

DISCOUNTED CASH-FLOW ANALYSIS

Objectives:

- Study determinants of incremental cash flows
- Estimate incremental after-tax cash flows from accounting data and use them to estimate NPV
- Introduce salvage value and additions to net working capital
- Determine sensitivity of NPV to changes in underlying assumptions (or sets of assumptions)

Determinants of Incremental Cash Flows

1. Initial Outlay (often CF_0)

- Purchasing and installing new plants and equipment
- Working capital (investments in inventory and accounts receivable)
- After-tax cash flow from sale of old machine (capital gains taxes)

2. After-Tax Incremental Cash Flows (often CF_1 to CF_N)

- Changes in revenue minus expenses
- Changes in labor and materials; changes in overhead rates
- Changes in working capital
- Depreciation tax shields
- *Do not include interest expenses (they go into the discount rate)*

3. Terminal Cash Flows (often CF_N)

- Salvage value (taxable gains and losses over book value)
- Recovery of working capital (selling off inventory, getting paid)
- Clean-up costs

General Advice Regarding Calculation of Incremental Cash Flows **Identify cash flows (not accounting profits)**

- A. Only after-tax cash flows are relevant
- B. Estimate cash flows on an incremental basis
 - 1) Incremental cash flows equal the cash flows with the project minus cash flows without the project
 - 2) Pitfalls in estimating incremental cash flows
 - Include all incidental effects: What happens to the demand for Ford's existing minivans when they introduce a new minivan?
 - Include working capital requirements such as investments in inventory and investments in accounts receivable
 - Sunk costs are not incremental
 - Opportunity costs are relevant
 - *Financing decisions are separate – do not include interest costs*
- C. Treat inflation consistently – discount nominal cash flows with nominal discount rates; discount real cash flows with real discount rates

Focus of the chapter: identifying the relevant cash flows, i.e. *what* should be discounted when calculating NPV.

1. Discount Cash Flows, not Accounting Profits

- Projects are financially attractive because of the cash they generate, either for distribution to shareholders or for reinvestment in the firm.
- Recognize investment expenditures when they occur, not later when they show up as depreciation.

Example: Project Income Statement

Revenues	\$100
Depreciation (D)	20
All other costs	40
	<hr/>
NIBT	\$40
Taxes (40%)	16
	<hr/>
Project NI(PNI)	\$24
	<hr/>

Cash flow = PNI + Noncash expenses = PNI + Depreciation = \$24 + \$20 = \$44.

Example:

Sales	200,000
(Operating Expense)	<u>120,000</u>
	80,000
-Depreciation Expense	<u>20,000</u>
Operating Income (EBIT)	60,000
-Tax(40%)	<u>24,000</u>
Net Income	36,000

$$CF = (R - E) (1 - t) + D(t)$$

$$(200000 - 120000) (1 - .4) + 20000(.4)$$

$$= 48000 + 8000 = 56000$$

- Recognize cash flows when the company actually pays its bills and when it receives payments for goods and services provided.

December		June	
Sales	\$500,000	Sales	0
Less investment in accounts receivable	-500,000	Plus recovery of accounts receivable	+\$500,000
Cash flow	0	Cash flow	\$500,000

2. Discount Incremental Cash Flows

- A project's value depends on the *extra* cash flows that it produces.

Incremental Cash Flow = Cash Flow w/ Project - Cash Flow w/o Project

Example: XYZ Corporation sells 170,000 cars a year at a profit of \$1,000 per car. It is considering a proposal to produce 200,000 cars of a new model that can be sold at a profit of \$1,100 per car. Find the incremental cash flows that must be used to value the proposal.

Incremental Cash Flow = Cash Flow w/ Project - Cash Flow w/o Project =
 = \$1,100 × 200,000 - \$1,000 × 170,000 = \$50,000,000

- Remember:
 - i) Include all externalities.
 - the effects (positive or negative) of the new project on the firm's existing business (see previous example).
 - ii) Forget sunk costs.
 - sunk costs ≡ past and irreversible outflows.
 - example: expenses already incurred for research of the investment project shall not be taken into account when making the investment decision.
 - iii) Include opportunity costs.
 - opportunity cost ≡ benefit or cash flow forgone as a result of an action.
 - example: land under new manufacturing facility could otherwise be sold.
 - iv) Include changes in net working capital.
 - net working capital ≡ current assets - current liabilities.
 - example: starting new production requires creation of an inventory of raw materials. The money tied up in the inventory has to be taken into account.
 - v) Beware of allocated overhead costs.

Example: A firm is considering an investment in a new manufacturing plant. The site already is owned by the company, but existing buildings would need to be demolished. Which of the following should be treated as incremental cash flows?

- The market value of the site.
- The market value of the existing buildings.
- Demolition costs and site clearance.
- The cost of a new access road put in last year.
- Lost cash flows on other projects due to executive time spent on the new facility.
- Future depreciation of the new plant.

3. Treat Inflation Consistently.

- Discount nominal cash flows using nominal interest rate.
- Discount real cash flows using real interest rate.

4. Separate Investment and Financing Decisions

Incremental Cash Flows (*ignoring taxes*)

A supermarket is deciding whether to install a sushi vending machine. The vending machine costs \$250,000 in year 0 and has a salvage value of \$0 at the end of 5 years.

	Year 0	1	2	3	4	5
Sales		250,000	300,000	300,000	250,000	250,000
Operating Expenses		200,000	200,000	200,000	200,000	200,000
Depreciation		50,000	50,000	50,000	50,000	50,000
Pre-tax Profit		0	50,000	50,000	0	0
0%		0	0	0	0	0
After-tax Profit		0	50,000	50,000	0	0

Ignoring taxes, what are the cash flows of this project?

	Year 0	1	2	3	4	5
Investment	-250,000					
Sales		250,000	300,000	300,000	250,000	250,000
Operating Expenses		200,000	200,000	200,000	200,000	200,000
Cash Flows	-250,000	50,000	100,000	100,000	50,000	50,000

Formula for Calculating Cash Flows

Total Cash Flow = cash flow from investment in plant and equipment
+ cash flow from investments in working capital
+ cash flow from operations

Working Capital = investments in inventory and accounts receivable
(increase in working capital \ negative cash flow)

Note: cash flow is measured by the change in working capital rather than the level of working capital

Cash Flow from Operations = revenues - cash expenses - taxes paid
= after-tax profit + depreciation
= (revenues - cash expenses) x (1 - tax rate)
+ (depreciation x tax rate)

Note: three equivalent ways to calculate cash flows ops.

Calculating Cash Flow from Operations

A project generates revenues of \$1,000, has cash expenses of \$600, and depreciation charges of \$200 in a particular year. The firm's tax rate is 35%. What is the firm's net income?

Profit Before Taxes = revenues - cash expenses - depreciation = \$200

*Taxes = profit before taxes * tax rate = \$70*

Profit After Taxes = profit before taxes - taxes = \$130

What is the firm's cash flow from operations?

Method 1 = revenues - cash expenses - taxes paid
= \$1000 - \$600 - \$70 = \$330

Method 2 = after-tax profit + depreciation
= \$130 + \$200 = \$330

Method 3 = (revenues - cash expenses) x (1 - tax rate) + (depr. x tax rate)
= (\$1000 - \$600) x (1 - .35) + (\$200 x .35) = \$330

Taxes and Cash Flows:

Depreciation

Depreciation is the accounting procedure by which the value of a firm's plant and equipment are revalued for the purpose of the firm's balance sheet.

Depreciation matters because it affects the taxes that firms must pay. The basic idea is simple:

- Straight-line depreciation – Divide the cost of the asset by the useful life of the asset (as stated by the IRS)
- Accelerated depreciation – See text for a general discussion

Depreciation matters for cash flows because it is treated as an expense and therefore reduces a firm's taxable income. This explains why the IRS determines depreciation rates.

Depreciation also matters for cash flows because it helps determine the capital gain or loss from the sale of plants and equipment.

MACRS Depreciation Schedule

Under the Modified Accelerated Cost Recovery System (MACRS), depreciation amounts in early years are higher than under straight-line and depreciation amounts in later years are lower. IRS determines the amounts.

Year	Mid-Quarter Convention, Placed in Service in First Quarter					
	Recovery Period					
	3-Year	5-Year	7-Year	10-Year	15-Year	20-Year
1	58.33	35.00	25.00	17.50	8.75	6.563
2	27.78	26.00	21.43	16.50	9.13	7.000
3	12.35	15.60	15.31	13.20	8.21	6.482
4	1.54	11.01	10.93	10.56	7.39	5.996
5		11.01	8.75	8.45	6.65	5.546
6		1.38	8.74	6.76	5.99	5.130
7			8.75	6.55	5.90	4.746
8			1.09	6.55	5.91	4.459
9				6.56	5.90	4.459
10				6.55	5.91	4.459
11				0.82	5.90	4.459
12-15					5.90	4.459
16					0.76	4.459
17-20						4.459
21						0.570

Taxes and Cash Flows: Depreciation Tax Shield

Depreciation is treated as an annual expense even though the company is not paying out any money. Because depreciation expenses reduce the company's pretax profit, they reduce the company's tax bill by the depreciation amount times the corporate tax rate.

- Pretax Profit = Revenues - Expenses - Depreciation
- Increasing depreciation by \$1.00 reduces taxes by \$0.35 ($\tau = 35\%$), and lower tax payments imply higher after-tax cash flows.

CF Operations with straight-line depreciation:

	Year 0	1	2	3	4	5
Capital Investment	5,000					
Revenues (grow 5%)		3,500	3,675	3,859	4,052	4,254
Expenses (grow 5%)		2,000	2,100	2,205	2,315	2,431
Depreciation		1,000	1,000	1,000	1,000	1,000
Pretax Profit		500	575	654	736	823
Tax 35%		175	201	229	258	288
Profit After Tax		325	374	425	479	535
CF Operations		1,325	1,374	1,425	1,479	1,535

**Taxes and Cash Flows:
Capital Gains**

Capital gains tax is a tax on the difference between the sale price and book value of an asset

Example: You purchased a share of AAPL fifteen years ago for \$5.00. Today a share trades for \$40.00. You have an unrealized capital gain of $(\$40 - \$5) = \$35$. If you sold the stock today you would realize a capital gain of \$35.00 and pay capital gains taxes on that amount.

Capital gains taxes affect incremental cash flows when old equipment is sold at either a capital gain or a capital loss.

Example: Corning purchases a new machine in year 0 to produce square Corningware. The machine costs \$5,000. In year 2, the machines are in short supply and Corning sells the machine for a cool \$7,500. Corning realizes a capital gain on the sale and must pay capital gains taxes.

**Taxes and Cash Flows:
Depreciation and Capital Gains**

What are the cash flows from the sale of the Corningware machine?

The book value of the machine in year 0 is \$5,000. Assume that the useful life of the machine is 5 years, that Corning applies straight-line depreciation, and that the corporate tax rate is 35%.

$$\begin{aligned} \text{Capital Gains (year 2)} &= \text{Sale Price} - \text{Book Value} \\ &= 7,500 - (5,000 - (2 * 1,000)) \\ &= 7,500 - 3,000 = \mathbf{4,500} \end{aligned}$$

$$\begin{aligned} \text{Capital Gains Tax (year 2)} &= \text{Capital Gains} * \tau_{\text{capital gains}} \\ &= 4,500 * 0.35 = \mathbf{1,575} \end{aligned}$$

$$\begin{aligned} \text{Salvage Value (year 2)} &= \text{Sale Price} - \text{Capital Gains Tax} \\ &= 7,500 - 1,575 = \mathbf{5,925} \end{aligned}$$

Change in Net Working Capital

When calculating financial cash flows, we need to adjust for the fact that accountants use depreciation to smooth capital spending over time.

Similarly, we need to adjust for the fact that accrual accounting matches revenues to costs when transactions occur, not necessarily when the revenues are received or the costs are incurred!

Since accrual accounting conventions determine taxes, we cannot easily undo them. Instead, we adjust for differences between the assumed and actual timing of costs and revenues via Net Working Capital.

Let's consider three examples based on a project with sales in years 1-4.

- First, the firm receives 10% of its revenues in the year after each sale (*account receivables*)
- Second, the firm incurs 25% of its variable costs in the year before each sale (*investment in inventory*).
- Third, *account receivables plus investments in inventory*

Capital Budgeting Illustration

Example:

Assume a project costs \$1200, has a life time of 10 years, and $SV = \$200$ at the end of 10 years. The company uses straightline depreciation and the corporate tax rate is 50%. A new machine generates revenue of \$400 yearly for the next ten years and has expenses of \$150 yearly for the next ten years. The cost of capital is 10%.

$$\text{Dep} = 1200 / 10 = 120 \text{ yearly}$$

$$CF_t = \$-1200$$

$$\begin{aligned} CF_{1-10} &= (R-E)(1-t) + D(t) \\ &= (400 - 150) (1-.5) + 120(.5) \\ &= 125 + 60 = \$185 \end{aligned}$$

$$CF_{t=10} = \text{Recovery of SV after tax} = 200(1-.5) = \$100$$

$$\begin{aligned} \text{PV of cash inflow} &= 185 [\text{PVIFA}(10\%, 10 \text{ y})] + 100 [\text{PVIF}(10\%, 10\text{y})] \\ &= 185 (6.145) + 100(.386) \\ &= 1136.825 + 38.6 = 1175.4 \end{aligned}$$

$$\text{NPV} = 1175.4 - 1200 = -24.6$$

$$\text{Pay Back} = 1200/185 = 6.5 \text{ years}$$

$$\text{IRR} = 185[\text{PVIFA}(10\text{y}, r\%) + 100[\text{PVIF}(10\text{y}, r\%)]] = 1200$$

IRR must be less than 10%

lets try 9%:

$$185(6.418) + 100(.4224) =$$

$$1187.3 + 42.3 = 1229.6$$

IRR is Between 9 and 10%

5. Capital Budgeting Illustration, Another example

Data on Proposed New Assets

MACRS class:	3-year
Economic life:	4 years
Price:	\$200,000
Freight and installation:	\$40,000
Salvage value:	\$25,000
Effect on NWC:	Increase inventories by \$25,000 and increase A/P by \$5,000
Revenues:	\$200,000/year (100,000 units at \$2/unit)
Costs excluding depreciation:	60% of sales
Tax rate:	40%
Cost of capital:	10%

Net Investment Outlay (t=0)

Price	(\$200,000)
Freight and installation	(40,000)
Increase in NWC	(20,000)
Net Outlay	<u>(\$260,000)</u>

Depreciation for Tax Purposes: Modified Accelerated Cost recovery System (MACRS)

Ownership Year	3-year	5-year	7-year	10-year
1	33%	20%	14%	10%
2	45	32	25	18
3	15	19	17	14
4	7	12	13	12
5		11	9	9
6		6	9	7
7			9	7
8			4	7
9				7
10				6
11				3
	<u>100%</u>	<u>100%</u>	<u>100%</u>	<u>100%</u>

Depreciable Basis = \$200,000 + \$40,000 = \$240,000

Depreciation Schedule (MACRS)

<u>Year</u>	<u>Factor</u>		<u>Depreciation</u>
1	33%	×240,000 =	\$79,200
2	45	×240,000 =	108,000
3	15	×240,000 =	36,000
4	7	×240,000 =	16,800

Year	0	1	2	3	4
Total Revenues		\$200.0	\$200.0	\$200.0	\$200.0
Operating costs excluding depreciation (60%)		120.0	120.0	120.0	120.0
Depreciation		<u>79.2</u>	<u>108.0</u>	<u>36.0</u>	<u>16.8</u>
Total Costs		<u>\$199.2</u>	<u>\$228.0</u>	<u>\$156.0</u>	<u>\$136.8</u>
EBT		\$ 0.8	(\$ 28.0)	\$44.0	\$63.2
Taxes (40%)		<u>0.3</u>	<u>(0.0)</u>	<u>17.6</u>	<u>25.3</u>
Net Income		\$ 0.5	(\$ 28.0)	\$ 26.4	\$ 37.9
Depreciation		<u>79.2</u>	<u>108.0</u>	<u>36.0</u>	<u>16.8</u>
Net operating cash flows		<u>\$ 79.7</u>	<u>\$ 80.0</u>	<u>\$ 62.4</u>	<u>\$ 54.7</u>
Equipment cost	(\$200)				
Installation	(40)				
Increase in inventories	(25)				
Increase in A/P	5				
Salvage value					25
Tax on (salvage - book)					(10)
Return on NWC					20
Net Cash Flows	<u>(\$260.0)</u>	<u>\$ 79.7</u>	<u>\$ 80.0</u>	<u>\$ 62.4</u>	<u>\$ 89.7</u>

$$\text{NPV} = -260 + 79.7/(1.1) + 80.0/(1.1)^2 + 62.4/(1.1)^3 + 89.7/(1.1)^4 \approx \text{\$13} < \mathbf{0}$$

$$\text{IRR: } -260 + 79.7/(1+\text{IRR}) + 80.0/(1+\text{IRR})^2 + 62.4/(1+\text{IRR})^3 + 89.7/(1+\text{IRR})^4 = 0$$

$$\text{IRR} = 7.63\% < \text{cost of capital}$$

Reject the project.

Capital Budgeting Illustration Continued: Inflation Effects

Assume that 5% inflation causes unit prices to increase, which causes sales revenues to increase by 5% annually. Inflation expectation has been included in the cost of capital.

Year	0	1	2	3	4
Total Revenues		\$210.0	\$220.5	\$231.05	\$243.1
Operating costs excluding depreciation (60%)		126.0	132.3	138.9	145.9
Depreciation (same as before)		<u>79.2</u>	<u>108.0</u>	<u>36.0</u>	<u>16.8</u>
Total Costs		<u>\$205.2</u>	<u>\$240.3</u>	<u>\$174.9</u>	<u>\$162.7</u>
EBT		\$ 4.8	(\$ 19.8)	\$56.6	\$80.4
Taxes (40%)		<u>1.9</u>	<u>(0.0)</u>	<u>22.6</u>	<u>32.2</u>
Net Income		\$ 2.9	(\$ 19.8)	\$ 34.0	\$ 48.2
Depreciation		<u>79.2</u>	<u>108.0</u>	<u>36.0</u>	<u>16.8</u>
Net operating cash flows		<u>\$ 82.1</u>	<u>\$ 88.2</u>	<u>\$ 70.0</u>	<u>\$ 65.0</u>
Equipment cost	(\$200)				
Installation	(40)				
Increase in inventories	(25)				
Increase in A/P	5				
Salvage value					25
Tax on salvage value					(10)
Return on NWC					20
Net Cash Flows	<u>(\$260.0)</u>	<u>\$ 82.1</u>	<u>\$ 88.2</u>	<u>\$ 70.0</u>	<u>\$ 100.0</u>

$$\text{NPV} = -260 + 82.1/(1.1) + 88.2/(1.1)^2 + 70/(1.1)^3 + 100/(1.1)^4 \approx \mathbf{\$9 > 0}$$

$$\text{IRR: } -260 + 82.1/(1+\text{IRR}) + 88.2/(1+\text{IRR})^2 + 70/(1+\text{IRR})^3 + 100/(1+\text{IRR})^4 = 0$$

$$\text{IRR} = \mathbf{11.46\% > \text{cost of capital}}$$

Accept the project.

Replacement: equal life time

$$\text{cash flow} = (\Delta R - \Delta E)(1-t) + \Delta D(t) \pm \Delta WK$$

Example:

ABC purchase a machine 5 years ago at \$7500 expected lifetime of 15 years (n=15) and salvage value of zero (SV=0) ; The company uses straight line depreciation.

Division Manager reports a new machine for \$12,000, n=10 which will increase sales from \$60,000 to \$70,000 per year and increase operating expenses from \$10,000 to \$17,000 per year . SV = 2000 at end of 10 yrs, old machine could be sold today for \$1000 . Corporate tax rate of 34%. Corporate cost of capital is 10%. New machine req. add net working capital of \$1000 at beginning of year zero. Should the company replace the existing machine with the new one?

$$CF_t = 0$$

1.	Cost of new machine	- 12000
2.	Selling old machine	1000
3.	NWK	-1000
4.	Tax effect (5000-1000)(.34)=	<u>+1360</u>
		-10640
		=====

$$\text{Dep. old} = \frac{7500}{15} = 500 \text{ yearly}$$

$$\text{Dep. new} = \frac{12000}{10} = 1200 \text{ yearly}$$

Book value = 10 x 500 = 5000 or

$$\begin{aligned} \text{acc. dep.} &= 5 \times 500 = 2500 \\ \text{BK} &= \text{Purchase} - \text{acc. dep.} \\ &= 7500 - 2500 = 5000 \end{aligned}$$

$$\text{AD} = 1200 - 500 = 700$$

Net cash flow 1-10

$$(3000)(1-.34) + 700(.34) = 1980 + 238 = \$2,218 \quad \text{yearly}$$

t = 10	Recovery SV	2000 (1 - 0.34) = 1320	
	Recovery WK		<u>1000</u>
			2320
			=====

$$\begin{aligned} \text{PV} &= \sum_{t=1}^n \frac{2218}{(1+10\%)^t} + \frac{2320}{(1+10\%)^{10}} \\ &= 2218(6.1446) + 2320(.3855) = \$14,523 \end{aligned}$$

$$\text{NPV} = 14523 - 10640 = \$3883$$

$$\text{Payback} = \frac{10640}{2218} \square 4.8 \text{ years}$$

$$\text{IRR} = \sum_{t=1}^n \frac{2218}{(1+r)^t} + \frac{2320}{(1+r)^{10}} - 10640 = 0$$

Problems

1. Brigham Industries is considering replacing one of its existing machines with a new, more automated, more efficient machine. The existing machine has a zero book value but it could be sold today for \$40,000. It has an estimated remaining life of ten years.

The new machine could be purchased for \$210,000. Installation costs would be an additional \$5,000. It is estimated that this new machine would increase output by 10%, increasing sales revenue by \$30,000 per year. In addition, this machine is expected to reduce operating costs by \$35,000 annually. If purchased, this new machine would be used for ten years, then sold for an estimated \$25,000. It would be depreciated using the seven year schedule below. If Brigham pays tax at a rate of 40% and the opportunity cost of capital is 15%, what is the NPV of this project?

Depreciable Basis = \$210,00 + \$5,000= \$215,000

Year	% of Dep. Invest.		Dep. Expense		Tax Shield
1	14%	× 215,000 =	30,100	× 0.40=	12,040
2	25%	× 215,000 =	53,750	× 0.40=	21,500
3	17%	× 215,000 =	36,550	× 0.40=	14,620
4	13%	× 215,000 =	27,950	× 0.40=	11,180
5	9%	× 215,000 =	19,350	× 0.40=	7,740
6	9%	× 215,000 =	19,350	× 0.40=	7,740
7	9%	× 215,000 =	19,350	× 0.40=	7,740
8	4%	× 215,000 =	8,600	× 0.40=	3,440

$$\text{year 0} \quad \left\{ \begin{array}{l} \text{new machine} = -210,000 \\ \text{installation cost} = -5,000 \\ \text{sell old machine} = +40,000 \\ \text{tax on sale of old machine} = \underline{-16,000} \\ \hline -191,000 \end{array} \right.$$

$$\text{years 1-10} \quad \left\{ \begin{array}{l} \text{higher revenue} = +30,000 \\ \text{lower operating costs} = \underline{-(-35,000)} \\ \hline 65,000 \\ \text{tax (40\%)} = \underline{-26,000} \\ \hline 39,000 \quad + \text{depreciation shields} \end{array} \right.$$

$$\text{year 10} \quad \{ \text{sale of new machine (after tax)} = (1 - 0.40) \times (\$25,000 - 0) = \$15,000$$

year	0	1	2	3	4
	-191,000	51,040	60,500	53,620	50,180
		39,000+12,040			
5	6	7	8	9	10
46,740	46,740	46,740	42,440	39,000	54,000

$$\text{NPV} = -191,000 + \frac{51,040}{1.15} + \frac{60,500}{(1.15)^2} + \frac{53,620}{(1.15)^3} + \dots + \frac{54,000}{(1.15)^{10}} = \$62,400$$

2. SSS is considering whether to become a distributor for a new line of frozen yogurt. Undertaking this project would require an initial cash outlay of \$300,000 to buy a refrigerated storage unit, and an investment in working capital (for inventory) of \$10,000. The storage unit would be depreciated using the 7 year MACRS schedule below. SSS plan to terminate this project after 5 years. It is estimated that they can sell the storage unit for \$50,000 at that time.

SSS estimates the Frozen Yogurt's sales would be \$150,000 each year and the operating expenses to be \$80,000 per year. SSS's marginal tax rate is 30%. The cost of capital is 15%. Find the NPV of this project.

7-year Depreciation Schedule

Depreciable Basis = \$300,000

Year	% of Dep. Invest.		Dep. Expense		Tax Shield
1	14%	× 300,000 =	42,000	× 0.30	12,600
2	25%	× 300,000 =	75,000	× 0.30	22,500
3	17%	× 300,000 =	51,000	× 0.30	15,300
4	13%	× 300,000 =	39,000	× 0.30	11,700
5	9%	× 300,000 =	27,000	× 0.30	8,100
6	9%	× 300,000 =	27,000	remaining book value at end of year } 5 = 27,000 + 27,000 + 12,000 = } \$66,000	
7	9%	× 300,000 =	27,000		
8	4%	× 300,000 =	12,000		

year 0 { buy storage unit = \$-300,000
 investment in WC = -10,000
 \$-310,000

years 1-5 { revenues: 150,000
 expenses: -80,000
 70,000
 taxes (30%) -21,000
 \$49,000 + dep. tax saving (see table)

year 5 { recovery of WC +10,000
 sale of storage unit +50,000
 tax savings from
 16,000 loss on disposal of storage unit (66k-50k) + 4,800
 \$64,800

year	0	1	2	3	4	5
net cash flow	-310,000	49,000 + 12,600 = 61,600	49,000 + 22,500 = 71,500	49,000 + 15,300 = 64,300	49,000 + 11,700 = 60,700	49,000 + 81,000 + 64,800 = 121,900
PV@15%	-310,000	53,565	54,064	42,278	34,705	60,606

NPV = \$-64,781 \Rightarrow **Reject the Project.**

Choosing Between Mutually Exclusive Projects

Two Sticky Situations:

1. Choice between long- and short-lived projects: *Should the firm save money today by investing in shorter-lived projects (such as less durable machinery)?*

\ This is a job for Equivalent Annual Annuities

2. Investment timing and replacement decisions:

\ *Should you invest in a new computer system today or invest in a new computer system next year?*

\ *When should existing machinery be replaced?*

REPLACEMENT WITH UNEQUAL LINES

Previously, we assume that new machine had a life equal to remaining life of existing one. Suppose we should make a decision based on two mutually exclusive with different lines. For example, A has life of 10 and B of 15. Most common way is to set up a series of “replacement chain” for machine A and B. This would be year 30 so it would be necessary to compare three chain cycles for A with two for B.

Example:

Suppose we want to replace a fully depreciated machine with a new one and the plant is still profitable and expected to work for many years but is not efficient as new one.

1088

To Summarize:

1. NPV of second A for the next 5 years = $9700 (3.7908) - 36100 = 617$
2. PV of $NPV_{A2} = 671$ (PVIF 10%, 5y) = $671 (.6209) = 417$
3. True $NPV_A = 671 + 417 = 1088$

Thus, A is better.

EQUIVALENT ANNUAL ANNUITY METHOD

The above example find chain is easy but suppose:

$$N_A = 8y$$

$$N_B = 11y$$

Requires an analysis of 88 years.

Suppose the Sandra Co. is planning to modernize its production facilities by replacement of its old fully depreciable machine. Two new models which are mutually exclusive are available.

End of Year	Change Cash Flow A	After tan B
0	-36100	-57500
1	9700	9500
2	9700	9500
3	9700	9500
4	9700	9500
5	9700	9500
6		9500
7		9500
8		9500
9		9500
10		9500

Cost of capital = 10%

Assume everything remains constant.

Most common use:

Find PV of each project assuming continuous replacement chains

1. Find NPV over its original life
2. Find annuity
3. Assume infinite replacement this its perpetuity value is $PV = \frac{A}{K}$

In our example, the five steps are:

1. $NPV_A = 671$
 $NPV_B = 874$
2. $671 = A [PVIF_A (10\%, 5y)]$

3.

$$A = \frac{671}{3.7908} = 177.01$$

$$874 = B [PVIF_A (10\%, 10y)]$$

$$B = \frac{874}{6.1446} = 142.24$$

4. $NPV_A = \frac{177.01}{.10} = 1770.10$

5.

$$NPV_B = \frac{142.24}{.10} = 1422.40$$

Choose A.

Another example:

Evaluation projects of unequal lines (long-short lined)

Project	Initial Cost	Life (N)	Cost of Capital	Annual Cash Flow
A	280	5y	10%	100
B	350	7y	12%	105

For Project A:

$$NPV_A = 100 [PVIF_A (10\%, 5y)] - 280$$

$$= 100 (3.7908) - 280 = 99.08$$

For Project B:

$$NPV_B = 105 [PVIF_A (12\%, 7y)] - 350$$

$$= 105 (4.5638) - 350 = 129.20$$

For Project A:

$$99.08 = A [PVIF_A (10\%, 5y)]$$

$$A = \frac{99.08}{3.7908} = 26.137$$

$$NPV_{A\infty} = \frac{A}{K} = \frac{26.137}{.10} = 261.37$$

For Project B:

$$129.20 = A [PVIF_A (12\%, 7y)]$$

$$A = \frac{129.20}{4.5638} = 28.310$$

$$NPV_{B\infty} = \frac{A}{K} = \frac{28.310}{.12} = 235.92$$

Thus select A over project B.

*Notice: to calculate PNPV of the project over its original life we do not have to have equal annual cash flows.

More Equivalent Annual Annuities

- Example:**
- You own a hotel in Bend. The location is killer but the place is run down.
 - Remodeling will cost you \$100,000 but generate cash flows of \$50,000 per year for the next five years.
 - Rebuilding will cost you \$300,000 but generate cash flows of \$66,000 per year for the next ten years.
 - Your opportunity cost of capital is 10%

Cash Flows:

Time	0	1	2	3	4	5	6	7	8	9	10
Remodel	-100	+50	+50	+50	+50	+50					
Rebuilding	-300	+66	+66	+66	+66	+66	+66	+66	+66	+66	+66

What is the NPV of remodeling your hotel?

$$NPV = -100,000 + \frac{50,000}{1.10} + \frac{50,000}{1.10^2} + \frac{50,000}{1.10^3} + \frac{50,000}{1.10^4} + \frac{50,000}{1.10^5}$$
$$= 89,550$$

OR

$$\begin{aligned}
 NPV_M &= 50,000 [PVIFA (10\%, 5y) - 100,000 \\
 &= 50,000 (3.791) - 100,000 \\
 &= 189,550 - 100,000 = 89,550
 \end{aligned}$$

What is the NPV of rebuilding your hotel?

$$\begin{aligned}
 NPV &= -300,000 + \frac{66,000}{1.10} + \frac{66,000}{1.10^2} + \frac{66,000}{1.10^3} + \dots + \frac{66,000}{1.10^{10}} \\
 &= 105,570
 \end{aligned}$$

OR

$$\begin{aligned}
 NPV_B &= 66,000 [PVIFA (10\%, 10y) - 300,000 \\
 &= 66,000 (6.145) - 300,000 \\
 &= 405,570 - 300,000 = 105,570
 \end{aligned}$$

- Summary:**
- Remodeling has an NPV of \$89,550
 - Rebuilding has an NPV of \$105,570

What's the problem? The projects have different lives!

<i>Time</i>	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>
<i>Remodel</i>	-100	+50	+50	+50	+50	+50					
<i>Rebuilding</i>	-300	+66	+66	+66	+66	+66	+66	+66	+66	+66	+66

Are we doomed? No way! Equivalent Annual Annuity to the rescue!

For Project M

$$NPV_F = A_F [PVIFA (5y, 10\%)]$$

$$89,550 = A_F (3.791)$$

$$A_F = 23,621$$

For Project B

$$NPV_G = A_G [PVIFA (10y, 10\%)]$$

$$105,570 = A_G (6.145)$$

$$A_G = 17,179$$

Thus, Project M (remodel) is better.

Additional Problem

Consider two machines that produce widgets. The machines are designed differently but have identical capacity and do exactly the same job.

The “spendy” version costs \$15,000, lasts three years, and only costs \$4,000 per year to run. The “cheapo” version costs \$10,000, lasts two years, and costs \$6,000 per year to run. ($r = 6\%$)

Step 1:

$$NPV_S = -15,000 - 4,000 [PVIFA (6\%, 3y)]$$

$$= -15,000 - 4,000 (2.673)$$

$$= -15,000 - 10,692 = -25,692$$

$$NPV_C = -10,000 - 6,000 [PVIFA (6\%, 2y)]$$

$$= -10,000 - 6,000 (1.833)$$

$$= -10,000 - 10,998 = -20,998$$

Step 2:

$$NPV_S = A_S [PVIFA (6\%, 3y)]$$

$$-25,692 = A_S (2.673)$$

$$A_S = -9,611.6$$

$$NPV_C = A_C [PVIFA (6\%, 2y)]$$

$$-20,998 = A_C (1.833)$$

$$A_C = -11,455.5$$

EQUIVALENT ANNUAL COSTS

We can illustrate this problem with a simple example. Imagine we are in the business of manufacturing stamped metal subassemblies. Whenever a stamping mechanism wears out, we have to replace it with a new one to stay in business. We are considering which of two stamping mechanisms to buy.

Machine A costs \$100 to buy and \$10 per year to operate. It wears out and must be replaced every two years. Machine B costs \$140 to buy and \$8 per year to operate. It lasts for three years and must then be replaced. Ignoring taxes, which one should we choose if we use a 10 percent discount rate?

Solution:

In comparing the two machines, we notice that the first is cheaper to buy, but it costs more to operate and it wears out more quickly. How can we evaluate these trade-offs? We can start by computing the present value of the costs for each:

Step 1

$$\begin{aligned} \text{Machine A: } PVA &= -100 - 10 [PVIF (1y, 10\%)] \\ &\quad - 10 [PVIF (2y, 10\%)] \\ &= -100 - 10 (.909) - 10 (.826) \\ &= -100 - 9.09 - 8.26 = -117.35 \end{aligned}$$

$$\begin{aligned} \text{Machine B: } PVB &= -140 - 8 [PVIF (1y, 10\%)] - 8 [PVIF (2y, 10\%)] \\ &\quad - 8 [PVIF (3y, 10\%)] \\ PVB &= -140 - 8 (.909) - 8 (.826) - 8 (.751) \\ &= -140 - 7.27 - 6.61 - 6.01 \\ &= -159.89 \end{aligned}$$

Notice that *all* the numbers here are costs, so they all have negative signs. If we stopped here, it might appear that A is more attractive because the PV of the costs is less. However, all we have really discovered so far is that A effectively provides two years' worth of stamping

service for \$117.36, whereas B effectively provides three years' worth for \$159.89. These costs are not directly comparable because of the difference in service periods.

We need to somehow work out a cost per year for these two alternatives. To do this, we ask: What amount, paid each year over the life of the machine, has the same PV of costs? This amount is called the **equivalent annual cost (EAC)**.

Step 2

For Machine A with $n=2$

$$-117.36 = A [\text{PVIF}_A (2y, 10\%)]$$

$$-117.36 = A (1.736)$$

$$A = -67.62$$

For Machine B with $n=3$

$$-159.89 = A [\text{PVIF}_A (3y, 10\%)]$$

$$-159.89 = A (2.487)$$

$$A = -64.29$$

Based on this analysis, we should purchase B because it effectively costs \$64.29 per year versus \$67.62 for A. In other words, all things considered, B is cheaper. In this case, the longer life and lower operating cost are more than enough to offset the higher initial purchase price.

Notice since cost of capital is the same for both machines, we do not need the third step to find the perpetuity cost.

$$\text{NPV}_{A\infty} = \frac{-67.62}{.10} = -676.2$$

$$\text{NPV}_{B\infty} = \frac{-64.29}{.10} = -642.9$$

Investment Timing Decision

Recall that Dewey, Cheatem, and Howe wants to invest in a computer system that will help them overcharge clients. The computer system costs \$30,000 today but will only cost \$25,000 next year. Assume that the present value of the additional client charges *at the time of installation* is a constant \$37,302. Should Dewey, Cheatem, and Howe wait a year to install Dell's new FleeceM 1000? ($r = 10\%$)

The NPV of installing the FleeceM 1000 this year is:

$$\text{NPV} = -\$30,000 + \$37,302 = \$7,302$$

The NPV of installing the FleeceM 1000 next year is:

$$NPV = \frac{-\$25,000}{1.1} + \frac{\$37,302}{1.1} = \$11,183$$

Therefore, they want to install the FleeceM 1000 next year.

For what future price of the Dell FleeceM 1000 are Dewey, Cheatem, and Howe indifferent between installing the computer today and installing it next year?

Again, the NPV of installing the FleeceM 1000 this year is:

$$NPV = -\$30,000 + \$37,302 = \$7,302$$

Setting the NPV of installing the FleeceM 1000 next year equal to the NPV of installing the FleeceM 1000 this year and solving for Price...

$$NPV = \frac{-\text{Price}}{1.1} + \frac{\$37,302}{1.1} = \$7,302 \Rightarrow \text{Price} = \$29,270$$

Therefore, for any future price less than \$29,270, the Dewey, Cheatem, and Howe will wait a year to install their new computer system.

Investment Timing Decision - Take 2

Now consider the optimal time to purchase a new computer system. Assume that the PV of the savings *at the time of installation* is a constant \$70,000, but that the cost of the computer declines over time. When should you buy?

Year of Purchase	Cost of Computer	PV of Savings	NPV @ Year of Purchase	Growth in NPV @ Year of Purchase	NPV Today (r = 10%)
0	-50	70	20		20.00
1	-45	70	25	$(25-20)/20 = 25.00\%$	22.73
2	-40	70	30	$(30-25)/25 = 20.00\%$	24.79
3	-36	70	34	$(34-30)/30 = 13.33\%$	25.54
4	-33	70	37	$(37-34)/34 = 8.82\%$	25.27
5	-31	70	39	$(39-37)/37 = 5.41\%$	24.22

You should buy the computer when the rate in increase in NPV (the return to waiting) falls below your discount rate. In this case, year three.

1. A widget manufacturer currently produces 200,000 units a year. It buys widget lids from an outside supplier at a price of \$2 a lid. The plant manager believes that it would be cheaper to make these lids than buy them. Direct production costs are estimated to be only \$1.50 a lid. The necessary machinery would cost \$150,000. It would be depreciated on a straight line basis over a ten year life to a zero salvage value. The plant manager estimates that the operation would require additional working capital of \$30,000, but argues that this sum can be ignored since it is recoverable at the end of the 10 years. If the company pays tax at a rate of 34% and the opportunity cost of capital is 15%, would you support the plant manager's proposal? State clearly any additional assumptions that you need to make.
2. The Army is asking for bids on multiple-use digitizing devices (MUDDs). The contract calls for 4 units to be delivered each year for the next 3 years. Labor and material costs are estimated to be \$10,000 per MUDD. Production space can be leased for \$12,000 per year. The project will require \$50,000 in new equipment, which is expected to have a salvage value of \$10,000 at the end of the project. Making MUDDs will mean a \$10,000 increase in net working capital. The tax rate is 34% and the required return is 15%. Assume straight-line depreciation.
3. A firm is comparing the purchase of two mutually exclusive machine investments. Machine F involves an investment of \$40,000 and would produce annual net cash flows after taxes of \$12,000 for five years. Machine H would require an investment of \$100,000 and would produce annual cash flows after taxes of \$30,000 for seven years. Machine H is somewhat more risky and requires a cost of capital of 12%, compared to 10% for Machine F. Which machine should be selected?

$$1. \text{ Dep} = \frac{150}{10} = 15\text{k}$$

$$CF = (R-E)(1-t) + D(t)$$

$$\text{Expenses yearly} = 200,000 \times 2 = 400,000$$

If it buys

$$CF = (-400,000)(1-.34) = -264\text{k yearly}$$

$$PV \text{ of } CF = \sum_{t=1}^{10} \frac{-264}{(1.15)^t} = -264 (5.019) = -1325\text{k}$$

If it produces

$CF_t = 0$	Cost	-150	
	Working Capital		<u>-30</u>
	Total	-180	

$$\text{Expenses yearly} = 1.5 \times 200,000 = 300,000$$

$$CF_{t=1-10} = (-300)(1-.34) + 15(.34) = -198 + 5.1 = -192.9$$

$$t=10 \quad \text{recovery of working capital} \quad +30$$

$$NPV = -192.9 [\text{PVIFA} (10\text{y}, 15\%)] + 30 [\text{PVIF}(10\text{y}, 15\%)] = 180$$

$$= -1127.9 + 3.7 = -1124 - 180 = -1304.2$$

Thus produce vs. buy

Or alternatively:

$$\text{Saving per unit if is produced} = 2 - 1.15 = .50$$

$$\text{Total savings per year} = .5 \times 200,000 = \$100,000$$

$$CF_{t=1-10} = 100,000 (1-.34) + 15,000 (.34) = 66,000 + 5,100 = 71,100 \text{ per year}$$

$CF_t = 0$	Cost	-150	
	Working Capital		<u>-30</u>
	Total	-180	

$$t=10 \quad \text{recovery of working capital} \quad +30$$

$$\Delta NPV = 71,100 [\text{PVIFA} (10\text{y}, 15\%)] + 30,000 [\text{PVIF} (10\text{y}, 15\%)] - 180,000$$

$$= 71,100 (5.84) + 30,000 (.1229) - 180,000$$

$$= 415,224 + 3,687 - 180,000 = \$238,911$$

Thus produce

2.

Year	0	1	2	3
Purchase	-50,000	0	0	0
Additional NWC	-10,000			
OCF		CF	CF	CF
Recovery of NWC				+10,000
Recovery of SV (1-t): 10k(1-.34)				6,600
TOTAL	-60,000	CF	CF	CF+16,600

$$NPV = CF [PVIFA (3y, 15\%) + 16,600 [PVIF (3y, 15\%) = 60,000$$

$$CF [2.283] + 16,600 (.658) = 60,000$$

$$2.283CF + 10,923 = 60,000$$

$$2.283CF = 49,077$$

$$CF = \$21,497$$

$$Dep = \frac{50000}{3} = 16,667$$

$$CF = EBIT (1-t) + D$$

$$21,497 = EBIT (1-t) + 16,667$$

$$EBIT (1-t) = 4,830$$

$$(S - OC - D) (1-.34) = 4,830$$

$$OC = 12,000 + 4 (10,000) = 52,000$$

Year	0	1	2	3
OCF		CF	CF	CF

$$(S - 52,000 - 16,667) (.66) = 4,830$$

$$(S - 68,719) (.66) = 4,830$$

$$.66S - 45,355 = 4,830$$

$$.66S = 50,185$$

$$S = 76,038$$

$$Sales Permit = \frac{76,038}{4} = \$19,009 \text{ per MUDD}$$

3.

Step 1

$$NPV_F = 12,000 [PVIFa (5y, 10\%)] - 40,000$$

$$= 12,000 (3.791) - 40,000 = 45,492 - 40,000 = 5,492$$

$$NPV_H = 30,000 [PVIFa (7y, 12\%)] - 100,000$$

$$= 30,000 (4.564) - 100,000 = 136,920 - 100,000 = 36,920$$

Step 2

$$5492 = A [\text{PVIFa}(5\text{y}, 10\%)]$$

$$A_F = \frac{5492}{3.791} = 1449$$

$$6920 = A [\text{PVIFa}(7\text{y}, 12\%)]$$

$$A_H = \frac{36920}{4.564} = 8089$$

Step 3

$$NPV_{\infty}(F) = \frac{1449}{.10} = 14,490$$

$$NPV_{\infty}(FH) = \frac{8089}{.12} = 67,412$$

Conclusion: Machine H is preferable.