Precautionary Saving and the Marginal Propensity To Consume Out of Permanent Income

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Abstract

Because the budget constraint implies that consumption must eventually fully adjust to permanent shocks, intuition suggests that consumption-smoothers will have an immediate marginal propensity to consume of one out of permanent shocks. However, this paper shows that if consumers are impatient and experience both transitory and permanent income shocks, the immediate marginal propensity to consume out of permanent shocks is strictly less than one, because buffer-stock savers have a target wealth-to-permanent-income *ratio*; for a consumer starting at the target ratio, a positive shock to permanent income moves their actual wealth-to-permanent-income ratio below the target, temporarily boosting the saving rate.

Keywords: Precautionary saving, buffer-stock saving, consumption, marginal propensity to consume, permanent income

JEL Classification Codes: D81, D91, E21

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1 Introduction

Rigorous theoretical understanding of the effects of uncertainty on the marginal propensity to consume (MPC) out of transitory shocks to income is surprisingly recent: Kimball (1990a, 1990b) showed that for standard utility functions, the introduction of uncertainty in noncapital income increases the MPC at a given level of consumption, but not necessarily at a given level of wealth; and Carroll and Kimball (1996) show that the introduction of uncertainty causes the MPC to rise at any given level of wealth, but to increase more for consumers at lower levels of wealth.[1]

The response of consumption to *permanent* shocks to income (henceforth, the MPCP) is also an important question, for both micro and macroeconomic analysis of tax policies and business cycles, and for microeconomic analysis of inequality (in both consumption and income).[2] Yet no paper in the new literature has performed a general analysis of the MPCP, despite the presence of a large body of evidence from a variety of sources suggesting that permanent shocks are empirically quite large at the household level (MaCurdy (1982); Abowd and Card (1989); Carroll and Samwick (1997); Jappelli and Pistaferri (1999)).

There appears to be a presumption in the literature that in a model with impatient consumers the MPCP must equal one. Aside from its strong intuitive appeal to economists steeped in the proposition that consumption equals permanent income, this presumption draws its principal theoretical backing from Deaton (1991), who shows that in a model of impatient, liquidity-constrained consumers, when shocks to permanent income are the only form of income uncertainty, consumers who begin with zero wealth will exhibit an MPCP of one. This is because these consumers always set their consumption equal to their actual income. There seems to be a compelling explanation for this behavior: It is impossible to permanently insulate consumption from a permanent shock, and if consumption does not adjust immediately and fully, it will eventually need to adjust *more than* one-for-one to make up for the initial period of less-than-full adjustment. Consumption-smoothers will prefer to adjust fully now rather than less-than-fully now and more-than-fully later.

It turns out, however, that Deaton's result relies critically on the specific setup of his model. After deriving some new results that bolster Deaton's conjecture that in his specific model wealth tends to fall toward the absorbing state of zero where the MPCP is indeed one, this paper shows that if the model is parameterized in a way that allows for empirically realistic transitory as well as permanent

[1] This result is a direct implication of the concavity of the consumption function that Carroll and Kimball (1996) prove.

[2] By 'permanent shocks to income' I mean shocks to noncapital income; I will use the terms permanent income and permanent noncapital income interchangably in this paper, except where doing so might cause confusion because of the ambiguity the term 'permanent income' can have when consumers receive both capital and noncapital income.

shocks, and if consumers are impatient in the relevant sense, the MPCP can be substantially (though not enormously) less than one. The essential intuition comes from the target-saving behavior that emerges when consumers are both prudent and impatient. An increase in the level of permanent income leaves the *target* ratio of wealth to permanent income unchanged, but for a given level of initial wealth, a positive shock to permanent income reduces the ratio of actual wealth to permanent income, thus inducing the consumer to increase his saving rate because his wealth-to-permanent income ratio is now lower relative to its target. Thus consumption does not move up by the full amount of the income shock; the reciprocal logic holds for negative shocks to permanent income.

This paper is organized as follows. The first section sets up the model and demonstrates that a problem with both transitory and permanent shocks to noncapital income can be restated in a form such that all stock and flow variables are expressed as ratios to permanent noncapital income. The second section derives an expression for the marginal propensity to consume out of permanent shocks and explains qualitatively why it can be different from one. This section proceeds to show the relationship between the formula and Deaton's results, then derives a formula that applies to the more general model with both transitory and permanent shocks. Because the exact value of the MPCP out of permanent shocks cannot be determined except by simulation methods, the fourth section presents simulation results and finds that the marginal propensity to consume out of permanent shocks tends to fall in the range from 0.80 to 0.95 for a plausible range of parameter values of the general model with both transitory and permanent shocks. This section also shows that behavior of the ergodic population of consumers is very close to behavior of a single consumer with wealth equal to its target value, indicating that the computationally burdensome task of simulation may be unnecessary for the analysis of models in this class.

2 The Model

Consider a consumer solving the maximization problem

$$\begin{aligned}
V_t(X_t, P_t) &= \max_{\{C_s\}_t^T} \; u(C_t) + E_t\left[\sum_{s=t+1}^{T} \beta^{s-t} u(\tilde{C}_s)\right] \qquad (1) \\
&\text{s.t.} \\
W_t &= X_t - C_t, \\
X_{t+1} &= RW_t + Y_{t+1}, \\
Y_{t+1} &= P_{t+1}\epsilon_{t+1}, \\
P_{t+1} &= GP_tN_{t+1},
\end{aligned}$$

where W_t indicates the consumer's wealth at the end of period t, which accrues interest at gross rate $R = (1 + r)$ between periods; X_{t+1} indicates the level of the consumer's 'cash-on-hand,' the sum of beginning-of-period assets plus current-period noncapital income Y_{t+1}; actual noncapital income Y_{t+1} equals permanent noncapital income P_{t+1} multiplied by a mean-one transitory shock $\epsilon_{t+1}, E_t[\tilde{\epsilon}_{t+1}] = 1$;[3] and permanent noncapital income P_{t+1} is equal to its previous value, multiplied by a growth factor G, and modified by a mean-one shock N_{t+1}, $E_t[\tilde{N}_{t+1}] = 1$.[4] This problem is essentially identical to problems that have been analyzed in a number of papers on 'buffer-stock saving' beginning with Carroll (1992); it differs from the problem analyzed by Deaton (1991) primarily because liquidity constraints are absent. As usual, the recursive nature of the problem allows us to rewrite the problem as:

$$V_t(X_t, P_t) \;=\; \max_{\{C_t\}} \; u(C_t) + \beta E_t\left[V_{t+1}(\tilde{X}_{t+1}, \tilde{P}_{t+1})\right]. \tag{2}$$

As written, the problem has two state variables, the level of permanent income P_t and the level of cash-on-hand X_t. If utility is of the Constant Relative Risk Aversion (CRRA) form $u(C) = C^{1-\rho}/(1-\rho)$ it is possible to normalize all variables by the level of permanent income P_t and thereby to effectively reduce the number of state variables to one. Specifically, defining lower-case $x_t = X_t/P_t$, $c_t = C_t/P_t$, and so on, consider the problem in the second-to-last period of life,

$$\begin{aligned}
V_{T-1}(X_{T-1}, P_{T-1}) &= \left(\frac{1}{1-\rho}\right) \max_{\{C_{T-1}\}} \; C_{T-1}^{1-\rho} + E_{T-1}[\beta \tilde{X}_T^{1-\rho}] \\
&= \left(\frac{1}{1-\rho}\right) \max_{\{c_{T-1}\}} \; (P_{T-1}c_{T-1})^{1-\rho} + E_{T-1}[\beta(\tilde{P}_T\tilde{x}_T)^{1-\rho}] \\
&= \left(\frac{1}{1-\rho}\right) P_{T-1}^{1-\rho} \max_{\{c_{T-1}\}} \; c_{T-1}^{1-\rho} + E_{T-1}[\beta(G\tilde{N}_T x_T)^{1-\rho}] \\
&= P_{T-1}^{1-\rho} v_{T-1}(x_{T-1})
\end{aligned}$$

[3]The notational convention is that stochastic variables have a $\sim$ over them when their expectation is being taken, but not otherwise, on the grounds that equations where the expectation is being taken are equations where the time period from which the equation is being viewed is well-specified. Hence we write $P_{t+1} = GP_tN_{t+1}$ but if we need the period-t expectation we would write $E_t[\tilde{P}_{t+1}] = GP_tE_t[\tilde{N}_{t+1}]$.

[4]Note that the definition of permanent income here differs from Deaton's (1992) definition (which is often used in the macro literature), in which permanent income is the amount that a perfect foresight consumer could spend while leaving total (human and nonhuman) wealth constant.

where we define $v_T(x_T) = x_T^{1-\rho}/(1-\rho)$ and

$$\begin{aligned} v_t(x_t) &= \max_{\{c_t\}} \; u(c_t) + \beta E_t[(G\tilde{N}_{t+1})^{1-\rho} v_{t+1}(\tilde{x}_{t+1})] && (3) \\ &\text{s.t.} \\ w_t &= x_t - c_t, && (4) \\ x_{t+1} &= \left(\frac{R}{GN_{t+1}}\right) w_t + \epsilon_{t+1}. && (5) \end{aligned}$$

Note that equation (3) has only a single state variable, x_t, and recusion on this equation yields a 'normalized' value function function for any period prior to $T-1$. The full value function $V_t(X_t, P_t)$ is recovered simply from $V_t(X_t, P_t) = P_t^{1-\rho} v_t(X_t/P_t)$. The first order condition is

$$c_t^{-\rho} = R\beta E_t[(G\tilde{N}_{t+1})^{-\rho} \tilde{c}_{t+1}^{-\rho}], \tag{6}$$

and Carroll (1996) shows that the problem defines a contraction mapping if the 'impatience condition' originally derived by Deaton (1991)

$$R\beta E_t[(G\tilde{N}_{t+1})^{-\rho}] < 1 \tag{7}$$

holds, so that as the horizon recedes the consumption function $c_t(x_t)$ approaches an invariant function $c(x)$ which we define as the infinite-horizon solution to the problem, and which will be used as the object of analysis in the rest of this paper; we will refer to the term on the LHS of (7) as the 'coefficient of impatience.'

Some important conclusions can be drawn simply from the fact that the model can be rewritten in ratio form. The first is that because the level of consumption can be rewritten as $C_t = c(x_t)P_t$ for some invariant function $c(x)$, the only way the elasticity of consumption with respect to permanent income P_t can be different from one is if there is a correlation between P_t and x_t. But of course such a correlation does exist: Both P_t and x_t are influenced by the realization of the stochastic shock to permanent income N_t. Furthermore, both will reflect residual effects of the previous shocks to permanent income, $N_{t-1}, N_{t-2}, \ldots$. It is these effects of the permanent shocks on the cash-on-hand to permanent-income *ratio* that will be the key to understanding the results below.

Another important insight is that if the distribution of x_t is ergodic (which to my knowledge remains an important unproven conjecture,[5] but which both intuition and simulations suggest is true), then *eventually* the infinite-horizon MPCP must be one because ergodicity of x_t means that the expectation as of time t of x_s as

[5]Similar ergodicity results can be found in Laitner (1992) and Aiyigari (1994), but both of these papers assume liquidity constraints so their results are not directly applicable.

$s \to \infty$ is the same for any particular realizations of $N_t, N_{t-1}, N_{t-2}, \ldots$, implying that as $s \to \infty$ the time-t expectation of $c(x_s)P_s$ depends only on the level of P_t.

But the 'marginal propensity to consume' out of a shock has traditionally been defined as the immediate effect, not the total eventual effect, and so we now turn to the question of how consumption is affected in period t by the contemporaneous realization of the shock to permanent income N_t.

3 The Marginal Propensity to Consume Out Of Permanent Income

3.1 The Perfect Foresight Case

As a baseline for comparison, it is useful to derive the relationship between consumption and permanent income in the perfect foresight framework.[6] A standard result in consumption theory is that in the infinite horizon perfect foresight version of the model above (i.e. a version in which $\epsilon_t = N_t = 1 \, \forall \, t$), the level of consumption is given by

$$C_t \;=\; (1 - R^{-1}(R\beta)^{1/\rho}) \left[RW_{t-1} + \left(\frac{P_t}{1 - G/R} \right) \right]. \tag{8}$$

While strictly speaking there is no such thing as a 'shock' to permanent income in the perfect foresight model, it is of course possible to calculate how consumption would change with a change in permanent income. The answer is given by

$$\left(\frac{dC_t}{dP_t} \right) \;=\; \left(\frac{1 - R^{-1}(R\beta)^{1/\rho}}{1 - G/R} \right), \tag{9}$$

which I will refer to henceforth as the MPCP for the perfect foresight model. This quantity is less than one if

$$\begin{aligned} G/R &< R^{-1}(R\beta)^{1/\rho} && (10) \\ G &< (R\beta)^{1/\rho} && (11) \\ 1 &< R\beta G^{-\rho}. && (12) \end{aligned}$$

Notice that this is exactly the opposite of the 'impatience' condition (7). The interpretation is that in the perfect foresight framework, only the *patient* consumers

[6] Results would be similar in the certainty equivalent framework obtained under quadratic utility, or in a model with Constant Absolute Risk Aversion with a nonstochastic component of income growing at rate G.

have an MPCP of less than one. This makes intuitive sense: Patient consumers prefer to consume more in the future than in the present, so they do not spend all of the increase in income today.

Although this perfect foresight framework is often presented as the formalization of Milton Friedman's (1957) Permanent Income Hypothesis, the model implies that consumption responds one-for-one to a change in permanent income P_t only if $(R\beta)^{1/\rho} = G$. For plausible parameter values the model can easily predict an MPCP of anywhere between 0 and 6 (see the table below for a paramterization that implies an MPCP of 6). This observation casts doubt upon the proposition that it is appropriate to treat the perfect foresight model as a formalization of Friedman (1957). For an argument that the buffer-stock model with impatient but prudent consumers considered below is a much better match to Friedman's original description of the PIH model, see Carroll (2001).

3.2 The Response to Permanent Income Shocks

The natural definition of the MPCP in a model with shocks is the derivative of C_{t+1} with respect to N_{t+1}, given an initial level of savings $W_t = w_t P_t$,

$$\begin{aligned} \frac{dC_{t+1}}{dN_{t+1}} &= \frac{dP_{t+1}c(x_{t+1})}{dN_{t+1}} && (13) \\ &= \frac{d}{dN_{t+1}}\left[GP_tN_{t+1}c(\frac{R}{GN_{t+1}}w_t + \epsilon_{t+1})\right]. && (14) \end{aligned}$$

This equation reveals a minor conceptual difficulty: the effect of N_{t+1} on C_{t+1} depends not only on the value of w_t but also on the realization of ϵ_{t+1}, and so in principle there are two 'state variables' (other than the scaling variable P_t) that determine the *ex post* MPCP. However, since ϵ_{t+1} is an i.i.d. random variable, it is easy and intuitive to calculate the 'expected MPCP' as

$$\begin{aligned} E_t\left[\frac{d}{d\tilde{N}_{t+1}}GP_t\tilde{N}_{t+1}\tilde{c}_{t+1}\right] &= GP_tE_t\left[\tilde{N}_{t+1}\frac{dc(\tilde{x}_{t+1})}{d\tilde{N}_{t+1}} + c(\tilde{x}_{t+1})\right] \\ &= GP_tE_t\left[\tilde{N}_{t+1}c'(\tilde{x}_{t+1})\frac{d\tilde{x}_{t+1}}{d\tilde{N}_{t+1}} + c(\tilde{x}_{t+1})\right] \\ &= GP_tE_t\left[\tilde{N}_{t+1}c'(\tilde{x}_{t+1})\frac{d}{d\tilde{N}_{t+1}}\left(\left(\frac{R}{G\tilde{N}_{t+1}}\right)w_t + \tilde{\epsilon}_{t+1}\right) + c(\tilde{x}_{t+1})\right] \\ &= GP_tE_t\left[c(\tilde{x}_{t+1}) - c'(\tilde{x}_{t+1})\left(\frac{R}{G\tilde{N}_{t+1}}\right)w_t\right], && (15) \end{aligned}$$

or, since $E_t[\tilde{P}_{t+1}] = GP_t$, define $\bar{P}_{t+1}$ as 'expected permanent income' and rewrite

(15) as

$$E_t\left[\frac{d}{d\tilde{N}_{t+1}} GP_t\tilde{N}_{t+1}\tilde{c}_{t+1}\right] = \bar{P}_{t+1}E_t\left[c(\tilde{x}_{t+1}) - c'(\tilde{x}_{t+1})\left(\frac{R}{G\tilde{N}_{t+1}}\right)w_t\right] \quad (16)$$

which leads to the natural definition of the MPCP, $\chi(w_t)$, as the expression multiplying the expected level of permanent income,

$$\chi(w_t) = E_t\left[c(\tilde{x}_{t+1}) - c'(\tilde{x}_{t+1})\left(\frac{R}{G\tilde{N}_{t+1}}\right)w_t\right]. \quad (17)$$

3.3 The Deaton Case (Permanent Shocks Only)

Note first how this expression maps into Deaton's (1991) finding that for consumers who begin with zero wealth the marginal propensity to consume out of P_{t+1} is one. Such consumers have $w_t = 0$ and therefore the second term on the RHS in equation (16) drops out. Deaton also assumed that there were no transitory shocks to income, so that $\epsilon_{t+1} \equiv 1$. Finally, his consumers were sufficiently impatient so that their consumption at $c(x)$ was equal to one at $x = 1$. Hence the MPCP was given by $\chi(0) = E_t[c(1)] = 1$.

To really understand Deaton's result, it is necessary to recall why it must be that $c(1) = 1$.[7] Consider the first order condition for the unconstrained optimization problem,

$$c(x_t)^{-\rho} = R\beta E_t[(G\tilde{N}_{t+1})^{-\rho}c(\tilde{x}_{t+1})^{-\rho}]. \quad (18)$$

The consumer will be constrained at $c_t = x_t = 1$ iff the marginal utility of consuming 1 (which is $1^{-\rho} = 1$) is greater than the marginal utility of saving $w_t = 0$, i.e. if

$$\begin{aligned} 1 &> R\beta E_t[(G\tilde{N}_{t+1})^{-\rho}c((R/G\tilde{N}_{t+1}) * 0 + 1)^{-\rho}] \\ 1 &> R\beta E_t[(G\tilde{N}_{t+1})^{-\rho}] \end{aligned} \quad (19)$$

where the second line follows from the first because with $w_t = 0$, $x_{t+1} = \epsilon_{t+1} = 1 = x_t$. But notice that equation (19) is identical to the impatience condition which we have already imposed, equation (7). Thus in imposing the impatience condition we guarantee Deaton's result that a consumer with zero wealth who experiences only permanent shocks will remain at zero wealth forever. Zero wealth is an absorbing state.

[7] The following is intended as a loose intuitive argument rather than a rigorous derivation; in particular it mixes logic from the constrained and unconstrained optimization problems. See Deaton (1991) for the rigorous version.

What Deaton was unable to prove, but conjectured must be true, was that a liquidity-constrained consumer who starts with positive wealth will always eventually run down that wealth to reach the absorbing state of zero wealth. Consider the accumulation equation for wealth,[8]

$$w_{t+1} \quad = \quad (R/GN_{t+1})w_t + 1 - c(1 + (R/GN_{t+1})w_t). \tag{20}$$

Carroll and Kimball (1999) show that the marginal propensity to consume out of transitory income in a problem with liquidity constraints is always greater than the MPC in the unconstrained case. We also know, from combining Kimball (1990a) and Carroll and Kimball (1996), that the MPC in the unconstrained case with noncapital income risk is greater than the MPC without noncapital income risk. But from (8) we know that the MPC in the unconstrained case with no uncertainty is

$$\underline{c} = (1 - R^{-1}(R\beta)^{1/\rho}), \tag{21}$$

and so the Carroll and Kimball (1996) results tell us that

$$\begin{aligned} c(1 + (R/GN_{t+1})w_t) \quad &> \quad c(1) + \underline{c}(R/GN_{t+1})w_t & (22) \\ &= \quad 1 + \underline{c}(R/GN_{t+1})w_t & (23) \end{aligned}$$

where the equality uses $c(1) = 1$. Substituting in equation (20),

$$\begin{aligned} w_{t+1} \quad &< \quad (R/GN_{t+1})w_t - \underline{c}(R/GN_{t+1})w_t \\ &< \quad (R/GN_{t+1})w_t(1 - \underline{c}). & (24) \end{aligned}$$

From this we have (substituting (21) into (24))

$$\begin{aligned} w_{t+1} \quad &< \quad (R/GN_{t+1})w_t R^{-1}(R\beta)^{1/\rho} \\ &= \quad [(R\beta)^{1/\rho}/GN_{t+1}]w_t \end{aligned}$$

implying

$$E_t[w_{t+1}] \quad < \quad E_t[(R\beta)^{1/\rho}/G\tilde{N}_{t+1}]w_t. \tag{25}$$

But note that if $\tilde{N}_{t+1}$ is lognormally distributed then the impatience condition (7) implies that the expression in brackets on the RHS of equation (25) is less than one, implying

$$E_t[w_{t+1}] \quad < \quad w_t. \tag{26}$$

[8]Substitute (5) into (4) and roll forward one period.

Thus, at any positive level of wealth $w_t > 0$, wealth is expected to fall toward zero. Note that this condition does not guarantee that wealth ever reaches zero in finite time, because in principle it is possible (though arbitrarily improbable) to draw an arbitrarily long sequence of low draws of N_t. On the other hand, equation (26) does rule out the possibility that Deaton raised (but doubted) that some positive level of wealth $\underline{w}$ could exist such that if $w_t > \underline{w}$ the consumption rule might never allow wealth to fall below $\underline{w}$, thus preventing the consumer from ever reaching the absorbing state of $w_t = 0$. Hence, in Deaton's model, the wealth ratio falls toward zero, and if it ever reaches zero, the MPCP equals one ever after.

3.4 The General Case (Transitory and Permanent Shocks)

In the real world households experience both transitory and permanent shocks to their incomes. A natural supposition might be that since nothing can be done to insulate consumption in the long run against the permanent shocks, the presence or absence of transitory shocks should not affect the MPC out of permanent shocks. This section shows otherwise.

Consider the behavior of consumption around the 'target' level of wealth $\bar{w}$ defined as the level of wealth such that $E_t[\tilde{w}_{t+1}] = w_t$.[9]

$$\begin{aligned}
w_{t+1} &= (R/GN_{t+1})w_t + \epsilon_{t+1} - c((R/GN_{t+1})w_t + \epsilon_{t+1}) && (27)\\
E_t[\tilde{w}_{t+1}] &= E_t[(R/G\tilde{N}_{t+1})]w_t + 1 - E_t\left[c((R/G\tilde{N}_{t+1})w_t + \tilde{\epsilon}_{t+1})\right] \\
\bar{w} &= \bar{w}E_t[(R/G\tilde{N}_{t+1})] + 1 - E_t\left[c((R/G\tilde{N}_{t+1})\bar{w} + \tilde{\epsilon}_{t+1})\right] \\
E_t\left[c((R/G\tilde{N}_{t+1})\bar{w} + \tilde{\epsilon}_{t+1})\right] &= 1 + \bar{w}\left(E_t[(R/G\tilde{N}_{t+1})] - 1\right). && (28)
\end{aligned}$$

Now recall that Carroll and Kimball (1996) have shown that the marginal propensity to consume under uncertainty is strictly greater than the MPC in the corresponding perfect certainty model, and therefore we know that $c'(x_{t+1}) > \underline{c}$ where as above $\underline{c} = 1 - R^{-1}(R\beta)^{1/\rho}$. Using these facts in the formula for $\chi(\bar{w})$ gives

$$\begin{aligned}
\chi(\bar{w}) &= E_t\left[c(\tilde{x}_{t+1}) - c'(\tilde{x}_{t+1})\left(\frac{R}{G\tilde{N}_{t+1}}\right)w_t \ \ |w_t = \bar{w}\right] && (29)\\
&< 1 + (E_t[R/G\tilde{N}_{t+1}] - 1)\bar{w} - \underline{c}E_t[R/G\tilde{N}_{t+1}]\bar{w} && (30)\\
&= 1 + (E_t[R/G\tilde{N}_{t+1}](1 - \underline{c}) - 1)\bar{w} && (31)\\
&= 1 + (E_t[R/G\tilde{N}_{t+1}](R^{-1}(R\beta)^{1/\rho}) - 1)\bar{w} && (32)\\
&= 1 + (E_t[(R\beta)^{1/\rho}/G\tilde{N}_{t+1}] - 1)\bar{w}. && (33)
\end{aligned}$$

[9] Carroll (1996) proves that such a target will exist if consumers satisfy the impatience condition, and Carroll (1992) shows that average actual wealth in a population of simulated consumers is close to the target.

But as noted above, if $\tilde{N}_{t+1}$ is lognormally distributed, the impatience condition (7) implies that $E_t[(R\beta)^{1/\rho}/G\tilde{N}_{t+1}] < 1$ and thus that $\chi(\bar{w}) < 1$ if $\bar{w} > 0$. Hence, at the target level of wealth the MPCP is strictly less than one.

We can also say something about how $\chi(w_t)$ varies with the level of wealth. Its derivative with respect to wealth is given by

$$\begin{aligned} \left(\frac{d}{dw_t}\right)\chi(w_t) &= E_t\left[c'(\tilde{x}_{t+1})(R/G\tilde{N}_{t+1}) - c'(\tilde{x}_{t+1})(R/G\tilde{N}_{t+1}) - c''[\tilde{x}_{t+1}](R/G\tilde{N}_{t+1})^2\right] \\ &= E_t[-c''(\tilde{x}_{t+1})(R/G\tilde{N}_{t+1})^2]. \end{aligned} \tag{34}$$

But Carroll and Kimball (1996) prove that for problems in the class considered here the consumption function is strictly concave, $c''(x) < 0$, and since $(R/G\tilde{N}_{t+1})^2$ is certainly positive, equation (34) implies that the marginal propensity to consume out of permanent shocks is increasing in the level of wealth.

These results appear to be the most that can be said analytically about the characteristics of $\chi(w_t)$. To obtain quantitative results for the average behavior of a population of consumers behaving according to the optimal rule it is necessary to turn to simulations.

4 Simulation Results

Table 1 presents simulation results for the average value of χ (labelled "Mean χ") that arises in steady-state among a population of consumers all behaving according to the model outlined above, under a baseline set of parameter values and a variety of alternatives.

The baseline calibration of the income process is taken from Carroll (1992), who finds that household-level data from the *Panel Study of Income Dynamics* are reasonably well characterized by the assumption that N_t is lognormally distributed with standard deviation $\sigma_N = .10$, while the process for transitory income has two parts: With probability p, income is zero, and with probability $(1-p)$ the transitory shock ϵ_t is equal to $1/(1-p)$ times the value of a shock drawn from a lognormal distribution with standard deviation $\sigma_\epsilon = .10$ and mean value one, so that $E_t[\tilde{\epsilon}_{t+1}] = 1$ as assumed above. Income growth at the household level is roughly $G = 1.03$.

The baseline calibration for the interest rate and time preference rate are commonly-used values in macroeconomics, $R = 1.04$, $\beta = 0.96$. The baseline coefficient of relative risk aversion is $\rho = 3$, in the middle of the range from 1 to 5 generally considered plausible.

The first row of the table presents results for the baseline parameter values. The main result is found in the column labelled "Mean χ"; for comparison, the table

Deviations[a]	Impatience[b]	Mean c	Mean w	Mean c'	Mean χ	χ_{∞}^{PF}	χ_{40}^{PF}
None (Baseline)	0.969	1.012	0.620	0.235	0.873	4.053	1.355
$\beta = 0.98$	0.989	1.017	0.805	0.160	0.896	3.364	1.276
$\beta = 0.90$	0.909	1.009	0.440	0.370	0.848	6.181	1.615
R = 1.02	0.950	1.000	0.547	0.276	0.857	N/A[c]	1.477
R = 1.06	0.988	1.031	0.772	0.180	0.898	1.806	1.255
G = 1.02	0.998	1.030	1.009	0.122	0.918	2.027	1.232
G = 1.04	0.941	1.005	0.515	0.301	0.855	N/A[c]	1.494
$\rho = 1$	0.979	1.005	0.231	0.319	0.936	4.160	1.367
$\rho = 4$	0.978	1.016	0.839	0.200	0.857	4.040	1.353
$\sigma_N = 0.05$	0.927	1.006	0.497	0.315	0.853	4.053	1.355
$\sigma_N = 0.12$	0.994	1.020	0.838	0.158	0.900	4.053	1.355
p = 0.0005	0.969	1.006	0.320	0.292	0.920	4.053	1.355
p = 0.05	0.969	1.034	1.470	0.177	0.787	4.053	1.355
$\sigma_\epsilon = 0.05$	0.969	1.012	0.582	0.239	0.874	4.053	1.355
$\sigma_\epsilon = 0.15$	0.969	1.014	0.681	0.228	0.870	4.053	1.355
σ_N=0, G = 1.00	0.998	1.063	1.585	0.067	0.963	1.013	1.027
σ_N=0, G = 1.01	0.969	1.020	0.670	0.209	0.882	1.351	1.123
σ_N=0, G = 1.02	0.941	1.011	0.539	0.282	0.860	2.027	1.232

Notes: [a]This column indicates parameters that differ from the baseline. The baseline values are $R = 1.04, \beta = 0.96, G = 1.03, \rho = 3, \sigma_N^2 = 0.1, \sigma_\epsilon = 0.1, p = 0.005$. The first row presents results when all parameters are at their baseline values.

[b]This column calculates the value of the impatience coefficient defined in equation (7).

[c]The infinite horizon perfect foresight solution is not well defined for this configuration of parameter values because $R \leq G$.

Table 1: The MPCP χ For Baseline And Alternative Parameter Values

also presents, where applicable,[10] the MPCP implied by the perfect foresight infinite horizon version of the model (labelled "χ_{∞}^{PF}"), and from a perfect foresight model for a consumer of average age (45) who has twenty years of work and twenty years of retirement ahead (labelled "χ_{40}^{PF}").[11]

Under the baseline parameter values, the population-average value of χ is about 0.87. As the remainder of the table shows, the population-average value of χ is between about 0.8 and 0.95 for all parametric configurations.

In addition to χ, the table presents population-average values of each of the terms that made up χ from (17), reproduced here for convenience:

$$\chi(w_t) = E_t\left[c(\tilde{x}_{t+1})] - c'(\tilde{x}_{t+1})\left(\frac{R}{G\tilde{N}_{t+1}}\right) w_t\right]. \tag{35}$$

Recall that at the target level of $\bar{w}$ equation (28) tells us that

$$E_t\left[c(\tilde{x}_{t+1})\right] \;=\; 1 + \bar{w}\left(E_t[R/G\tilde{N}_{t+1}] - 1\right). \tag{36}$$

Since $E_t[R/G\tilde{N}_{t+1}]$ will generally be a number close to one, this first term in the $\chi(w)$ expression could be substantially different from one only if consumers ended up holding large values of w. But since they are impatient by assumption, they are not likely to end up with large values of w. This reasoning is confirmed by the column of the table labelled "Mean c," which finds values very close to 1 for all parametric combinations.

Thus, most of the variation in the average value of χ across parametric choices is attributable to differences the $E_t[-c'(\tilde{x}_{t+1})(R/G\tilde{N}_{t+1})w_t]$ term. Making consumers more patient has two effects on this term: On the one hand, it increases the level of target wealth $\bar{w}$, which would reduce $\chi(w_t)$; on the other hand, the MPC c' declines with the level of wealth, which would tend to increase $\chi(w_t)$. The near-constancy of χ indicates that these two effects are of roughly offsetting magnitude across different parametric choices.

The relative stability of χ for the buffer-stock model contrasts sharply with the MPCP for the infinite horizon perfect foresight model, for which the MPCP is always greater than 1.8 in the first panel of the table, and rises as high as 6.2. The reason the MPCP in the PF model is always greater than one is that our consumers all satisfy the impatience criterion; inspection of (9) will verify that the MPCP must be greater than one if the impatience criterion is satisfied. This makes sense; impatient

[10]For the infinite horizon MPCP to exist, the condition $R > G$ must hold, but this condition is not required to solve the stochastic model.

[11]The assumption is that upon retirement, total noncapital income (including Social Security and pension income) drops permanently to about 70 percent of preretirement salary, a calibration that roughly matches empirical evidence for the U.S.

perfect-foresight consumers, upon learning that their income will be higher forever, will tend to increase their consumption by more than the increase in current income. However, what may not have been obvious *ex ante* is how *much* greater than 1 the MPCP typically is in the PF model. Results for the finite-horizon perfect foresight model are less extreme than for the infinite horizon version, but even in the finite-horizon model the MPCP is always at least 1.2 in the upper panel of the table.

The second panel of the table (the last three rows) presents results when the permanent shocks are shut down and income growth is reduced; the most important result is for the case where there is no growth at all in income, so that $(R\beta)^{1/\rho} \approx G$, which, as noted earlier, is the condition that guarantees an MPCP of 1 in the perfect foresight model (the actual MPCP reported in the table is slightly above 1 because $R\beta$ is slightly below 1 for the baseline values $(R, \beta) = (1.04, 0.96)$). In the absence of permanent shocks, the impatience condition is (barely) satisfied and the stochastic version of the model can be solved with transitory shocks, generating an average χ of about 0.96.

The remaining two rows of the panel show the consequences when the expected growth rate of income rises to 1 percent and 2 percent: The PF MPCP increases sharply, to slightly over 2 when $G = 1.02$; in the finite-horizion PF model, the MPCP rises to slightly over 1.2. In contrast, χ falls to 0.86 in the stochastic version of the model. This experiment highlights the interesting point that the relationship between impatience and the MPCP is of opposite *sign* in the stochastic and nonstochastic versions of the model.

The principal message from the table is that if consumers are impatient but prudent, optimal behavior implies an immediate MPC out of permanent shocks that is somewhat less than one (but not enormously less) for a wide variety of parameter values. More broadly, the value of the MPCP is much less sensitive to parameter values in the stochastic version of the model than in the perfect foresight version.

A final point deserves elaboration. The theoretical results derived in section 3 applied only at the target level of wealth. Yet table 1 shows that the conclusions reached for the target level of wealth hold for populations distributed according to the ergodic distributions. Since finding the ergodic distributions requires a considerable amount of extra work, it would be worthwhile to see whether results at exactly the target levels of wealth are a good proxy for results from the ergodic populations.

Table 2 presents the main statistics of interest, calculated both as an average across the ergodic populations and at the target value of x or s (depending on the argument of the function). The message of the table is simple: The target values are always very close to the population-average values among the ergodic distribution. This suggests that theoretical work along the lines of that conducted in section 3 is likely to be both qualitatively and quantitatively a good guide to the behavior of an entire population. Since many more propositions can be proven for the target level

Deviations[a]	Impatience[b]	Wealth w Mean	Wealth w Target	MPC c' Mean	MPC c' Target	MPCP χ Mean	MPCP χ Target
None (Baseline)	0.969	0.620	0.602	0.235	0.230	0.873	0.871
$\beta = 0.98$	0.989	0.805	0.764	0.160	0.154	0.896	0.894
$\beta = 0.90$	0.909	0.440	0.433	0.370	0.370	0.848	0.847
R = 1.02	0.950	0.547	0.535	0.276	0.267	0.857	0.855
R = 1.06	0.988	0.772	0.733	0.180	0.173	0.898	0.897
G = 1.02	0.998	1.009	0.929	0.122	0.112	0.918	0.917
G = 1.04	0.941	0.515	0.505	0.301	0.296	0.855	0.854
$\rho = 1$	0.979	0.231	0.221	0.319	0.313	0.936	0.935
$\rho = 4$	0.978	0.839	0.811	0.200	0.196	0.857	0.855
$\sigma_N = 0.05$	0.927	0.497	0.488	0.315	0.313	0.853	0.851
$\sigma_N = 0.12$	0.994	0.838	0.785	0.158	0.147	0.900	0.899
p = 0.0005	0.969	0.320	0.303	0.292	0.259	0.920	0.918
p = 0.05	0.969	1.470	1.388	0.177	0.172	0.787	0.780
$\sigma_\epsilon = 0.05$	0.969	0.582	0.573	0.239	0.239	0.874	0.874
$\sigma_\epsilon = 0.15$	0.969	0.681	0.648	0.228	0.215	0.870	0.868
σ_N=0, G = 1.00	0.998	1.585	1.398	0.067	0.065	0.963	0.961
σ_N=0, G = 1.01	0.969	0.670	0.652	0.209	0.200	0.882	0.881
σ_N=0, G = 1.02	0.941	0.539	0.529	0.282	0.274	0.860	0.859

Notes: [a]This column indicates parameters that differ from the baseline. The baseline values are $R = 1.04, \beta = 0.96, G = 1.03, \rho = 3, \sigma_N^2 = 0.1, \sigma_\epsilon = 0.1, p = 0.005$. The first row presents results when all parameters are at their baseline values.

[b]This column calculates the value of the impatience coefficient defined in equation (7).

Table 2: Population-Mean Results Versus Results At Target Values

of wealth than for the behavior of the ergodic population, and since it is possible to obtain quantitative results for the target values of a model without simulating, this suggests that future theoretical and quantitative work with this model may be able to dispense with simulation altogether, considerably reducing the computational demands of working with this class of models.

5 Conclusion

Intuition suggests that rational forward-looking consumers should have a marginal propensity to consume of one out of permanent shocks. This paper shows that while this intuition is not correct, or even close to correct, for the canonical infinite horizion perfect-foresight version of the optimization model, it is approximately right for the 'buffer-stock' version of the model that arises when consumers are impatient and have a standard precautionary saving motive. The reason the MPCP is somewhat less than one in the buffer-stock model is that an increase in permanent income reduces the ratio of wealth to permanent income, thus (temporarily) increasing the amount of precautionary saving. Simulations show that across a wide range of assumptions about the degree of impatience, the marginal propensity to consume out of permanent shocks is generally in or near the range from 0.80 to about 0.95.

The results in this paper are important for three reasons. First, empirical evidence from household surveys indicates that households experience large permanent shocks to their incomes of precisely the kind studied here, and no existing paper has provided a general theoretical analysis of the effects of these kinds of shocks on consumption. Second, the sharp contrast between the results for the stochastic and nonstochastic models, and the fact that the results for the stochastic model are much more plausible, provides another reason economists should avoid using the perfect foresight model for quantitative analysis. Finally, the paper provides a formal justification (that many economists probably did not know was lacking in the traditional perfect foresight framework) for the usual assumption that permanent increases or decreases in taxes should result in consumption responses of roughly the same size (though the scrupulous policy adviser should warn that the response is likely to be modestly less than one-for-one).

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