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An integrated production-inventory-marketing model under inflationary conditions for deteriorating items

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Abstract: The integration of production and marketing policies is one of the key factors of successful business operation. Moreover, inventory policies for deteriorating items are very sensitive to different marketing strategies and rising inflation rate directly affects the financial situation of industries. Realising the importance of such scenario, an integrated production-inventory-marketing model has been developed under inflationary conditions for deteriorating items; where demand is considered as a function of insertions in a media, as it is well-established fact that 'marginal effect of advertisement on sales is proportional to the unrealised potential of the market'. The model jointly optimises the production lot size and the number of insertions by maximising the net profit. A simple to use solution procedure has been developed to determine these parameters. Further, the model has been extended for multiple media case and a general form of demand function has been developed. The model is then formulated as a mathematical programming problem. The results have been validated with numerical examples followed by sensitivity analysis.

Keywords: inventory; production; media insertion; deterioration; inflation.

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1 Introduction

In the present scenario, great emphasis has been on the aspect of customer demands, as the market is totally customer oriented. In the literature, a lot of work has been published for controlling the production-inventory problems of deteriorating and non-deteriorating items under different assumptions, but the interaction of production, inventory together with marketing parameters (like frequency of advertisement, price elasticity) in demand function has received little attention. However, in the present scenario, advertisement has a very significant influence on demand; as one of the simple basic observation, which cannot be ignored, is that 'advertisement is undertaken in order to sell product'. The dependence of demand on varying marketing conditions, like the frequency of advertisement, fluctuation in prices, price elasticity, etc. has so far received a very little attention in the study of inventory management. [Ladany and Sternlieb \(1974\)](#) considered the effect of price variation on demand. [Subramanyam and Kumaraswamy \(1981\)](#) considered the effect of price elasticity as well as frequency of advertisement on the demand assuming a linear relationship. Aggarwal and Aylawadi (1988) formulated an economic order quantity (EOQ) model by taking demand as function of promotional efforts and price. Further, Aggarwal et al. (1993) extended the model by taking demand as a function of four parameters: price, quality, promotional efforts and distribution. McTavish and Goyal (1998) presented a model by incorporating price elasticity, frequency of advertisement and the batch quantity. None of these models incorporate the practical situation because in practice, every organisation would like to fetch the demand up to its saturation level through advertisement. [Goyal and Gunasekaran \(1995\)](#) developed an integrated production-inventory-marketing model for deteriorating items taking demand as a function of price per unit product and the advertisement frequency. Kapur et al. (2001) in their paper considered demand function dependent on the number of insertions in media assuming that the marginal effect of advertisement on sales is proportional to the unrealised potential of the market and jointly optimised the number of insertions and production lot size. They also modified the model to include frequency of insertions in different media.

In real life situations, there is inventory loss by deterioration and the inventory value is dependent on the product value at the time of evaluation. Generally, deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, pilferage, etc. that results in decreasing the usefulness of the original one. For items such as steel, hardware, glassware and toys, the rate of deterioration is low in which case, there is little need for considering deterioration in the determination of the economic lot size. However, some

items such as food items, pharmaceuticals, chemicals, blood, alcohol, gasoline and radioactive chemical deteriorate rapidly over time. Thus, the loss from deterioration should not be ignored. At first, Ghare and Schrader (1963) presented an EOQ model for deteriorating items assuming exponential decay. Covert and Phillip (1973) extended the model with the assumption of Weibull distribution deterioration. Later, there are several interesting papers related to deterioration such as Dave and Patel (1981), Hollier and Mark (1983), Sachan (1984), Chung and Ting (1994), Hargia and Benkherouf (1994), Hargia (1995) and Chakrabarti and Chaudhuri (1997). Recently, [Samanta and Roy \(2004\)](#) developed a production inventory model, where distribution of the time to deterioration of an item follows exponential distribution.

From a financial standpoint, an inventory represents a capital investment and must compete with other assets for a firm's limited capital funds. The effects of inflation are not usually considered when an inventory system is analysed because most people think that inflation would not influence the inventory policy to any significant degree. Due to high inflation, the financial situation has changed in many developing countries. Besides this, inflation also influences demand of certain products. As inflation rate increases, the value of money goes down which erodes the future worth of savings and forces one for more current spending. These spending may be on clothes, accessories, peripherals or daily household items that give rise to demand of these items. As a result, while determining the optimal inventory policy, the effect of inflation cannot be ignored. Many authors have developed different inventory models incorporating the concept of inflation under different assumptions. The fundamental result in the development of EOQ model with inflation is that of Buzacott (1975) who discussed EOQ model with inflation subject to different types of pricing policies. Several other interesting and relevant papers in this direction are Datta and Pal (1991), Hargia and Ben-Daya (1996) and Aggarwal et al. (1997). Later, Ray and Chaudhuri (1997), Chen (1998), Chung and Lin (2001), Wee and Law (2001) and Jaggi and Goel (2005) all have investigated the effects of inflation, time value of money and deterioration on inventory models. Further, [Moon et al. \(2005\)](#) developed EOQ model for deteriorating items under inflation and time discounting. Recently, [Jaggi et al. \(2006\)](#) investigated the optimal ordering policies for deteriorating items with inflation-induced demand.

However, none of the above models considered the effects of inflation and time value of money on a production-inventory-marketing model for deteriorating items. The present paper investigates the impact of marketing strategies on a production lot size model under inflation for deteriorating items. Here, demand is taken as a function of insertions in a media, as it is a well-established fact that 'the marginal effect of advertisement on sales is proportional to the unrealised potential of the market'. The main purpose is to see how marketing strategies i.e., advertisements in media can help the seller to attract more customers and also fetch the unrealised demand. An integrated production-inventory model is then formulated to maximise the net profit by jointly optimising number of insertions and production lot size. An algorithm is also provided in this regard. Further, the authors extend the model for multiple media; to include frequency of insertion in different media, as in reality, it is observed that companies explore more than one media for better effective exposure. The model is then formulated as mathematical programming problem. Problem has been solved using software What's Best followed by a sensitivity analysis.

2 Assumptions and notations

2.1 Assumptions

- 1 The demand for the product is assumed to be known and uniform which depends upon the number of insertions in media, denoted by ' $D = D(N)$ ', demand due to N insertions in media per cycle.
- 2 The authors assume that production occurs at a constant rate. R denotes the annual production rate. The authors further assume that no backorders are allowed and that the production rate exceeds the demand rate or $R > D$.
- 3 The advertisements are placed at equal time intervals in a planning period.
- 4 Shortages are not allowed.
- 5 There exists a saturation level to the market.
- 6 Advertisement awareness is transferred into sales without any time lag.
- 7 Inflation rate is constant over the planning horizon.
- 8 A DCF approach is used to consider the various costs at various times.
- 9 Planning horizon is infinite.
- 10 Lead-time is negligible.
- 11 A constant fraction of the on-hand inventory deteriorates per unit time.
- 12 There is no repair or replenishment of the deteriorated items during the inventory cycle.

2.2 Notations

$D = D(N)$ demand rate per unit time

$I(t)$ the inventory level at time t

Q economic production quantity in each cycle

$A(t)$ ordering cost per order (Rs./order) at any time t

$P(t)$ selling price of the item per unit (Rs.) at any time t

$C(t)$ purchasing cost of the item per unit (Rs.) at any time t

$Ca(t)$ cost of putting an advertisement in media (Rs./insertion) at any time t

I inventory carrying charge per unit time

α constant inflation rate

s opportunity cost (discount rate)

K constant representing the difference between the discount (cost of capital) and the inflation rate i.e., $K = s - \alpha$

θ the deterioration rate per unit time ($0 \leq \theta \leq 1$)

T	length of each replenishment cycle
N	frequency of advertisements
\bar{D}	maximum demand rate if the total market potential is realised due to advertisement
d_o	initial demand without any advertisement
r	rate at which demand approaches the maximum-demand (\bar{D}) in response to media insertion.

3 Model formulation for optimal insertions for single media under inflation

In this section, a production inventory system for deteriorating items is considered with demand as a function of insertion only in single media without any budget restriction under inflationary conditions. The authors' objective is to jointly optimise the number of insertions and cycle length by maximising net profit.

3.1 Mathematical formulation

The demand is governed by the following function:

$$D(N+1) - D(N) = r[\bar{D} - D(N)] \quad (3.1)$$

under the assumption that 'the marginal effect of advertisement on sales is proportional to the unrealised potential of the market demand without any delay'. The solution of the above difference equation, under the condition that at $N = 0$, $D(N) = d_o$ (initial demand), keeping other attributes like price, quantity, etc. at constant level, is given by:

$$D = d_o (1-r)^N + \bar{D} (1 - (1-r)^N) \quad (3.2)$$

i.e., $D = \bar{D} - (\bar{D} - d_o)(1-r)^N$

It is important to note that for the existence of the above demand function for any product, the parameters viz. initial demand (d_o), maximum demand (\bar{D}) and rate of saturation of demand (r) must be known, if not, then these parameters can be estimated using the past data.

The authors assume that the planning horizon is infinite and consists of replenishment cycles of equal length. Hence, the reorder times over the planning horizon are $T_i = iT$ ($i = 0, 1, 2, \dots$). Also $T_i - T_{i-1} = T$.

Each replenishment cycle could be divided into two periods. During the period t_1 , the inventory level increases due to production, less demand and deterioration until the maximum inventory level at $t = t_1$ is reached. After t_1 , production stops and the inventory level decreases due to demand and deterioration. Thus, the differential equations for the instantaneous state of $I(t)$ over $(0, T)$ are given by:

$$\frac{dI(t)}{dt} + \theta I(t) = R - D \quad 0 \leq t \leq T_1 \quad (3.3)$$

and

$$\frac{dI(t)}{dt} + \theta I(t) = -D \quad T_1 \leq t \leq T \quad (3.4)$$

The solution of equation (3.3) is using condition, $I(t) = 0$ at $t = 0$ is

$$I(t) = \frac{R-D}{\theta} \left(1 - e^{-\theta t} \right) \quad 0 \leq t \leq t_1 \quad (3.5)$$

and the solution of equation (3.4) is using condition, $I(t) = 0$ at $t = T$ is

$$I(t) = \frac{D}{\theta} \left(e^{\theta(T-t)} - 1 \right) \quad t_1 \leq t \leq T \quad (3.6)$$

Equating the equations (3.5) and (3.6) at $t = t_1$, we have

$$\frac{R-D}{\theta} \left(1 - e^{-\theta t_1} \right) = \frac{D}{\theta} \left(e^{\theta(T-t_1)} - 1 \right) \quad (3.7)$$

which gives

$$t_1 = \frac{1}{\theta} \log \left(1 + \frac{D}{R} \left(e^{\theta T} - 1 \right) \right)$$

and the optimal order quantity can be calculated using the following equation:

$$Q = R \cdot t_1 \quad (3.8)$$

Moreover, the present model has been developed under inflationary conditions. Hence, one simple way of modelling is to assume α , the constant rate of inflation.

Let $C(t)$ be the unit purchase cost of an item at time t , which becomes through the inflation of cost:

$$C(t + \delta t) \text{ at time } t + \delta t,$$

where $C(t + \delta t) = C(t) + \alpha C(t) \delta t$

$$\Rightarrow \frac{C(t + \delta t) - C(t)}{\delta t} = \alpha C(t)$$

As $\delta t \rightarrow 0$, we get $\frac{dC(t)}{dt} = \alpha C(t)$

The solution of the above differential equation along with the boundary condition at time $t = 0$, $C(t) = C_0$ is

$$C(t) = C_0 e^{\alpha t} \quad (3.9)$$

where C_0 is the cost at time zero.

Similarly, the selling price, ordering costs, out-of-pocket inventory carrying cost and advertisement cost at any time t are

$$\begin{aligned} P(t) &= P_0 e^{\alpha t} \\ A(t) &= A_0 e^{\alpha t} \\ IC(t) &= IC_0 e^{\alpha t} \\ Ca(t) &= Ca_0 e^{\alpha t} \end{aligned} \quad (3.10)$$

where

P_0 is the selling price at time zero

A_0 is the ordering cost at time zero

IC_0 is the inventory carrying cost at time zero

Ca is the advertisement cost at time zero.

The present worth of net profit for the first replenishment cycle, $Z_1(T, N)$ is given by sales revenue – ordering cost – purchasing cost – inventory holding cost – advertisement cost.

By using the ‘DCF’ approach, the present worth of the various costs for the first replenishment cycle is as follows:

- 1 present worth of the sales revenue for the first cycle, SR_1 is

$$\begin{aligned} SR_1 &= \int_0^T DP(t) e^{-st} dt \\ &= \int_0^T DP_0 e^{\alpha t} e^{-st} dt \\ &= P_0 \int_0^T D e^{-Kt} dt \\ &= \frac{P_0 D (1 - e^{-Kt})}{K} \end{aligned} \quad (3.11)$$

where $K = s - \alpha$

- 2 present worth of ordering cost for the first cycle, Co_1 is

$$Co_1 = A_0 \quad (3.12)$$

- 3 present worth of purchase cost for the first cycle, Cp_1 is

$$Cp_1 = QC_0 \quad (3.13)$$

- 4 present worth of the inventory holding cost for the first cycle, Ch_1 is

$$\begin{aligned}
Ch_1 &= I \left\{ \int_0^{t_1} I(t)C(t)e^{-st} dt + \int_{t_1}^{T_1} I(t)C(t)e^{-st} dt \right\} \\
&= I \left\{ \int_0^{t_1} I(t)C_0 e^{\alpha t} e^{-st} dt + \int_{t_1}^{T_1} I(t)C_0 e^{\alpha t} e^{-st} dt \right\} \\
&= IC_0 \left\{ \int_0^{t_1} I(t)e^{-Kt} dt + \int_{t_1}^{T_1} I(t)e^{-Kt} dt \right\} \\
&= IC_0 \left\{ \int_0^{t_1} \frac{R-D}{\theta} (1-e^{-\theta t}) e^{-Kt} dt + \int_{t_1}^{T_1} \frac{D}{\theta} (e^{\theta(T-t)} - 1) e^{-Kt} dt \right\} \\
&= IC_0 \left\{ \left(\frac{R-D}{K\theta(K+\theta)} (\theta + Ke^{-(K+\theta)t_1} - (K+\theta)e^{-Kt_1}) \right) \right. \\
&\quad \left. + \left(\frac{D}{\theta} \left(\frac{e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{-KT} - e^{-Kt_1}}{K} \right) \right) \right\}
\end{aligned} \tag{3.14}$$

5 present worth of the advertisement cost for the first cycle, Ca_1 , is

$$CA_1 = Ca_0 N. \tag{3.15}$$

Consequently, the present worth of the net profit function for the first cycle, $Z_1(T, N)$ is given by:

$$Z_1(T, N) = SR_1 - Co_1 - Cp_1 - Ch_1 - CA_1 \tag{3.16}$$

Now, the present worth of total profit of system over infinite planning horizon is

$$\begin{aligned}
Z(T, N) &= Z_1(T, N) \sum_{i=0}^{\infty} e^{-KiT} \\
&= \left[P_0 \frac{D(1-e^{-KT})}{K} - A_0 - QC_0 - IC_0 \right. \\
&\quad \left(\frac{R-D}{K\theta(K+\theta)} (\theta + Ke^{-(K+\theta)t_1} - (K+\theta)e^{-Kt_1}) \right) \\
&\quad \left. - IC_0 \left(\frac{D}{\theta} \left(\frac{e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{-KT} - e^{-Kt_1}}{K} \right) \right) - Ca_0 N \right] \\
&\quad * \frac{1}{(1-e^{-KT})}
\end{aligned} \tag{3.17}$$

where $D = \bar{D} - (\bar{D} - d_o)(1-r)^N$ and $K = s - \alpha$.

3.2 Solution procedure

Since $Z(T, N)$ is an exponential function, it is difficult to prove mathematically that $Z(T, N)$ is a concave function. Therefore, the concavity of total profit function has been established graphically (Appendix).

Hence, the necessary and sufficient conditions for finding the optimum value of T (for a fixed N) is

$$\frac{dZ(T, N)}{dT} = 0$$

$$\frac{d^2Z(T, N)}{dT^2} \leq 0$$

(It is difficult to prove mathematically, but numerically, it has been verified)

Now, $\frac{dZ(T, N)}{dT} = 0$ implies

$$\left[\begin{aligned} & P_0 \frac{DKe^{-KT}}{K} - C_0RB - IC_0 \\ & \left(\frac{R-D}{K\theta(K+\theta)} \left(-K(K+\theta)e^{-(K+\theta)t_1}B + K(K+\theta)e^{-Kt_1}B \right) \right) \\ & -IC_0 \left(\frac{D}{\theta} \left(\frac{\theta e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} \right. \right. \\ & \quad \left. \left. + \frac{e^{\theta T} (-(K+\theta)e^{-(K+\theta)t_1}B + (K+\theta)e^{-(K+\theta)T})}{K+\theta} \right) \right. \\ & \quad \left. \left. + \frac{(-Ke^{-KT} + Ke^{-Kt_1}B)}{K} \right) \right] * E \end{aligned} \right] \quad (3.18)$$

$$+ \left[\begin{aligned} & P_0 \frac{D(1-e^{-KT})}{K} - A_0 - QC_0 - IC_0 \\ & + \left(\frac{R-D}{K\theta(K+\theta)} \left(\theta + Ke^{-(K+\theta)t_1} - (K+\theta)e^{-Kt_1} \right) \right) \\ & -IC_0 \left(\frac{D}{\theta} \left(\frac{e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{-KT} - e^{-Kt_1}}{K} \right) \right) - Ca_0N \end{aligned} \right] * F = 0$$

where

$$E = \frac{1}{(1-e^{-KT})}$$

$$F = \frac{dE}{dT} = \frac{-Ke^{-KT}}{(1-e^{-KT})^2}$$

$$t_1 = \frac{1}{\theta} \log \left(1 + \frac{D(e^{\theta T} - 1)}{R} \right)$$

$$B = \frac{dt_1}{dT} = \frac{DT}{R \left(1 + \frac{D(e^{\theta T} - 1)}{R} \right)}$$

Now, equation (3.18) can be solved for optimal T using the software What's Best, for a fixed N . The optimal value of order quantity and profit can be obtained using equations (3.8) and (3.17) respectively using the proposed algorithm.

In order to jointly optimise Q and N , the following algorithm is proposed:

- 1 Put $N = 1$.
- 2 Calculate $D = \bar{D} - (\bar{D} - d_o)(1 - r)^N$, T^* and $Z(N)$.
- 3 If $Z(N) > Z(N - 1)$, increment the value of N by 1 and go to Step 2. Otherwise, go to Step 4.
- 4 Current value of N is optimal and corresponding value of D^* , T^* and $Z^*(N)$ can be calculated.

Table 3.1 Optimal solution

N	Demand (year)	T (days)	Q (per cycle)	Profit
0	700	52.4	101.2	630,438
1	1,248	138.3	480.4	958,367
2	1,725	172.5	830.5	1,281,236
3	2,140	199.0	1,189.9	1,571,214
4	2,502	222.9	1,558.7	1,828,304
5	2,816	245.6	1,934.4	2,054,536
6	3,089	267.6	2,313.9	2,252,389
7	3,327	289.2	2,694.1	2,424,418
8	3,534	310.4	3,072.3	2,573,121
9	3,714	331.1	3,446.0	2,700,873
10	3,870	351.4	3,813.1	2,809,899
11	4,006	371.3	4,172.0	2,902,251
12	4,125	390.7	4,521.3	2,979,810
13	4,228	409.6	4,860.1	3,044,285
14	4,318	428.0	5,187.8	3,097,219
15	4,396	445.9	5,504.2	3,140,001
16	4,464	463.2	5,809.1	3,173,872
17	4,523	480.1	6,102.7	3,199,939
18	4,574	496.5	6,385.2	3,219,188
19	4,619	512.4	6,657.0	3,232,490
20	4,658	527.9	6,918.6	3,240,618
21	4,692	542.9	7,170.5	3,244,253
22	4,721	557.6	7,413.1	3,243,994

3.3 Numerical example

Let $\bar{D} = 4,918$ units, $d_o = 700$ units, $r = 0.13$, $s = 0.12$, $\alpha = 0.10$, $K = s - \alpha = 0.02$, $\theta = 0.10$, $P_0 = \text{Rs. } 70/-$ per item, $A_0 = \text{Rs. } 100$ per order, $C_0 = \text{Rs. } 50/-$ per item, $I = 20\%$, $Ca_0 = \text{Rs. } 1,000/-$ per advertisement and $R = 7,500$ per year.

Using these parametric values, the optimal solution of the basic model with single media under inflation is obtained as $N^* = 21$, $T^* = 542.9$ days, $Q^* = 7,171$ units approx. and $Z^*(T, N^*) = \text{Rs. } 3,244,253$. Results are computed and tabulated in Table 3.1 using the algorithm proposed in Section 3.2.

3.4 Sensitivity analysis

The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision-making. Using the numerical example given in the preceding section, the effects of changing the parameters θ and α i.e., deterioration rate and inflation rate on the optimal replenishment policy are studied by changing their respective values. The results are summarised in Tables 3.2 and 3.3.

Table 3.2 Effects of changing the deterioration rate θ on the optimal replenishment policy

θ	N	T (in days)	Q (per cycle)	Profit	Demand (per year)	$D - do$	$(D - do)/N$
0.10	21	542.9	7,170	3,244,253	4,692	3,992	190
0.20	20	445.1	5,936	2,969,172	4,658	3,958	198
0.30	19	381.1	5,105	2,740,058	4,619	3,919	206

Table 3.3 Effects of changing the inflation rate α on the optimal replenishment policy

α	N	T (in days)	Q (per cycle)	Profit	Demand (per year)	$D - do$	$(D - do)/N$
0.04	18	374.7	4,789	694,181	4,574	3,874	215
0.06	19	416.4	5,384	971,041	4,619	3,919	206
0.08	20	469.9	6,142	1,533,531	4,658	3,958	198
0.10	21	542.9	7,170	3,244,253	4,692	3,992	190

From Table 3.2, it is observed as the deterioration rate (θ) increases, there is considerable decrease in the cycle length (which suggests less number of insertions for deteriorating items) and profit. Moreover, the capturing rate of unsaturated sales per insertion increases.

Further, Table 3.3 suggests that as inflation rate (α) increases, there is a significant increase in the cycle length (which suggests more number of insertions under inflationary conditions) and profit, whereas capturing rate of unsaturated sales.

4 Optimal insertions in multiple media under inflation

In the previous section, a production-inventory model for deteriorating items under inflationary conditions has been formulated where demand is taken as the function of

insertion in single media only. In this section, the authors develop a production-inventory model for deteriorating items where demand is taken as a function of insertion in different medias, as in real practice, companies explore more than one media for better effectiveness. The authors' objective is to jointly optimise the number of insertions in each media and cycle length by maximising net profit over a finite planning horizon under inflationary conditions. Here, the authors assume that the awareness generated due to advertisements is translated into sales without any time lag, though the response rate for different media may vary.

4.1 Mathematical formulation

Let p_i be the proportion of the potential market demand influenced by the advertisement in the i th media. We can write the following equation using the assumption that 'the marginal effect of advertisement in the i th media is proportional to the size of the potential unsaturated market influenced by the media'.

$$D_i(N_i + 1) - D_i(N_i) = r_i \{ p_i(\bar{D} - d_o) - D_i(N_i) \} \quad (4.1)$$

where

$D_i(N_i)$ additional sales due to N_i insertions in the i th media

$D_i(N_i + 1)$ additional sales due to $N_i + 1$ insertions in the i th media

r_i proportionality constant, which may be defined as the rate at which sales approaches saturation level through the i th media.

Solving the difference equation (4.1) with the initial condition $D_i(0) = 0$, we get

$$D_i(N_i) = p_i(\bar{D} - d_o) \{ 1 - (1 - r_i)^{N_i} \} \quad (4.2)$$

The additional demand through n media is

$$D = \sum_{i=1}^n D_i(N_i) = \sum_{i=1}^n p_i(\bar{D} - d_o) [1 - (1 - r_i)^{N_i}] \quad (4.3)$$

The total demand (D_T) using $\sum p_i = 1$ is

$$D_T = d_o + \sum_{i=1}^n p_i(\bar{D} - d_o) [1 - (1 - r_i)^{N_i}] \quad (4.4)$$

$$\text{i.e., } D_T = \bar{D} - (\bar{D} - d_o) \sum_{i=1}^n p_i (1 - r_i)^{N_i}$$

Now, using the same approach as used in the previous section, the problem of jointly optimising the number of insertions in each media and optimal order cycle, which maximises the net profit over a finite planning horizon in deteriorating production-inventory system under inflationary conditions, can be formulated as a mathematical programming problem:

$$\begin{aligned}
 \text{Max } Z_H(T, N) = & \left[\begin{aligned} & P_0 \frac{D_T(1-e^{-KT})}{K} - A_0 - QC_0 - IC_0 \\ & \left(\frac{R-D_T}{K\theta(K+\theta)} \left(\theta + Ke^{-(K+\theta)t_1} - (K+\theta)e^{-Kt_1} \right) \right) \\ & - IC_0 \left(\frac{D_T}{\theta} \left(\frac{e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{-KT} - e^{-Kt_1}}{K} \right) \right) \\ & - \sum_{i=1}^n Cai_0 N_i \end{aligned} \right] \quad (4.5) \\
 & * \frac{1}{(1-e^{-KT})}
 \end{aligned}$$

where

$$D_T = \bar{D} - (\bar{D} - d_o) \sum_{i=1}^n p_i (1-r_i)^{N_i}$$

Subject to

$$N_i \geq 0$$

$$T \geq 0$$

The above problems can be solved using any optimisation software e.g., What's Best, Lingo, Solver.

4.2 Numerical example and sensitivity analysis

Let us consider a hypothetical example for insertion in four types of media ($n = 4$):

$$\begin{aligned}
 & \bar{D} = 110,000 \text{ units, } d_o = 17,000 \text{ units, } s = 0.12, \alpha = 0.10, K = s - \alpha = 0.02, \\
 & \theta = 0.010, A_0 = \text{Rs. } 1,000 \text{ per order, } C_0 = \text{Rs. } 500/- \text{ per unit,} \\
 & P_0 = \text{Rs. } 700/- \text{ per unit, } I = 0.20, Ca1_0 = \text{Rs. } 10,000/-, Ca2_0 = \text{Rs. } 9,000/-, \\
 & Ca3_0 = \text{Rs. } 8,500/-, Ca4_0 = \text{Rs. } 105,000/-, p_1 = 0.25, p_2 = 0.20, p_3 = 0.20, \\
 & p_4 = 0.35, r_1 = 0.24, r_2 = 0.277, r_3 = 0.21, r_4 = 0.29 \text{ and } R = 150,000
 \end{aligned}$$

Using these parametric values, the optimal solution is obtained as $N_1^* = 15$, $N_2^* = 13$, $N_3^* = 16$, $N_4^* = 13$, $T^* = 160.72$ days, $Q^* = 48,080$ units approx. and $Z^*(T, N^*) = \text{Rs. } 990,182,570$.

Further, the effects of changing the parameters θ and α i.e., deterioration rate and inflation rate on the optimal replenishment policy are studied by changing their respective values. The results are summarised in Tables 4.1 and 4.2.

Table 4.1 Effects of changing the deterioration rate θ on the optimal replenishment policy

θ	$N1$	$N2$	$N3$	$N4$	T	Q
0.10	16	14	18	14	216.9	65,289
0.20	15	13	17	14	177.3	53,516
0.30	15	13	16	13	153.0	46,271
<i>Profit</i>	<i>Demand</i>		<i>D - do</i>		<i>(D - do)/N</i>	
990,182,570	108,977		91,977		1,484	
970,969,640	108,739		91,739		1,555	
954,805,116	108,539		91,539		1,606	

Table 4.2 Effects of changing the inflation rate α on the optimal replenishment policy

α	$N1$	$N2$	$N3$	$N4$	T	Q
0.04	14	12	16	13	144.1	42,992
0.06	15	13	16	13	160.7	48,082
0.08	15	13	17	14	182.9	54,880
0.10	16	14	18	14	216.9	65,289
<i>Profit</i>	<i>Demand</i>		<i>D - do</i>		<i>(D - do)/N</i>	
237,522,116	108,315		91,315		1,660	
320,592,560	108,539		91,539		1,606	
487,483,376	108,739		91,739		1,555	
990,182,570	108,977		91,977		1,484	

The following inferences are made from Tables 4.1 and 4.2:

- It is observed from Table 4.1 that as the deterioration rate (θ) increases, there is considerable decrease in cycle length (which suggests less number of insertions in different medias for deteriorating items) and profit, whereas capturing rate of unsaturated sales per insertion increases significantly.
- Moreover, as inflation rate (α) increases, there is significant increase in the cycle length (which suggests more number of insertions in different medias under inflationary conditions) and profit. But capturing rate of unsaturated sales per insertion decreases.

5 Conclusions

In this paper, inventory models has been developed for determining the optimal inventory and marketing policy for production inventory system of deteriorating items under inflationary conditions with demand as a function of insertions in media, under the assumption that the 'the marginal effect of advertisement on sales is proportional to the unrealised potential of the market'. An algorithm is also provided in this regard. Further, the authors extend the model for multiple media and develop a general form of demand function by incorporating the number of insertions in different medias, as in real practice, it is observed that companies explore more than one media for better effectiveness. Result

shows that as deterioration rate increases, cycle length squeezes and unsaturated sales decrease, but capturing rate of unsaturated sales per insertion increases. Whereas, when inflation rate increases, cycle length and unsaturated sales increases, but capturing rate of unsaturated sales per insertion decreases. The proposed model can assist managers in making important inventory and marketing decisions of deteriorating items under inflationary conditions.

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Appendix

The concavity

Theorem 1: $Z_H(T, N)$ is concave with respect to T

The present value of total profit $Z_H(T, N)$ over the infinite planning horizon is given by:

$$Z(T, N) = \left[P_0 \frac{D(1-e^{-KT})}{K} - A_0 - QC_0 - IC_0 \right. \\ \left. \left(\frac{R-D}{K\theta(K+\theta)} \left(\theta + Ke^{-(K+\theta)t_1} - (K+\theta)e^{-Kt_1} \right) \right) \right. \\ \left. - IC_0 \left(\frac{D}{\theta} \left(\frac{e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{-KT} - e^{-Kt_1}}{K} \right) \right) - Ca_0 N \right] * \frac{1}{(1-e^{-KT})}$$

where $D = \bar{D} - (\bar{D} - d_o)(1-r)^N$ and $K = s - a$.

Now

$$\frac{dZ(T, N)}{dT} = \left[\begin{array}{l} P_0 \frac{DKe^{-KT}}{K} - C_0RB - IC_0 \\ \left(\frac{R-D}{K\theta(K+\theta)} \left(-K(K+\theta)e^{-(K+\theta)t_1}B + K(K+\theta)e^{-Kt_1}B \right) \right) \\ -IC_0 \left(\frac{D}{\theta} + \frac{\theta e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{\theta T} (-(K+\theta)e^{-(K+\theta)t_1}B + (K+\theta)e^{-(K+\theta)T})}{K+\theta} \right. \\ \left. + \frac{(-Ke^{-KT} + Ke^{-Kt_1}B)}{K} \right) \end{array} \right] * E$$

$$+ \left[\begin{array}{l} P_0 \frac{D(1-e^{-KT})}{K} - A_0 - QC_0 - IC_0 \\ \left(\frac{R-D}{K\theta(K+\theta)} \left(\theta + Ke^{-(K+\theta)t_1} - (K+\theta)e^{-Kt_1} \right) \right) \\ -IC_0 \left(\frac{D}{\theta} \left(\frac{e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{-KT} - e^{-Kt_1}}{K} \right) \right) - Ca_0N \end{array} \right] * F$$

where

$$E = \frac{1}{(1-e^{-KT})}$$

$$F = \frac{dE}{dT} = \frac{-Ke^{-KT}}{(1-e^{-KT})^2}$$

$$t_1 = \frac{1}{\theta} \log \left(1 + \frac{D(e^{\theta T} - 1)}{R} \right)$$

$$B = \frac{dt_1}{dT} = \frac{DT}{R \left(1 + \frac{D(e^{\theta T} - 1)}{R} \right)}$$

And

$$\begin{aligned}
\frac{d^2 Z(T, N)}{dT^2} = & \left[\begin{aligned} & P_0 \frac{DK e^{-KT}}{K} - C_0 RB - IC_0 \\ & \left(\frac{R-D}{K\theta(K+\theta)} \left(-K(K+\theta)e^{-(K+\theta)t_1} B + K(K+\theta)e^{-Kt_1} B \right) \right) \\ & -IC_0 \left(\frac{D}{\theta} + \frac{\theta e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} \right. \\ & \quad \left. + \frac{e^{\theta T} (-(K+\theta)e^{-(K+\theta)t_1} B + (K+\theta)e^{-(K+\theta)T})}{K+\theta} \right. \\ & \quad \left. + \frac{(-Ke^{-KT} + Ke^{-Kt_1} B)}{K} \right) \end{aligned} \right] * F \\
+ & \left[\begin{aligned} & -P_0 \frac{DK^2 e^{-KT}}{K} - C_0 RG - IC_0 \\ & \left(\frac{R-D}{K\theta(K+\theta)} \left(K(K+\theta)^2 e^{-(K+\theta)t_1} B^2 - K(K+\theta)e^{-(K+\theta)t_1} G \right) \right. \\ & \quad \left. - K^2(K+\theta)e^{-Kt_1} B^2 + K(K+\theta)e^{-Kt_1} G \right) \\ & -IC_0 \left(\frac{D}{\theta} + \frac{\theta^2 e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} \right. \\ & \quad + \frac{\theta e^{\theta T} (-(K+\theta)e^{-(K+\theta)t_1} B + (K+\theta)e^{-(K+\theta)T})}{K+\theta} \\ & \quad + \frac{\theta e^{\theta T} (-(K+\theta)e^{-(K+\theta)t_1} B + (K+\theta)e^{-(K+\theta)T})}{K+\theta} \\ & \quad + \frac{e^{\theta T} ((K+\theta)^2 e^{-(K+\theta)t_1} B^2 - (K+\theta)e^{-(K+\theta)t_1} G - (K+\theta)^2 e^{-(K+\theta)T})}{K+\theta} \\ & \quad \left. + \frac{(K^2 e^{-KT} - K^2 e^{-Kt_1} B^2 + Ke^{-Kt_1} G)}{K} \right) \end{aligned} \right] * E \\
+ & \left[\begin{aligned} & P_0 \frac{D(1-e^{-KT})}{K} - A_0 - QC_0 - IC_0 \\ & \left(\frac{R-D}{K\theta(K+\theta)} \left(\theta + Ke^{-(K+\theta)t_1} - (K+\theta)e^{-Kt_1} \right) \right) \\ & -IC_0 \left(\frac{D}{\theta} \left(\frac{e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} + \frac{e^{-KT} - e^{-Kt_1}}{K} \right) \right) - Ca_0 N \end{aligned} \right] * F' \\
+ & \left[\begin{aligned} & P_0 \frac{DK e^{-KT}}{K} - C_0 RB - IC_0 \left(\frac{R-D}{K\theta(K+\theta)} \left(-K(K+\theta)e^{-(K+\theta)t_1} B + K(K+\theta)e^{-Kt_1} B \right) \right) \\ & -IC_0 \left(\frac{D}{\theta} \left(\frac{\theta e^{\theta T} (e^{-(K+\theta)t_1} - e^{-(K+\theta)T})}{K+\theta} \right. \right. \\ & \quad \left. + \frac{e^{\theta T} (-(K+\theta)e^{-(K+\theta)t_1} B + (K+\theta)e^{-(K+\theta)T})}{K+\theta} + \frac{(-Ke^{-KT} + Ke^{-Kt_1} B)}{K} \right) \end{aligned} \right] * F
\end{aligned}$$

where

$$t_1 = \frac{1}{\theta} \log \left(1 + \frac{D(e^{\theta T} - 1)}{R} \right)$$

$$B = \frac{dt_1}{dT} = \frac{DT}{R \left(1 + \frac{D(e^{\theta T} - 1)}{R} \right)},$$

$$G = \frac{d^2 t_1}{dT^2} = \frac{D(R + D(e^{\theta T} - 1)) - D^2 T \theta e^{\theta T}}{(R + D(e^{\theta T} - 1))^2},$$

and

$$F' = \frac{dF}{dT} = \frac{(1 - e^{-KT})^2 (K^2 e^{-KT}) - (-K e^{-KT}) 2(1 - e^{-KT})(K e^{-KT})}{(1 - e^{-KT})^4}$$

Since it is difficult to prove mathematically that $d^2 Z(T, N)/dT^2 < 0$, therefore, the concavity of total profit function has been established graphically with the help of different dataset. Moreover, it has also been verified numerically that $d^2 Z(T, N)/dT^2 < 0$ for different dataset.

Figure 1 The concavity of total profit function (see online version for colours)

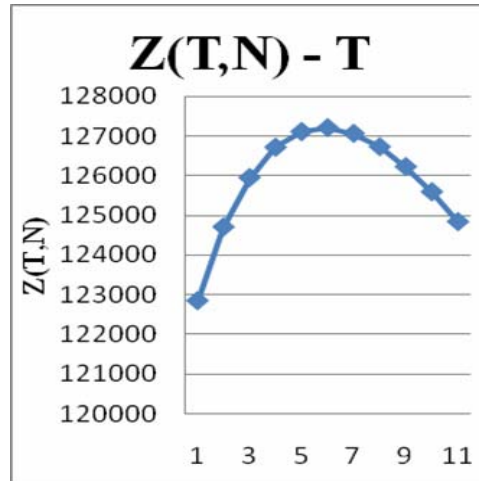
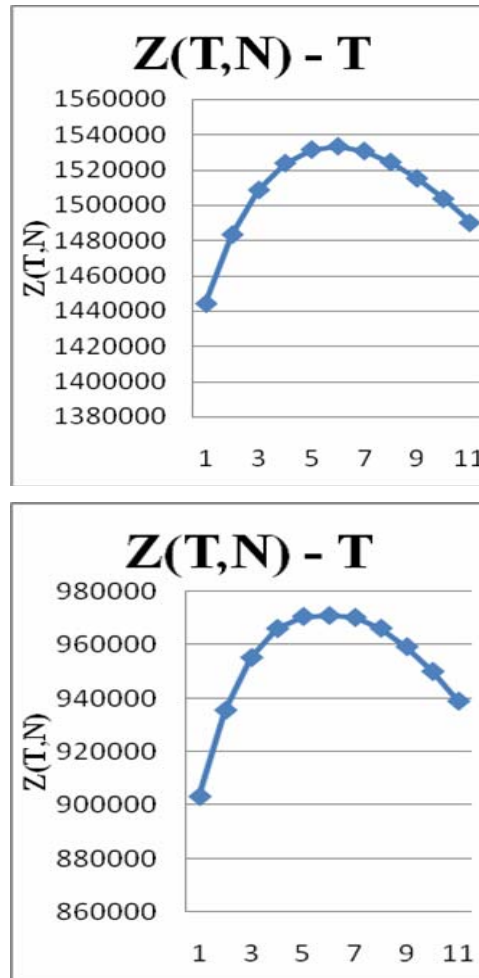


Figure 1 The concavity of total profit function (continued) (see online version for colours)



Thus, from the figures, it is clear that $Z(T, N)$ is concave with respect to T . This completes the proof.