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Advance of Dynamic Production-Inventory Strategy for Multiple Policies Using Genetic Algorithm

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Abstract: This study is mainly to cope with the problems when a firm faces time-varying demand and receives product from a single supplier who faces random supply. The supplier's availability may be affected by events such as natural disasters, labor strikes, manufacturing defects, machine breakdowns, or other events. A model is proposed in this research that considers a dynamic production-inventory environment: the exponential distribution of disruption, as well as the demand can be time dependent. The model explains production-inventory as a NP-hard problem, which using genetic algorithms is developed to minimize the average total cost to determinate the production cycles under various ordering policies. To evaluate the performance of the proposed algorithm, a numerical study has been conducted to compare the ordering policies under various demands in an extensive order. Based on the computational result, it can be seen that the optimal ordering policy not only should strike a balance between protecting against stockouts during disruptions but also maintaining low inventory levels of finished products and raw materials.

Key words: Time-varying demand, genetic algorithm (GA), production-inventory problem with backlog

INTRODUCTION

Despite the careful attention paid to inventory planning in a supply chain, supply disruptions are inevitable. Disruptions may come from a variety of sources including natural disasters, labor strikes, manufacturing defects, machine breakdowns, or other events; meanwhile they are typically infrequent and unpredictable. Even though the shortage of inventory incurred from supply disruptions can be backlogged, there still evolves the excess costs for handling inventory backlog. Also it increases the difficulty to maintain the suitable stock level of raw materials as the disruptions might happen in the production process. As result, an effective production-inventory policy not only should strike a balance between protecting against stockouts during disruptions but also maintaining low inventory levels of finished products and raw materials.

Most studies on inventory management are based on the economic lot sizes, but Parsons *et al.* (2004) explained the advantage of formulating the problem in terms of reorder intervals rather than in terms of lot sizes. They established three principal reasons for this: (1) The experience that production planning is more naturally centered around the frequency of production because it indicated the numbers of set-ups, the requests for tooling and fixtures and the demands on the material handling system, (2) the mathematical representation of the model

is simplified and (3) from a scheduling point of view it is often practical to keep reorder intervals constant in the face of minor changes to demand forecasts and to adjust lot sizes accordingly.

Moreover, traditionally the economic lot size for raw material purchase and manufacturing batch size are determined separately. However, when the raw material is used in production, its ordering quantities are dependent on the batch quantity of the product. Therefore, it is undesirable to separate the problem of economic purchase of raw material from economic batch quantity. As results, one should determine the optimum production cycle time associated with its raw material ordering quantities at the same time.

This study discusses a manufacturing system which the manufacturer uses raw material, received from an outside supplier, so to produce a finished product under an imperfect production condition with trended demand and shortage which will be completely backlogged. This research formulates the problem in terms of reproduction intervals rather than in terms of lot sizes; meanwhile, the production-inventory policy is cyclically modeled into four stages as production, consumption, shortage and reproduction stage (Fig. 1). By assuming the production disruptions are obeyed the exponential distribution, this study uses genetic algorithm to optimize the production cycles in a given time horizon to minimize the average inventory costs at each production cycle. As

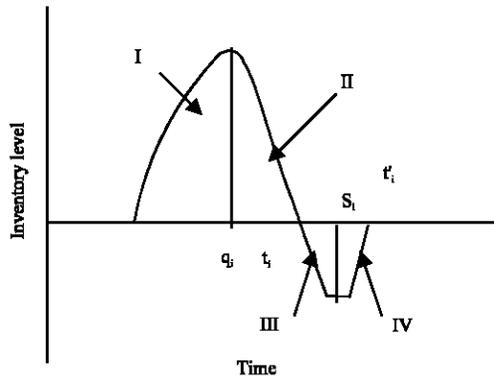


Fig. 1: The stages of EPQ system

the production cycles has determined, the (s, Q) inventory policy for raw material is also can be determined accordingly and the aggregative production plan can be generated.

After consulted the related literature and the consideration related important attribute, we also find the problem of the economic replenishment policy for an inventory item having a time-varying demand over a finite planning horizon has attracted the attention of researchers over the last 25 years since the publication of the research of Jumpol and Sarah (2006), which gave the analytic solution of the no-shortage case of a linear trend in demand. Donaldson used simple calculus to develop a computational procedure for finding the optimal reorder times over a finite planning horizon. Chyr *et al.* (2005) developed an approximate solution procedure for a linearly time-varying demand by using the Silver-Meal heuristic. None of these models permitted shortages in inventory.

Chan *et al.* (2007) developed an Economic Production Quantity (EPQ) model with a linearly trended demand and a uniform production rate with inventory shortages. Schweitzer and Seidmann (1991) questioned the assumption of uniform production rate and pointed out that the machine production rate should be flexible to adjust itself with the variability in the market demand. Giri and Chaudhuri (1999) discussed a production-inventory model in which the demand varies linearly with time, unit production cost is taken to be a function of the production rate and shortage in inventory are allowed and fully backlogged. Meanwhile, Sana *et al.* (2004) developed an Economic Production Lot Sizing model (EPLS) for a deteriorating item over a finite planning horizon with a linearly time-varying demand rate and a uniform production rate, allowing shortages which are completely backlogged. In the EPLS models of Zhou (1995, 1996), over a finite horizon with a linear trend

in demand and shortages, the production rate is adjusted at the beginning of each production cycle to cope with the increasing demand and the cost of adjusting the production rate depends linearly on the magnitude of change in the production rate.

From those studies discussed above, the production conditions were hypothesized under a perfect condition, that is, there were no any production disruption existed. But in reality, there are many possibilities that the production might incur disruption in the process caused by natural disasters, labor strikes, manufacturing defects, machine breakdowns, or other events. The first treatment of supply disruptions in the literature appears to be that of Meyer *et al.* (1979), who consider a production facility subject to stochastic disruptions and repairs. Items produced by the facility are stored in a capacitated buffer that sees constant, deterministic demand. Their model is descriptive rather than prescriptive, characterizing the stockout percentage for a given inventory policy rather than finding the optimal policy.

Parlar and Berkin (1991) present an EOQ-like model with deterministic demand but stochastic disruptions and repairs with the aim of finding the optimal order quantity. Parlar and Perry (1995) relax the Zero-Inventory Ordering (ZIO) assumption and consider random as well as deterministic yields. Both the reorder point and the waiting time between unsuccessful orders are decision variables, leading to what the authors call a (Q, r, T) inventory policy. Moinzadeh and Prabhu (1997) consider a model based on the EPQ model. They propose a continuous-review (s, S) policy, rather than a (Q, r) policy, since the inventory level may fall strictly below the reorder point during a disruption. Chen *et al.* (2006) considered the possible alternatives existed on the production environment which will incur imperfect production and quality variance and used fuzzy AHP method to evaluate the most critical alternative in order to determine the economic production quantity. In this study, we deal the supply disruption caused by machine breakdowns and manufacturing defects with backlog and assume this disruption obeys the exponential distribution.

Traditionally the economic lot size for raw material purchase and manufacturing batch size are determined separately. An ordering policy for raw materials to meet the requirements of a production facility under a fixed quantity, periodic delivery policy has been developed by Sarker and Parija (1994), Jamal and Sarker (1993), Golhar and Sarker (1992) and Sarker and Golhar (1993). They considered that the manufacturer is allowed to place only one order for raw material per finished product inventory cycle. In this case, a fixed quantity of finished

goods is to be delivered to the customer at the end of a fixed interval. This delivery pattern forces inventory build-up in a saw-tooth fashion during the production up-time. The on-hand inventory depletes sharply at regular intervals during the production down time until the end of the cycle.

MODEL FORMULATION AND PROBLEM DEFINITION

Assumptions: The assumptions, similar to Sana *et al.* (2004), of production-inventory model at each cycle consist of four stages shown as Fig. 1.

The initial stock at each cycle is zero and production starts at the very beginning of the cycle. As production continues, in production stage I, inventory begins to build up continuously after meeting demand with a defect rate incurred from imperfect production system which obeys exponential distribution. Production is stopped at a certain time q_i . The accumulated inventory is then gradually depleted and ultimately becomes zero due to consumption in consumption stage II. Production does not restart in stage III and inventory shortages continue to accumulate for some time. Thereafter, production restarts, in stage IV, at s_i and shortages are gradually cleared after meeting current demands and defects. The cycle ends with zero inventories at time t'_i . The other assumptions about this production-inventory model include:

- Production rate is finite and constant
- Defect rate is constant and obeys exponential distribution
- Shortages are allowed and are completely backlogged
- Time horizon is finite
- The finite time horizon is divided into a finite number of production cycles, each of equal duration
- Demand rate is linear in time

Problem definition and modeling: Facing the stochastic demand of high variety of products, the most important thing a manufacturer must consider is how to make a quick respond to customer requirement with efficiently scheduling its aggregative production plan as well as lowering the total cost of production. That is, a flexible production schedule to determine when to produce what product in how much quantity according to demand forecasting will be the most important decision which a manufacturer should make. Therefore, a practical scenario is described as:

The initial stock of the i -th cycle is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to build up after

meeting demand. Production stops at the time q_i . The accumulated inventory is just sufficient enough to satisfy demand over the interval $[q_i, t_i]$. Shortages accumulate over $[t_i, s_i]$. Production restarts at time s_i and the accumulated shortages are fully supplied during $[s_i, t_i]$ after meeting current demands (Fig. 1). The cycle ends with zero inventories. Therefore, the problem can be formulated to minimize the Average Total Cost (ATC) during the time horizon H as following:

$$\min ATC(n) = \frac{1}{H} \left\{ C_h \sum_{i=1}^n [\text{stock_p}(i) + \text{stock_c}(i)] + C_s \sum_{i=1}^n [\text{stock_s}(i) + \text{stock_rp}(i)] + C_d \sum_{i=1}^n \text{stock_d}(i) + nK \right\}^{s.t.}$$

$$\text{stock_p}(i) = \int_0^{q_i} p dt \times (1 - \int_0^{q_i} \lambda e^{-\lambda t} dt) - \int_0^{q_i} (a + bt) dt = \int_{q_i}^{t_i} (a + bt) dt = \text{stock_c}(i) \tag{1}$$

$$\text{stock_s}(i) = \int_{t_i}^{s_i} (a + bt) dt = \int_{s_i}^{t'_i} p dt \times (1 - \int_{s_i}^{t'_i} \lambda e^{-\lambda t} dt) - \int_{s_i}^{t'_i} (a + bt) dt = \text{stock_rp}(i) \tag{2}$$

$$\text{stock_d}(i) = \int_0^{q_i} p dt \int_0^{q_i} \lambda e^{-\lambda t} dt + \int_{s_i}^{t'_i} p dt \int_{s_i}^{t'_i} \lambda e^{-\lambda t} dt \tag{3}$$

$$t_i = r_i T_i + (1 - r_i) T_{i-1}, 0 < r_i < 1 \tag{4}$$

$$q_i = k_i t_i + (1 - k_i) T_{i-1}, 0 < k_i < 1 \tag{5}$$

$$s_i = d_i T_i + (1 - d_i) t_i, 0 < d_i < 1 \tag{6}$$

$$T_i = (H/n) i,$$

Where:

$$i = 1, 2, \dots, n \tag{7}$$

Meanwhile, the associated (R, Q) inventory policy for raw material at each production cycle can also be determined as:

$$Q_{rm}(i) = \int_0^{q_i} p dt + \text{stock_s}(i-1) \tag{8}$$

$$R = H/n \tag{9}$$

ALGORITHM FORMULATION

Genetic algorithms are stochastic search techniques based on the mechanism of natural selection and natural genetics, which requires little information to search effectively in a large or poorly understood search space. Some of the principal advantages of the genetic algorithm are versatility, flexibility, simplicity and efficiency. They have been employed to solve optimization problems across all disciplines and interests and their simplicity permits to solve difficult problems as NP-hard problems.

Applications of genetic algorithms in production planning and inventory management include assembly line balancing, buffer size optimization, production scheduling and manufacturing cell design. Hernández and Sür (1999) presented an application of genetic algorithms to obtain the reorder quantity for an incapacitated, no shortage allowed, single-item, single-level situation and lot sizing problem. Khouja *et al.* (1998) proposed the use of genetic algorithms to solve the economic lot size scheduling problem.

The proposed algorithm in this study uses genetic algorithms with the chromosome of real number type. Simulations are taken to determine the optimal number of cycles and the starting times to produce at each production cycle and reproduction cycle in order to minimize the average total cost of inventory system.

The GA-Approach procedure will follow as:

Step 1: An iteration approach is taken from 1 to a finite number n to determine the time interval of each cycle with the interval length equals H/n . Therefore, the optimal number of production cycles will be determined.

Step 2: Randomly pick up r_i to determine the t_i Eq. 4 in each production cycle T_i , where $T_i = (H/n) \times i$ Eq. 7.

Step 3: A local search, Newton's Method, has been used to find out the initial solution of q and s_i relative to t_i Eq. 5 and 6 which satisfied Eq. 1 and 2.

Step 4: A selection based on Roulette Wheel picks up two candidates from population.

Step 5: A crossover, by taking the average of two candidates to produce child 1, is taken under a given probability.

Step 6: Mutation is taken under a given probability with randomly initiating two random numbers to mutate child 2 from child 1 by child 2 = $a \cdot \text{child 1} + (1-a)\omega$, where σ and ω are random number range in $[0, 1]$.

Step 7: Only the best according to its fitness to the objective function is kept to maintain the population sizes constant.

Step 8: Repeat step 4 to 7 until the termination criterion of maximal iterations has achieved.

Step 9: Restart from step 1 for next n until finish.

Step 10: The ordering policy for raw material is calculated from Eq. 8 and 9 based on the optimal production cycles calculated from above.

COMPUTATION RESULTS

For illustrative purpose, a simple numerical example is tested under the operation environment of Windows XP Professional with Pentium III CPU, 846 MHz, 312MB RAM and encoding with MATLAB. Two numerical cases with a linear demand, formulated as $f(t) = 100 \pm 2t$, under a constant production rate equal to 200 with a mean disruption rate, λ equal to 0.05, which obeys exponential distribution in a time horizon equal to 30 days have been simulated and compared in this study. In both cases we set the cost of backlog, the holding cost for finished product, the holding cost for defects and the setup cost for production equal 20, 15, 15 and 100, respectively. For running GA, we set the population size equal to 5, probability of crossover equal to 0.95, probability of mutation equal to 0.3 and the stopping criteria include the maximal iterations of each cycle is 20 and the maximal iterations of finding initial solution is 40 and 50 for crossover and mutation.

After simulation of 30 day time horizon, in the increasing demand case, the optimal solution is the average inventory cost of 297 resulted from dividing the planning time horizon into 15 cycles (Fig. 2) and its relative ordering policy for raw material at each production is shown as Fig. 4. Meanwhile, in a decreasing demand case, the optimal solution is the average inventory cost of 252 resulted from dividing the planning time horizon into 24 cycles (Fig. 3) and its relative ordering policy for raw material at each production is shown as Fig. 5. The average CPU time for running number of cycles from 1 to 30 is 666 sec.

By comparing Fig. 2 and 3, since the shortage is allowed and fully backlogged in our production-inventory model, in both cases, the sensitivity of the holding costs for finished products and defects and the backlog cost for a longer period of time is much higher than the sensitivity of the setup cost increasing as the number of production cycles increased. As results, the average inventory cost

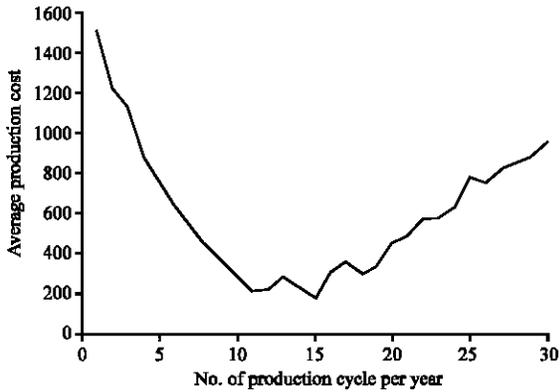


Fig. 2: The average cost of production-inventory model with increasing demand at different production cycle planning

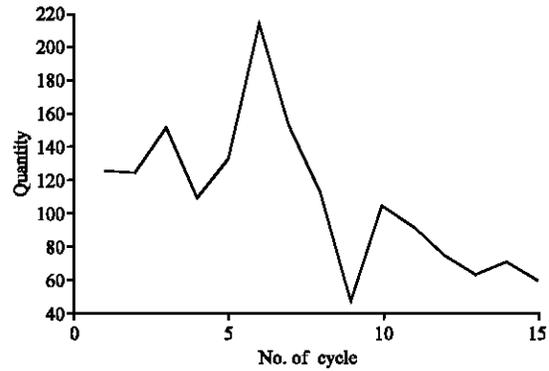


Fig. 4: The ordering policy for raw material with increasing demand with optimal production cycle planning

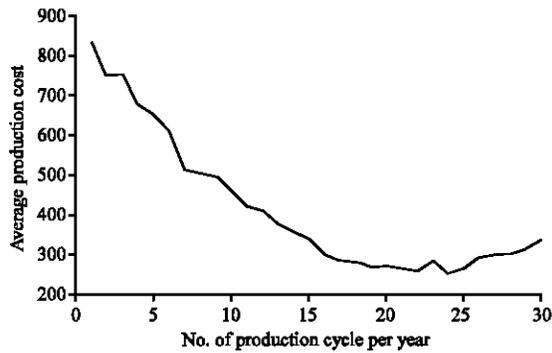


Fig. 3: The average total cost of production-inventory model with decreasing demand at different production cycle planning

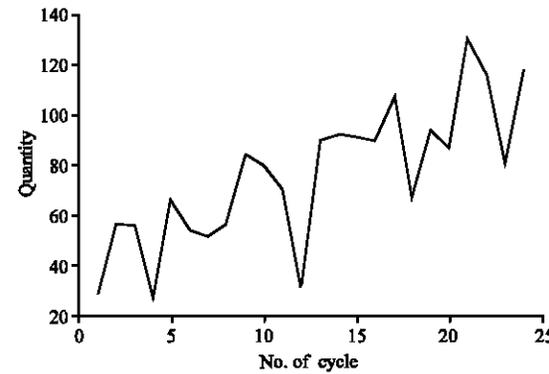


Fig. 5: The ordering policy for raw material with decreasing demand with optimal production cycle planning

has sharply decreased as the number of production cycles increased. But, as the setup cost effect is much sensitive than the effect of the costs, the average inventory cost increased again. Therefore the minimum average inventory cost can be searchable.

Since the demand is varied as time goes by, the optimal production cycles in increasing demand case are required less than they are in decreasing demand case. Meanwhile, the volatility due to production disruption is much more fluctuant in decreasing demand case than in increasing case. As shown in Fig. 4 and 5, the ordering quantity for inventory at each cycle is also much more fluctuant in decreasing case than in increasing case.

By formulated with the reproduction interval to search the optimal number of production cycle, the GA approached algorithm used in this study can provide not only the production periods in time horizon but also the requirement associated with each production cycle for raw material can be determined. As results, the inventory

policy and aggregate production plan can be generated on the basis of production cycles to minimize total inventory costs.

CONCLUSIONS

In this study, the genetic algorithm has been adopted to cope with the production-inventory problem with backlog in the real circumstance with time-varied demand and imperfect production due to the defects maybe exists in production disruption with exponential distribution. Based on the reproduction interval searching in a given time horizon, not only the number of production cycles could be optimized to generate a (R, Q) inventory policy, but also an aggregative production plan can be generated to minimize the total inventory cost. The result, at least a local optimum, is approachable to provider a comprehensive decision support to management.

This research is constrained on the equal duration of cycle periods divided from a finite planning time horizon and the demand rate is linear pattern. For future researches can consider using this production-inventory model on the dynamic duration of cycle periods in a finite or infinite time horizon but also consider using the different demand models for real time modification.

NOMENCLATURE

- $f(t) = a+bt$: b is the demand rate at time t where $a \geq b \neq 0$.
- p : Production rate, $p > a+bt$, $t \in [0, H]$
- λ : The average defect rate per unit time due to production disruption obeys exponential distribution
- C_h : Inventory holding cost per item per unit time
- C_s : Backlog cost per item per unit time
- C_d : Defect cost per item per unit time
- K : Setup cost per cycle
- H : Time horizon
- n : Number of cycles in $[0, H]$
- stock_p : The accumulated stock in stage of production
- stock_c : The accumulated stock in stage of consumption
- stock_s : The accumulated stock in stage of shortage
- stock_rp : The accumulated stock in stage of reproduction
- stock_d : The accumulated stock of defects due to production disruption
- $Q_{rm}(i)$: The required raw material quantity for ordering at cycle i
- $V(i)$: The stock level of raw material at the end of cycle i
- \overline{LT} : The average lead time for purchasing raw material
- σ_{LT} : The standard deviation of lead time for purchasing raw material
- $Z_{\alpha/2}$: The z -value of standard normal distribution under the confidence interval of α

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