

FIXED RATE MORTGAGE HOMEWORK PROBLEMS

1. Ms. Jones financed her home purchase with a fixed-rate 20-yr mortgage at 6%. The original loan balance was 400,000.00. With her monthly mortgage just paid her current loan balance is 301,903.98.
 - a. What is Jones's monthly payment to the bank?
 - b. How many months remain until the loan is paid off?
 - c. Jones would like to pay off her loan sooner. She has decided that she would like to pay off her loan in 10 years, and is willing to add \$A per month to her payment. What is the value for A?
2. Smith financed his home purchase with a conventional fixed-rate 30-yr mortgage at 9%. The original loan balance was 200,000.00. With his monthly mortgage just paid his current loan balance is 173,719.16.
 - a. What is Smith's monthly payment to the bank?
 - b. How many months remain until the loan is paid off?
 - c. Smith would like to pay off his loan sooner. He has decided that he can afford an extra 50 per month. How many months will it take to pay off his loan?
3. Consider a 15-year fixed-rate mortgage for 200,000 at 6.25%. Provide *continuous-time* answers:
 - a. What is the monthly payment?
 - b. What is the loan balance after 4 years, 3 months?
 - c. Suppose the remaining duration of the loan is 10 years and 9 months. How quickly will the loan be paid off if the 2000 is paid each month instead of the original monthly payment?
 - d. Suppose the remaining duration of the loan is 10 years and 9 months. How much must be added to the original monthly payment to pay off the loan in 5 years?
4. Consider a conventional fixed-rate 30-yr loan for 100,000 at 10%.
 - a. What is the total payment and the total interest paid over the life of the loan?
 - b. In a biweekly program, you pay half of the total monthly payment every two weeks until the loan is repaid. Assume biweekly compounding. In a biweekly program for this loan, when will it be paid off and what will be the total interest saved over the life of the loan?
5. Consider a conventional fixed-rate 30-yr loan for 500,000 at 12%. Assume a tax rate of 40%.
 - a. What are the LoanValue and TaxShieldValue when the discount rate equals the loan interest rate?
 - b. Answer part (a) if the loan will be paid off after 10 years.
6. Consider a conventional fixed-rate 20-yr loan for 100,000 at 9%. Assume a tax rate of 30%.
 - a. What are the LoanValue and TaxShieldValue when the discount rate equals the loan interest rate?
 - b. Answer parts (a) and (b) if the loan will be paid off after 5 years.
7. Your company is considering purchasing a machine that costs 1 million. The manufacturer offers to finance the purchase by lending you the purchase price for 10 years with annual interest payments of 4%. Principal of 1 million is paid at the end of year 10. The local bank will charge you 6% for such a loan. The tax rate is 40%. What is the value of the loan and the net purchase cost of the machine?
8. Your company is considering purchasing a machine that costs 10 million. The manufacturer offers to finance the purchase by lending you the purchase price for 6 years with annual interest payments of 3%. Principal of 10 million is paid at the end of year 6. The local bank will charge you 8% for such a loan. The tax rate is 40%. What is the value of the loan and the net purchase cost of the machine?

FIXED-RATE MORTGAGE HOMEWORK PROBLEM SOLUTIONS

1. a. $M = (0.005)(400,000)/[1 - (1.005)^{-240}] = 2865.72$.
 b. The ratio $LB(t)/LB(0) = 301,903.48/400,000$. We know that
 $LB(t)/LB(0) = [1 - (1.005)^{-(240-t)}]/[1 - (1.005)^{-240}]$.
 Thus, the number of months remaining is $(240-t) = 150$.
 Alternatively, $301,903.48 = [2865.72/0.005][1 - (1.005)^{-n}]$, which gives $n = 150$.
 c. We seek the value of A such that
 $[(2865.72 + A)/0.005][1 - (1.005)^{-120}] = 301,903.48$.
2. a. $M = 200,000(0.09/12)/[1 - (1 + 0.09/12)^{-360}] = 1609.25$.
 b. $173,719.16/200,000 = [1 - (1.0075)^{-n}]/[1 - (1.0075)^{-360}]$ and so $n = 222$.
 c. Seek n such that $173,719.16 = (1659.25/0.0075) [1 - (1.0075)^{-n}]$ and so $n = 206$.
3. a. 1712.16.
 b. $LB(4.25) = 160,833.17$.
 c. 8.68 years.
 d. 1409.01.
4. a. $TP = 877.57$. The difference between the total payments and the initial loan balance equals the total interest payment, and so it equals $= 360(877.57) - 100,000 = 215,925$.
 b. $100,000 = (877.57/2)*([1 - (1 + 0.10/26)^{-N}]/(0.10/26))$, which implies $N = 545$ or 21 yrs.
 Total interest payment is now $545(877.57/2) - 100,000 = 139,138$, a reduction of 35.6%.
5. a. $TP = 5143.06$. $IP(1) = 5000$. $PP(1) = 143.06$.
 $LoanPrincipalValue = 500,000 - 360(143.06)/1.01 = 449,008$.
 $LoanValue = TaxShieldValue = 0.40(449,008) = 179,603$.
 b. $LB(120) = 467,090$.
 $LoanPrincipalValue = 500,000 - 120(143.06)/1.01 - 467,090/(1.01)^{120} = 341,477$.
 $LoanValue = TaxShieldValue = 0.40(374,387) = 136,591$.
6. a. $TP = 899.73$. $IP(1) = 750$. $PP(1) = 149.73$.
 $LoanPrincipalValue = 100,000 - 240(149.73)/1.0075 = 64,332$.
 $LoanValue = TaxShieldValue = 0.30(64,332) = 19,300$.
 b. $LB(60) = 88,707$.
 $LoanPrincipalValue = 100,000 - 60(149.73)/1.0075 - 88,707/(1.0075)^{60} = 34,426$.
 $LoanValue = TaxShieldValue = 0.30(34,426) = 10,328$.
7. $i_a = 2.4\%$. $r = 3.6\%$.
 $LoanValue = (1 - 0.024/0.036)*[1,000,000 - 1,000,000/(1.036)^{10}] = 99,298$.
 So, the net purchase cost = 900,702.
NOTE: We can arrive at this answer by analyzing the incremental after-tax cash flow. The company's after-tax cost of funds = 3.6%. The after-tax interest rate (provided by the manufacturer) is 2.4%. The incremental after-tax cash flow here is $(3.6\% - 2.4\%)*(1M) = 12,000$ each year for 10 years. The PV of this incremental cash flow stream at the company's after-tax cost of funds of 3.6% is $12,000(1 - (1.036)^{-10})/0.036 = 99,298$, as before.
8. $i_a = 1.8\%$. $r = 4.8\%$.
 $LoanValue = (1 - 0.018/0.048)*[10M - 10M/(1.048)^6] = 1.532M$.
 So, the net purchase cost = 8.468M.
NOTE: We can arrive at this answer by analyzing the incremental after-tax cash flow. The company's after-tax cost of funds = 4.8%. The after-tax interest rate (provided by the manufacturer) is 1.8%. The incremental after-tax cash flow here is $(4.8\% - 1.8\%)*(10M) = 0.3M$ each year for 6 years. The PV of this incremental cash flow stream at the company's after-tax cost of funds of 4.8% is $0.3M(1 - (1.048)^{-6})/0.048 = 1.532M$, as before.