

## inventory investment

Interest in inventory investment's role in business cycle volatility goes back at least to John Maynard Keynes. This article examines some basic facts about aggregate inventory investment, emphasizing its highly volatile and pro-cyclical nature. It then outlines several approaches to modelling inventory behaviour, including a detailed discussion of the linear-quadratic model, and examines their implications for inventory investment's potential role in business cycle fluctuations. The article concludes with a discussion of the potential for progress in inventory control methods to have played a role in the decline in aggregate volatility since the mid-1980s.

Inventory investment is the change in the stocks of materials, works in process, and finished goods within a firm, industry, or entire economy over a specified period of time. Because in most instances the measure encompasses a variety of goods, it is usually measured in currency units, perhaps deflated (for example, in 1999 dollars). Occasionally, however, when highly disaggregated data are available, it can be measured in physical units (for example, Blanchard, 1983; Kahn, 1992).

In national income accounts, aggregate inventory investment is the difference between Gross Domestic Product (GDP) and final sales of domestic product. As a share of GDP it is tiny but highly volatile in modern industrial economies. In the post-war United States, for example, it averages 0.62 per cent of GDP, but has a standard deviation of 0.83 per cent. By comparison, fixed non-residential investment averages 10.6 per cent of GDP with a standard deviation of 1.2 per cent. (Data for these calculations come from the US National Income and Product Accounts, Table 5.)

Inventory investment is also highly pro-cyclical. For example, its correlation with real GDP growth in post-war US data is approximately 0.4, and very close to the correlation between fixed non-residential investment and real GDP growth. Also, the standard deviation of real GDP growth is substantially higher than that of final sales (4.0 versus 3.3 per cent), notwithstanding the fact that for more than half of the economy GDP and final sales are identical. Thus inventory investment 'adds' to the volatility of GDP growth in the accounting (though not necessarily causal) sense. Indeed, interest in inventory behaviour as a contributor to aggregate volatility goes back at least to Keynes (1936), and includes notable contributions by Metzler (1941) and Abramovitz (1950). Blinder (1981, p. 500) writes that 'to a great extent, business cycles are inventory fluctuations'.

The pro-cyclicality of inventory investment appears inconsistent with standard microeconomic models of inventory behaviour, particularly those that stress 'buffer stock' or 'production-smoothing' motives, as noted by, among others, Blinder (1986) and West (1986). And this was not the first puzzle brought to light by research on inventory behaviour. Some ten years earlier, Feldstein and Auerbach (1976) noted the persistence of inventory-sales ratios' deviations around their means (see also Ramey and West, 1999), particularly given the trivial adjustments needed to restore them to a (presumed) fixed target.

Researchers have also found inventory behaviour informative about the fundamental driving forces of business cycles (see West, 1990). For example, Blinder (1986), Eichenbaum (1989), Kydland and Prescott (1982), and Christiano (1988) hypothesize supply side disturbances to account for pro-cyclical inventory investment. Others (such as Ramey, 1991; Hornstein and Fischer, 2000) consider non-convexities such as fixed costs or downward-

sloping marginal cost. Kashyap, Stein and Wilcox (1994) argue for the importance of credit constraints. By contrast, Bils and Kahn (2000) argue that the counter-cyclical behaviour of inventory-sales ratios casts doubt on such supply side explanations, which imply counterfactually that inventories should be relatively tight (in relation to sales) during recessions and plentiful in expansions.

### The linear-quadratic model

The workhorse of applied inventory research is the linear-quadratic cost minimization model developed by Holt et al. (1960). The firm is assumed to face a stochastic demand process independent of its inventory and production decisions. Consequently, whether it is a competitive price-taker or has monopoly power, the firm can condition on its expected sales process and minimize costs, which take the form

$$E_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ c_1 y_{\tau} + c_2 y_{\tau}^2 + c_3 (h_{\tau} - h^*)^2 \right] \right\}, \quad (1)$$

subject to

$$h_{\tau} = h_{\tau-1} + y_{\tau} - s_{\tau}, \quad (2)$$

where  $y$  denotes production,  $s$  sales,  $h$  the end-of-period inventory stock,  $h^*$  the desired or ‘target’ stock, and  $\beta$  a discount factor. Some versions of the model include additional cost terms such as a cost of changing production. The target  $h^*$  is usually assumed to be either a constant or proportional to expected sales. In addition,  $c_1$  may be stochastic, and there may be additional additive stochastic terms (for example, materials prices).

A standard informational assumption is that production decisions at date  $t$  are based on period  $t-1$  information, with the implication that  $h_t$  is not controlled directly. Letting  $\theta = c_2/c_3$ , the solution to the problem (1)–(2) is:

$$E_{t-1}\{h_t\} = \lambda h_{t-1} + \lambda E_t \left\{ \sum_{\tau=t}^{\infty} (\beta\lambda)^{\tau-t} [\theta^{-1} h_{\tau}^* + \beta s_{\tau+1} - s_{\tau}] \right\} \quad (3)$$

where  $\lambda \in (0, 1)$  is the smaller root of  $\beta\lambda^2 - (1 + \beta + \theta^{-1})\lambda + 1$ . In the limiting case with  $\theta = c_2 = 0$  the solution is  $E_{t-1}\{h_t\} = h_t^*$ . If  $h^*$  is a constant, then the only motive for varying inventories is to smooth production. Durlauf and Maccini (1995) decisively reject this version of the model. It is worth noting that the solution (3) bears some similarity to another widely used model, the flexible accelerator of Lovell (1961) and others

$$h_t - h_{t-1} = \lambda_1 (h_t^* - h_{t-1}) - \lambda_2 (s_t - E_{t-1}\{s_t\}) + u_t,$$

where  $u_t$  is a disturbance term, and both  $\lambda_1$  and  $\lambda_2$  lie between zero and 1.

Both the linear-quadratic and flexible accelerator models, however, have a history of empirical difficulties. Blinder (1986) pointed out that, for the pure production-smoothing model (with  $h^*$  a constant), the model counterfactually implies that the variance of sales exceeds that of production. West (1986) showed that a more general variance inequality implied by the model with a target proportional to expected sales is also violated in US manufacturing data. Moreover, among studies of similar data, there is disagreement in the literature on the magnitudes, and even the signs, of key parameters. For example, Ramey (1991) finds negatively sloped marginal cost, in contrast to most other studies. West (1986) finds a relatively small cost of inventory deviations from their target. West and Wilcox (1994) find that obtaining

precise estimates of the linear-quadratic model may be problematic with realistic sample sizes.

Regarding the flexible accelerator, Feldstein and Auerbach (1976) estimated small values for both  $\lambda_1$  and  $\lambda_2$ , which is paradoxical because a small  $\lambda_1$  implies large adjustment costs, but a small  $\lambda_2$  implies that sales surprises are largely offset by within-period production responses. Their proposed solution is a target ratio that itself adjusts slowly over time. They do not provide a strong theoretical foundation for their ‘target adjustment’ model, however. One theme of the alternative approaches discussed in the next section is the effort to base inventory models on more rigorous microfoundations in the hope of resolving the empirical puzzles.

### Other approaches

Motivated by the empirical difficulties described above, researchers have examined a number of alternative approaches to modelling inventory behaviour. One, the so-called ‘stockout-avoidance’ model, provides a rigorous microfoundation for the target stock. Building on Karlin and Carr (1962), Kahn (1987; 1992) considers a firm that faces a non-negativity constraint on its inventories, and must commit to production and pricing decisions each period before observing potential sales, or ‘demand’  $x_t$ . Consequently, sales equal the minimum of  $x_t$  and the stock available  $h_{t-1} + y_t$ . If we let  $F$  denote the distribution function for  $x$ , profit maximization implies

$$p_t(1 - F(h_{t-1} + y_t)) - c_t + \beta E_t\{c_{t+1}\}F(h_{t-1} + y_t) = 0,$$

where  $p$  is price and  $c$  is marginal cost. Then, if demand uncertainty is multiplicative, for example, and  $p$  and  $c$  (and hence the markup) are constant, the firm will set  $h_{t-1} + y_t$  proportional to expected demand  $E_{t-1}\{x_t\}$ . In addition, positive serial correlation in demand results in the variance of production exceeding the variance of sales.

Another important implication of this approach is that inventory-sales ratios depend on price–cost markups. Bils and Kahn (2000) show, in a model in which expected sales are increasing in the stock available, that the optimal inventory–sales ratio is a function of the markup and a discount rate  $\beta E_t\{c_{t+1}/c_t\}$ . They argue that the counter-cyclical behaviour of the inventory–sales ratio implies a counter-cyclical markup, or, equivalently, pro-cyclical marginal cost.

An alternative approach builds on the work of Scarf (1960), who modelled inventory behaviour with fixed ordering costs. Scarf provided conditions under which inventories would fluctuate between a fixed upper and lower bound, which he dubbed ‘S’ and ‘s’ respectively – hence the moniker (S,s) model. (The conditions, such as i.i.d. orders, are quite restrictive, however.) Caplin (1985) showed that this model implies that the variance of orders exceeds the variance of sales, and Hornstein and Fisher (2000) extended this approach to a general equilibrium setting. Hall and Rust (2000) provide some empirical support for ‘generalized’ (S,s) behaviour where the two limits depend on the spot price of the good in inventory. While the existence of fixed costs at the microeconomic level is well established, their importance for aggregate inventory behaviour at business cycle frequencies remains a matter of debate.

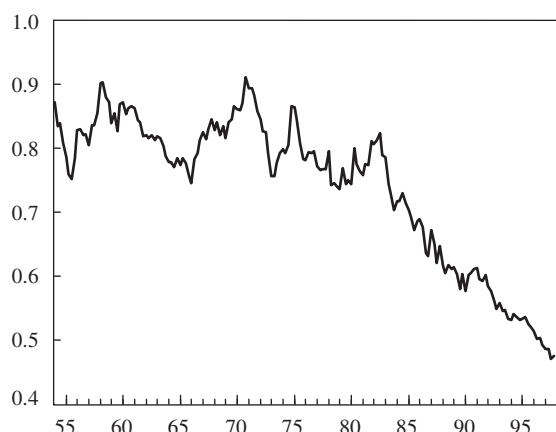


Fig. 1 Inventory-sales ratio, durable goods, USA, 1954–1998. *Note:* Inventories and sales are in chained 2000 dollars, *Source:* US National Income and Product Accounts. Durable goods inventories are from Table 5.7.6A, and final sales are from Table 1.2.6

### Inventories and the great moderation

Recently attention has again turned to inventory behaviour as a possible explanation for the dramatic reduction in aggregate volatility, which in the United States dates from approximately 1984 (McConnell and Perez-Quiros, 2000). Kahn, McConnell M. and Perez-Quiros (2002) show that reduced volatility is most pronounced in the durable goods sector, and for production more than for sales. At the same time, that sector has experienced large declines in inventory–sales ratios, as shown in the accompanying Figure 1, and reduced volatility of inventory investment. They also provide a model in which improved information about demand shocks results in reduced output volatility.

While there is much anecdotal evidence of efforts to improve inventory control by techniques such as ‘just-in-time’ management, there remains nonetheless considerable debate over the importance of inventories in increased aggregate stability.

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### See also

< xref = D000212 > dynamic programming;  
 < xref = S000473 > s-S models.

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**Index terms**

adjustment costs  
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