income level. Thus, X^* satisfies the two conditions

$$\mathbb{P}(X^* \leq G_T) = 1,$$

$$\mathbb{P}(X^* < G_T) > 0.$$

By absence of arbitrage opportunities, it follows that the present value of X^* (i.e. the value of the optimal wealth process) is strictly less than the present value of G_T at any point of the accumulation period strictly before retirement. Hence, the level of affordable income with the probability-maximising strategy is strictly less than ri_{asp} until retirement.

A.3 Optimal Portfolio Strategy

The optimal wealth at date $t \leq T$ is

$$\begin{aligned} W_t &= \mathbb{E}_t \left[\frac{M_T}{M_t} X^* \right] \\ &= ri_{ess} \mathbb{E}_t \left[\frac{M_T}{M_t} \beta_T \right] \\ &\quad + [ri_{asp} - ri_{ess}] \mathbb{E}_t \left[\frac{M_T}{M_t} \beta_T \mathbb{1}_{E_0} \right] \\ &= ri_{ess} \beta_t + [ri_{asp} - ri_{ess}] \beta_t \mathbb{Q}_t^\beta(E_0), \end{aligned}$$

where $\mathbb{Q}_t^\beta(E_0)$ denotes the probability of E_0 under \mathbb{Q}^β conditional on \mathcal{F}_t .

To compute the probability of E_0 , we write β_T as

$$\beta_T = \beta_t \exp \left[\int_t^T \left[r_s + \sigma_{\beta,s} \lambda_{\beta,s} - \frac{\sigma_{\beta,s}^2}{2} \right] ds + \int_t^T \boldsymbol{\sigma}'_{\beta,s} d\mathbf{z}_s \right].$$

Assume that the Sharpe ratios of the MSR portfolio and the GHP are constant and that the volatility of the GHP is constant too. Then,

$$\begin{aligned} M_T \beta_T &= M_t \beta_t \exp \left[-\frac{1}{2} \left[\lambda_{MSR}^2 + \sigma_\beta^2 - 2\sigma_\beta \lambda_\beta \right] T \right. \\ &\quad \left. - \int_t^T [\boldsymbol{\lambda}_s - \boldsymbol{\sigma}_{\beta,s}]' d\mathbf{z}_s \right]. \end{aligned}$$

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Consider the process

$$\mathbf{z}_t^\beta = \mathbf{z}_t + \int_0^t [\boldsymbol{\lambda}_s - \boldsymbol{\sigma}_{\beta,s}] ds.$$

By Girsanov's theorem, it is a Brownian motion under \mathbb{Q}^β . We have

$$M_T \beta_T = M_t \beta_t \exp \left[\frac{1}{2} \left[\lambda_{MSR}^2 + \sigma_\beta^2 - 2\sigma_\beta \lambda_\beta \right] T - \int_t^T [\boldsymbol{\lambda}_s - \boldsymbol{\sigma}_{\beta,s}]' d\mathbf{z}_s^\beta \right]. \quad (\text{A.5})$$

The stochastic integral in this expression can be rewritten as

$$\int_t^T [\boldsymbol{\lambda}_s - \boldsymbol{\sigma}_{\beta,s}]' d\mathbf{z}_s^\beta = \int_t^T u_s dB_s,$$

where the process B is defined by

$$\begin{aligned} B_0 &= 0, \\ dB_s &= \frac{1}{u} [\boldsymbol{\lambda}_s - \boldsymbol{\sigma}_{\beta,s}]' d\mathbf{z}_s^\beta, \\ u &= \sqrt{\lambda_{MSR}^2 + \sigma_\beta^2 - 2\sigma_\beta \lambda_\beta}. \end{aligned}$$

It follows that $M_T \beta_T$ is log-normally distributed conditional on \mathcal{F}_t under \mathbb{Q}^β , so that the optimal wealth at date t is given by

$$\begin{aligned} W_t &= r_{ess} \beta_t + [r_{asp} - r_{ess}] \beta_t \mathcal{N}(Z_t), \\ Z_t &= \frac{1}{\eta_t} \left[\ln \frac{\beta_0}{h M_t \beta_t} - \frac{1}{2} \eta_t^2 \right], \\ \eta_t &= u \sqrt{T-t}. \end{aligned} \quad (\text{A.6})$$

By applying Ito's lemma, we obtain

$$\begin{aligned} dW_t &= r_{ess} d\beta_t + [r_{asp} - r_{ess}] [\mathcal{N}(Z_t) d\beta_t \\ &\quad + \beta_t n(Z_t) dZ_t] + [\dots] dt, \end{aligned} \quad (\text{A.7})$$

where the terms in dt do not matter here. Let $\mathbf{w}_{\beta,t}$ denote the portfolio fully invested

in the GHP. Matching the diffusion terms

in both sides of Equation (A.7), we have

$$\begin{aligned} W_t \boldsymbol{\sigma}_t \mathbf{w}_t^* &= r_{ess} \beta_t \boldsymbol{\sigma}_t \mathbf{w}_{\beta,t} + [r_{asp} - r_{ess}] \beta_t \boldsymbol{\sigma}_t \mathbf{w}_{\beta,t} \\ &\quad + [r_{asp} - r_{ess}] \frac{\beta_t n(Z_t)}{\eta_t} [\boldsymbol{\lambda}_t - \boldsymbol{\sigma}_{\beta,t}], \end{aligned}$$

so the optimal portfolio is given by

$$\begin{aligned} \mathbf{w}_t^* &= \frac{r_{ess} \beta_t}{W_t} \mathbf{w}_{\beta,t} \\ &\quad + \frac{[r_{asp} - r_{ess}] \beta_t}{W_t} \left[\mathcal{N}(Z_t) \mathbf{w}_{\beta,t} \right. \\ &\quad \left. + \frac{n(Z_t)}{\eta_t} [\boldsymbol{\sigma}_t^{-1} \boldsymbol{\lambda}_t - \mathbf{w}_{\beta,t}] \right]. \end{aligned}$$

Rearranging terms, we obtain

$$\begin{aligned} \mathbf{w}_t^* &= \varphi_t \mathbf{w}_t^{go} + [1 - \varphi_t] \mathbf{w}_{\beta,t}, \\ \varphi_t &= \frac{[r_{asp} - r_{ess}] \beta_t}{\eta_t W_t} n(Z_t). \end{aligned}$$

This equation expresses the optimal portfolio as a combination of two funds, namely the growth-optimal portfolio and the GHP.

One can also view the optimal portfolio as a combination of the MSR portfolio, the cash account and the GHP, by noting that the growth-optimal portfolio can be rewritten as

$$\mathbf{w}_t^{go} = \frac{\lambda_{MSR}}{\sigma_{MSR,t}} \mathbf{w}_{MSR,t}.$$

The MSR portfolio is fully invested in the risky assets, and its composition is given by

$$\mathbf{w}_{MSR,t} = \frac{\boldsymbol{\sigma}_t^{-1} \boldsymbol{\lambda}_t}{\mathbf{1}' \boldsymbol{\sigma}_t^{-1} \boldsymbol{\lambda}_t},$$

where $\mathbf{1}$ is a conforming vector of ones.

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Finally, to get the expression for the coefficient φ_t given in the text, it suffices to note that by Equation (A.6), the quantity Z_t is

$$Z_t = \mathcal{N}^{-1} \left(\frac{W_t - r_{ess}\beta_t}{[r_{asp} - r_{ess}]\beta_t} \right).$$

B. Stochastic Model for Monte-Carlo Simulations

This section describes the stochastic model and the parameter values used in stochastic simulations of target date funds and GBI strategies. The risk factors involved in this process are:

- the nominal term structure, which impacts the Goal Price Index for constant or deterministic replacement income cash flows;
- the real term structure, which replaces the nominal one in the case of inflation-indexed cash flows;
- the value of the stock index used as the PSP in target date funds and GBI strategies;
- the value of the bond index that serves as a safe building block in deterministic target date funds;
- the consumer price index, which impacts the Goal Price Index when income is indexed on inflation.

B.1 Nominal and Real Term Structure

For simplicity, we adopt a one-factor model for each term structure, namely the model of Vasicek (1977). The risk factor is the short-term rate, which follows a mean-reverting process:

$$dr_t = a[b - r_t]dt + \sigma_r dz_{rt}.$$

Here, z_r is a standard Brownian motion. The price of interest rate risk is a constant λ_r , which must be negative for long-term bonds to have a positive expected excess returns over short-term bills.

The dataset used to estimate nominal rate parameters consists of the nominal zero-coupon rates calculated by the method of Gürkaynak, Sack, and Wright (2007) and currently available on the website of the Federal Reserve at <https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html>, which we complete with the secondary market rate on three-month Treasury bills, also available from the Fed at <https://www.federalreserve.gov/releases/h15/>. All series are sampled at the monthly frequency.

There are five parameters to estimate: the speed of mean reversion a , the long-term mean b , the short-term volatility σ_r , the price of risk λ_r , and the initial value r_0 . Estimates are revised at each date a simulation is run in order to achieve consistency between the model-implied term structure and the observed one at the start date. To ensure that estimates do not vary too fast over time, we use a mixture of historical estimation over a moving window, which by nature implies stable estimates, and calibration, which implies a good fit between the model-implied and the observed yield curves.

Specifically, b is estimated as the mean of the nominal three-month rate or the two-year real rate over 20 years. The maturity of two years is taken in the case of the real term structure because it is the shortest available in the sample. a and σ_r are chosen in such a way that the model-implied long-term volatility and one-year autocorrelation match the sample moments of the short-term rate estimated over the same time frame. Formally, the long-term moments are defined as the limits of the volatility and the one-year autocorrelation as time goes to infinity (and therefore

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memory of initial conditions is lost):

$$\sigma_\infty = \lim_{t \uparrow \infty} \sqrt{\mathbb{V}[r_t]},$$

$$\rho_\infty = \lim_{t \uparrow \infty} \text{Corr}[r_{t-12}, r_t].$$

It can be shown that the long-term moments implied by the Vasicek model are

$$\mu_\infty = b, \quad \sigma_\infty = \frac{\sigma_r}{\sqrt{2a}}, \quad \rho_\infty = e^{-a},$$

so that the parameters can be recovered from the long-term moments as

$$a = -\log \rho_\infty, \quad b = \mu_\infty,$$

$$\sigma_r = \sigma_\infty \sqrt{2a}.$$

Moments ρ_∞ , μ_∞ and σ_∞ are estimated as the sample moments over the estimation period.

Parameters λ_r and r_0 are estimated by minimising the sum of squared differences between the model-implied zero-coupon yields and the "observed" ones.²⁰ In the Vasicek model, the zero-coupon rate of maturity u is given by

$$y(0, u) = \frac{D(u)}{u} r_0 - \frac{E(u)}{u},$$

with

$$D(u) = \frac{1 - e^{-au}}{a},$$

$$E(u) = \left[b - \frac{\sigma_r \lambda_r}{a} \right] [D(u) - u] + \frac{\sigma_r^2}{2a^2} \left[u - 2D(u) + \frac{1 - e^{-2au}}{2a} \right].$$

Table i shows the parameter estimates obtained through this procedure for the first day of January of the past five years. The most stable estimate is that of σ_n which is a volatility parameter, and is thus more robust to the choice of the sample than a first-order moment. Long-term mean estimates decrease over time because interest rates have been following a decreasing trend since the early 1980s. Because long-term mean is estimated over the 20 years preceding the calibration date, the decreasing pattern in rates implies that the initial short-term rate is always less than the long-term mean. Due to the mean reversion in the short rate process, this initial condition implies that a rise in interest rates is simulated.

Figure i displays the observed and model-implied yield curves on 2 January 2018. The fit is good for the nominal term structure and average for the real one. A better adjustment of the observed curve would be achieved by minimising the sum of

20 - Zero-coupon rates are not directly observed in practice, and they must be inferred from the market prices of Treasuries. See Gürkaynak, Sack, and Wright (2007).

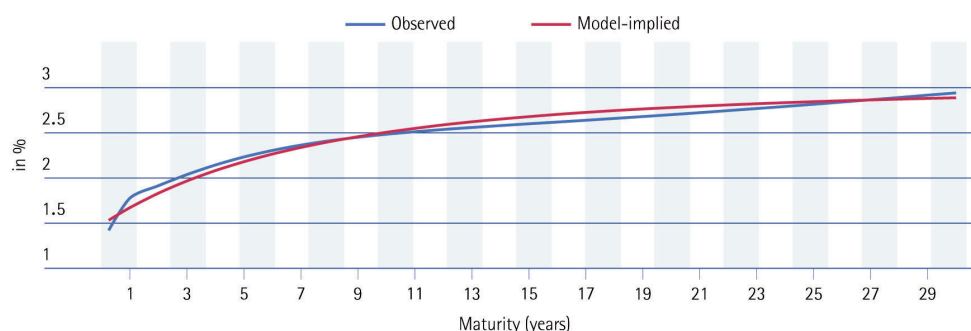
Table i: Examples of nominal and real interest rate parameter values.

Reference date	Nominal					Real				
	a	b	σ_r	λ_r	r_0	a	b	σ_r	λ_r	r_0
1 Jan. 2018	0.2313	0.0192	0.0137	-0.2199	0.0145	0.8523	-0.0045	0.0153	-0.6486	-0.0071
1 Jan. 2017	0.2063	0.0213	0.0136	-0.2941	0.0053	0.7410	-0.0020	0.0173	-0.5832	-0.0242
1 Jan. 2016	0.1950	0.0235	0.0135	-0.2499	0.0030	0.5134	0.0008	0.0161	-0.4184	-0.0086
1 Jan. 2015	0.1908	0.0262	0.0136	-0.1755	0.0007	0.5133	0.0022	0.0163	-0.2361	-0.0079
1 Jan. 2014	0.2200	0.0283	0.0142	-0.3868	-0.0024	0.6340	0.0033	0.0175	-0.5669	-0.0391

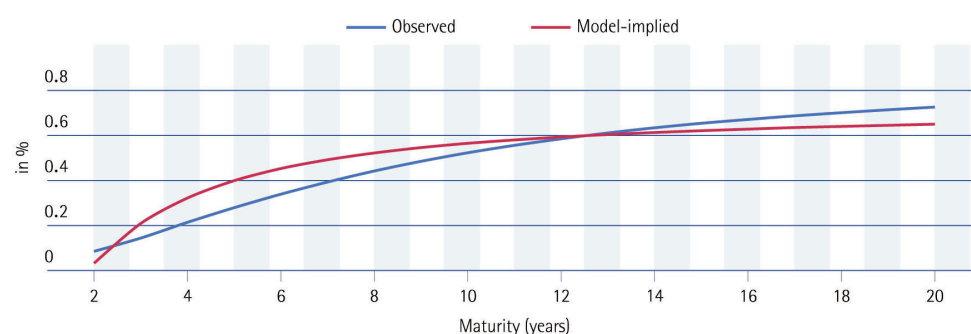
These parameter values for the Vasicek model have been estimated by the historical-calibration procedure described in the text.

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Figure i: Comparison of observed and model-implied zero-coupon yield curves on January 2, 2018.
(a) Nominal term structure.



(b) Real term structure.



"Observed" zero-coupon rates on 2 January 2018 are inferred from the market prices of Treasuries. Model-implied rates are the zero-coupon rates implied by the Vasicek model with parameters estimated by the mixed historical-calibration procedure described in the text over the period from January 1998 through January 2018.

squared errors over the five parameters, not just r_0 and λ_r . In any case, the Vasicek model has only one factor, so it cannot accurately match any arbitrary shape of the yield curve.

B.2 Stock and Bond Indices

The stock and the bond indices are modelled as diffusion processes with a constant volatility and a constant expected excess return over the nominal short-term rate. For instance, the stock index evolves as

$$\frac{dS_t}{S_t} = [r_t + \sigma_S \lambda_S] dt + \sigma_S dz_{St},$$

where z_S is a Brownian motion correlated to the Brownian motions of the nominal and the real short-term rates.

Sharpe ratios and volatilities are set to the values listed in Section 4.2, and they are borrowed from Merrill Lynch's capital market assumptions for 2017. They are summarised in Table ii.

B.3 Consumer Price Index

The consumer price index is a Geometric Brownian motion, the drift of which represents the annualised inflation rate:

$$\frac{d\Phi_t}{\Phi_t} = \pi dt + \sigma_\Phi dz_{\Phi,t}.$$

Expected inflation and the volatility of unexpected inflation are estimated from year-to-year monthly returns on the US price index for all items and all urban consumers, seasonally adjusted. The sample period goes from February 1978 to May 2017. Parameter values are displayed in Table ii.

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Table ii: Summary of constant parameter values.

	Sharpe ratio	Expected return (%)	Volatility (%)
Stock index	0.395	—	16.20
Bond	0.234	—	6.40
Consumer price index	—	2.20	1.20

	Correlations (%)			
	Real rate	Stock index	Bond index	Consumer price index
Nominal rate	0	0	−58.8	0
Real rate		0	0	0
Stock index			−14.8	0
Bond index				0

"Constant parameter values" are held constant from one simulation to the other, regardless of the date at which interest rate parameters are estimated.

B.4 Correlations

Instantaneous correlations are the correlations between the innovations to the various processes. They are set to the neutral value of zero, with two exceptions. The first is the correlation between the bond index and the nominal short-term rate, estimated as the empirical correlation between changes in the three-month Treasury bill rate and the returns on the BofA Merrill Lynch AAA US Treasury/Agency Master index between April 1978 and May 2017: the value is −0.588. The second non-zero correlation is the correlation between the stock and the bond indices, which is borrowed from Merrill Lynch's capital market assumptions for 2017 and is −14.75%.

C. Sensitivity of Simulation Analysis to Parameter Values

The simulated distribution of future replacement income under a given strategy (target date fund or GBI strategy) depends on the model for asset returns and the assumed parameter values. In this section, we let parameters vary to check the robustness of the base case results from Section 7.1.

C.1 Equity Parameters

Table iii shows the impact of equity volatility, by holding the risk premium or the Sharpe ratio constant. In the former case, changes in the Sharpe ratio offset changes in volatility to generate a constant expected excess return, and in the latter case, the risk premium changes by the same factor as the volatility.

Performance metrics, that is the expected funding ratio and the probabilities of reaching aspirational goals, appear to be more sensitive to the risk premium than to the volatility, both for the target date fund and the GBI strategy. When the risk premium is held constant, the probability of increasing the level of income by 50% ranges from 67.8% to 88.6% for the GBI strategy, but when the risk premium changes proportionally with volatility, the probability varies from 41.2% to 86.5%. These numbers show that it is particularly useful to seek to improve the performance of the performance-seeking building block in the GBI strategy.

Risk metrics, on the other hand, are sensitive to equity volatility, up to one important exception: the probability for the GBI strategy to experience at least

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one annual loss greater than 20% is consistently close to zero, and becomes truly positive only when the stock index becomes extremely risky, with a volatility of 32.4%. Even in this pessimistic case, the failure probability remains less than 3%. This is not so for the target date fund: the probability of missing the annual target at least once is 15.6% in the base case, which may not appear exceedingly high, but it jumps to 61.9% if volatility is assumed to be 24.3%. These results illustrate the lack of robustness of the deterministic target date fund with respect to parametric assumptions.

C.2 Interest Rate Parameters

To test for the effect of interest rate

parameters, Table iv displays the results of the *ex-ante* analysis for different estimation dates. The base case is 2 January 2018, and the estimation procedure is repeated at the beginning of January 2000, 2005, 2010 and 2015.²¹ The older the parameter values, the higher the simulated interest rates: indeed, interest rates have been decreasing since the beginning of the 1980s, so estimated long-term means and initial values for the short rate process have been decreasing too.

While success probabilities and the other metrics do depend on how interest rate parameters are set, the dependence does not appear to be very large. This applies

21- The exact day is the first day in the month that is not Saturday or Sunday.

Table iii: Effect of equity volatility on simulated distribution of the funding ratio.

	Target date fund						
	Base case	Change in equity volatility – Constant risk premium			Change in equity volatility – Constant Sharpe ratio		
		/2	x1.5	x2	/2	x1.5	x2
Expected funding ratio (%)	225.1	225.3	224.9	224.8	154.4	328.4	479.3
Success probability (%)							
130%	89.7	98.4	84.0	80.9	75.6	92.8	93.8
150%	81.1	93.3	75.0	71.8	52.6	87.5	89.4
200%	56.1	63.1	54.1	53.4	13.8	73.2	79.6
Volatility of annual changes (%)	10.8	6.7	15.5	20.4	6.6	15.9	21.3
Probability of annual loss > 20% (%)	15.6	0.1	62.0	88.5	0.3	51.6	78.4
Worst annual loss (%)	35.6	21.2	49.4	60.4	23.0	48.2	58.5
Goal-based investing strategy							
Expected funding ratio (%)	209.9	212.6	205.8	200.6	140.8	311.8	456.2
Success probability (%)							
130%	86.2	96.4	80.5	77.7	64.9	90.0	91.3
150%	75.4	88.2	70.5	67.7	40.7	83.9	86.4
200%	50.2	53.8	49.4	49.0	9.1	68.2	74.8
Volatility of annual changes (%)	12.1	6.9	17.5	23.0	6.4	18.8	26.2
Probability of annual loss > 20% (%)	0.0	0.0	0.1	2.7	0.0	0.0	1.9
Worst annual loss (%)	18.6	14.4	20.6	26.6	15.1	20.5	24.7

In the base case, equity volatility is 16.2% and the Sharpe ratio is 0.395, so the equity risk premium is 6.4%. Volatility is then divided by 2, multiplied by 1.5 and multiplied by 2, to take on the values 8.1%, 24.2% and 32.4%. To keep the equity risk premium constant, the Sharpe ratio is multiplied by 2, divided by 1.5 and divided by 2. When the Sharpe ratio is held constant, the risk premium changes by the same factor as the equity volatility, thus decreasing to 3.2%, or increasing to 9.6% or 12.8%. Interest rate parameters are estimated on 1 January 2018, and the investor plans to retire in 2038. The goal-based investing strategy is designed to cap the annual loss in the level of affordable income to 20% per year at most, and it has the same allocation to the performance-seeking portfolio as the target date fund at the beginning of each year.

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to both strategies. For instance, the probability for the GBI strategy to reach a funding level of 150% varied between 71.9% in January 2010 and 82.7% in January 2000. For the target date fund, the bounds were attained on the same dates, at 74.8% and 88.9%. As a general comment, the three probabilities change in the same way from one date to the other for both strategies. The volatility of changes in the level of affordable income, the loss probability and the worst annual loss are not substantially impacted either.

Table iv: Effect of interest rate parameters on simulated distribution of funding ratio.

	Target date fund				
	Base case – 1 Jan. 2018	3 Jan. 2000	3 Jan. 2005	1 Jan. 2010	1 Jan. 2015
Expected funding ratio (%)	225.1	284.4	236.8	201.9	248.5
Success probability (%)					
130%	89.7	94.6	90.8	86.2	93.7
150%	81.1	88.9	83	74.8	86.5
200%	56.1	71.1	59.7	46.9	64.8
Volatility of annual changes (%)	10.8	12.2	10.8	10.4	11.4
Probability of annual loss > 20% (%)	15.6	20.3	14.4	13.2	15.9
Worst annual loss (%)	35.6	38.8	35.7	36.1	35.8
	Goal-based investing strategy				
	Base case – 1 Jan. 2018	3 Jan. 2000	3 Jan. 2005	1 Jan. 2010	1 Jan. 2015
Expected funding ratio (%)	209.9	248.0	217.4	196.0	227.2
Success probability (%)					
130%	86.2	90.6	87.1	83.8	89.6
150%	75.4	82.7	77.1	71.9	80.4
200%	50.2	61.2	52.7	45.5	57.1
Volatility of annual changes (%)	12.1	13.1	12.1	11.6	12.6
Probability of annual loss > 20% (%)	0.0	0.0	0.0	0.0	0.0
Worst annual loss (%)	18.6	18.9	18.7	18.5	18.7

Interest rate parameters are successively estimated on the specified dates by the procedure described in Appendix B, holding other parameters fixed at their base case values. The investor starts to accumulate at the calibration date and plans to retire 20 years later, in 2038, 2020, 2025, 2030 or 2035.

Appendices

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About EDHEC-Risk Institute

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001.

This institute now boasts a team of close to 50 permanent professors, engineers and support staff, as well as 39 research associates from the financial industry and affiliate professors.

The Choice of Asset Allocation and Risk Management and the Need for Investment Solutions

EDHEC-Risk has structured all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds. EDHEC-Risk Institute has also developed an ambitious portfolio of research and educational initiatives in the domain of investment solutions for institutional and individual investors.

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Seven research programmes have been conducted by the centre to date:

- Investment Solutions in Institutional and Individual Money Management
- Equity Risk Premia in Investment Solutions
- Fixed-Income Risk Premia in Investment Solutions
- Alternative Risk Premia in Investment Solutions
- Multi-Asset Multi-Factor Investment Solutions
- Reporting and Regulation for Investment Solutions
- Technology, Big Data and Artificial Intelligence for Investment Solutions

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in the City of London in the United Kingdom; Nice and Paris in France.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- ETF, Indexing and Smart Beta Investment Strategies, in partnership with *Amundi*
- Regulation and Institutional Investment, in partnership with *AXA Investment Managers*
- Asset-Liability Management and Institutional Investment Management, in partnership with *BNP Paribas Investment Partners*
- New Frontiers in Risk Assessment and Performance Reporting, in partnership with *CACEIS*
- Exploring the Commodity Futures

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Risk Premium: Implications for Asset Allocation and Regulation,
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- Infrastructure Equity Investment Management and Benchmarking, in partnership with *Meridiam and Campbell Lutyens*

- Risk Allocation Framework for Goal-Driven Investing Strategies, in partnership with *Merrill Lynch Wealth Management*

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The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and global conferences.

To ensure the distribution of its research to the industry, EDHEC-Risk also provides professionals with access to its website, <https://risk.edhec.edu>, which is entirely devoted to international risk and asset management research. The website is aimed at professionals who wish to benefit from EDHEC-Risk's analysis and expertise in the area of applied portfolio management research. Its quarterly newsletter is distributed to more than 150,000 readers.

EDHEC-Risk Institute. Key Figures, 2014-2015

Number of permanent staff	48
Number of research associates & affiliate professors	36
Overall budget	€6,500,000
External financing	€7,025,695
Nbr of conference delegates	1,087
Nbr of participants at research seminars and executive education seminars	1,465

About EDHEC-Risk Institute

Research for Business

The Institute's activities have also given rise to executive education and research service offshoots. EDHEC-Risk's executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of asset-liability management for institutions and individuals, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of risk and investment management.

As part of its policy of transferring know-how to the industry, in 2013 EDHEC-Risk Institute also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.

EDHEC-Risk Institute Publications and Position Papers (2015-2018)



EDHEC-Risk Institute Publications (2015–2018)

2018

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- O'Kane, D. Initial Margin for Non-Centrally Cleared OTC Derivatives (June).

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