



LESSON 4

MAKING THE CALCULATIONS

Constructing an Amortization Schedule

There are several possible ways to construct the payment schedule when interest is calculated on a declining balance. There are tables that can be consulted and calculators that contain the formulae required to make the calculation when provided with the basic details of loan amount, interest rate and number of instalments. The following mathematical formula can also be used to calculate the loan payments and to construct an amortization schedule.

$$\text{instalment payment} = \frac{PV \times i \times (1 + i)^n}{(1 + i)^n - 1}$$

where  $i$  = interest rate per payment period  
 $n$  = number of payments  
 $PV$  = principal amount of the loan

Once the instalment payment is calculated with the above formula, then the amount that goes toward interest and principal can be determined for each payment period. The amount that goes toward interest is the nominal interest rate times the balance at the beginning of the period. The rest of the payment (the payment minus the amount going toward interest) is payment of principal.

For example, the amortization schedule for a three-month \$100 loan, with 2 percent monthly interest, would be calculated as follows:

1. Use the formula above to determine the monthly payment:

$$\text{Payment} = 100 \times \frac{0.2 \times (1 + .02)^3}{(1 + .02)^3 - 1} = 100 \times \frac{(.02 \times 1.0612)}{0.0612} = 34.68$$

2. Calculate the interest to be paid in the first payment:

$$\$100.00 \times 0.02 = \$2.00 \text{ interest.}$$

3. Subtract the interest from the first payment to see how much principal is paid with the first payment:

$$\$34.68 - \$2 = 32.68$$

4. Subtract the first principal payment from the outstanding balance to determine the new outstanding balance:

$$100 - 32.68 = 67.32$$

Steps 2-4 can be repeated for each payment to construct the amortization schedule shown in Lesson 2.

## Calculating Effective Interest Rates

The following examples show the step-by-step process used to calculate the effective interest rate for different repayment scenarios. In all three cases, the original loan amount is \$100, the loan period is three months with monthly payments, and the nominal interest rate is 2 percent per month.

The attached spreadsheet can be used to help with the calculations.

### *Example 1: Interest Based on Original Loan Amount*

- a) Enter into the spreadsheet (Table 1a), the loan amount ( $PV$ ), the loan period ( $n$ ) and the nominal interest rate ( $i$ ):
  - $PV = 100$
  - $n = 3$
  - $i = 2\%$
- b) The principal paid per month, the monthly amount of interest and the total monthly payment can be seen, calculated as below:
  - Principal:  $100/3 = 33.33$  per month
  - Amount of interest:  $(0.02 \times 100) = 2.00$  per month
  - Total monthly payment:  $35.33$
- c) The effective interest rate is displayed:  $2.97\%$  per month.

### *Example 2: Interest Deducted Up Front from Original Loan Amount*

- a) Determine the amount to be paid in interest on an amortized basis.
  - Enter into the spreadsheet (Table 2) the original loan amount, the number of payment periods and the nominal monthly interest rate:
    - $PV = 100$
    - $N = 3$
    - $i = 2$
  - The amortized monthly payment is 34.68.
  - If 34.68 were paid each month, then the total paid would be \$104.04 ( $34.68 \times 3$ ), of which \$100 is principal. So the balance of \$4.04 is interest, which can be seen in the spreadsheet.
- b) The spreadsheet also sets out the actual monthly payment and the amount the borrower actually received:
  - Actual Monthly Payment is -33.33 (the actual monthly payment is only principal because the interest is being subtracted up front from the original loan amount, i.e.  $100/3 = 33.33$ )
  - Amount borrower receives is 95.96 (\$4.04 interest is subtracted from original loan amount)
- 3) The effective interest rate is displayed:  $2.08\%$  per month

**Example 3: Loans with Compensating Balances**

The borrower receives a loan of \$100, but \$25 of his own money has to be kept on deposit for the life of the loan. The borrower loses the income that he could have earned on his \$25 (which the lending institution can now earn). Furthermore, effectively, the borrower only gets \$75 in new money to use because his own \$25 is on deposit. His \$25 is not lost but returned to him when he makes his final payment, effectively reducing the amount of that payment by \$25.

a) Input the following assumptions into the spreadsheet (Table 3):

$$\begin{aligned} PV &= 100 \\ N &= 3 \\ i &= 2 \end{aligned}$$

and the amortized monthly payment shown will be 34.68 as before.

So the borrower pays 34.68 in each period but he is also incurring an additional cost in the form of lost income on his deposit. Because he is paying 2 percent per month for the loan, we can assume that his \$25 deposit would be earning at least 2 percent if he didn't have to keep it on deposit. (It is earning 2 percent for the bank if they lend it to another borrower.) Thus, he is actually forfeiting 2 percent of \$25 (0.50) each month in lost income. In effect his monthly payments are \$35.18 (34.68 + 0.50).

When his last payment is due, he gets his \$25 returned to him, meaning that he effectively pays only \$10.18 (35.18 - 25) that month. So his monthly payments are 35.18, 35.18, and 10.18.

b) The spreadsheet will display the effective interest rate of this loan.

Because the payment amounts vary, the effective interest rate is calculated with the internal rate of return function in the spreadsheet. The flow of funds is:

- Amount received by borrower = 75
- First payment = (35.18)
- Second payment = (35.18)
- Third payment = (10.18)

The internal rate of return (IRR) will display as 4.3%.

Thus the effective interest rate on the loan is 4.3 percent.

Postscript

**How Calculators Calculate the Effective Interest Rate**

The effective rate is determined through an iterative process carried out by a calculator. Given the three variables (present value, number of payment periods, and amount of payment), the effective interest rate is adjusted through trial and error until the result of the equation below is as close as possible to the present value (amount borrower receives) of the loan.

Using a three-month, \$100 loan with monthly payments of \$34.68 and no additional costs, the calculator continually adjusts the value of  $i$  (interest) until the sum of the Formula column equals the present value of the loan (\$100 in this case).

Payment Period	Payment	Formula
1	34.68	$\frac{34.68}{(1 + i)^1}$
2	34.68	$\frac{34.68}{(1 + i)^2}$
3	34.67	$\frac{34.68}{(1 + i)^3}$
Total present value (amount borrower receives):		100.00

In this case, the value of  $i$  which correctly completes the equation is 0.02, meaning the loan has an effective monthly interest rate of 2 percent.