

# The safety stock and inventory cost paradox in a stochastic lead time setting

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## Abstract

We study a stochastic lead-time problem motivated by real world global shipping data. Replenishment quantities are generated by the Order-Up-To policy which aims to achieve a strategic availability target. We show that unlike the constant lead-time case, minimum safety stocks do not always lead to minimum costs under stochastic lead-times.

**Keywords:** order up to policy, stochastic lead time, order crossover

## Introduction

Global sourcing often allows access to low-cost supply but is often associated with long and variable lead times (Blackburn 2012). These longer and more variable lead times bring with them a number of complications and potential pitfalls, from both cost and service perspectives (Stalk 2006). We add to the literature on planning with stochastic lead times by formulating and testing a calculation of safety stock that reflects these real-world complications. Our method allows for order crossover and correlation between pipeline inventory and replenishment orders, factors that are often ignored. Using a linear modification of the familiar Order-Up-To (OUT) ordering policy we find a solution that always results in lower inventory holding and backlog costs.

Practically, we have tracked and analyzed logistics data for global supply chains for both major forwarders and retailers and were struck by the violations of the lead time normality assumption – see Figure 1. When lead times is highly variable we may also have issues with order crossover, where shipments are received in a different sequence from which they were dispatched. Most inventory models do not allow for this order crossover, yet variable shipment delays, clerical errors, and random custom inspections can easily delay a shipment long enough for others to pass it.

From the analytical perspective, two prescriptions for inventory management are widely disseminated. These approaches either use an average (or maximum) lead time in the constant lead time reorder point solution or assume that the demand is normally distributed and then use the mean and variance of a random sum of random variables to determine the reorder point (Feller, 1966). We show that neither approach is well-suited to global supply chains with long transit times and multiple hand-offs due to the multi-modal inventory distribution.

This paper develops an exact theoretical treatment of the impact of the stochastic lead times with order crossover on the probability density function of the net stock levels. As we progressed in our investigations, we also began questioning the well-known assertion (Kaplan 1970) that the OUT model is always a good fit for global supply chains. We find that, when there is order crossover, better economic performance is possible when the ordering strategy

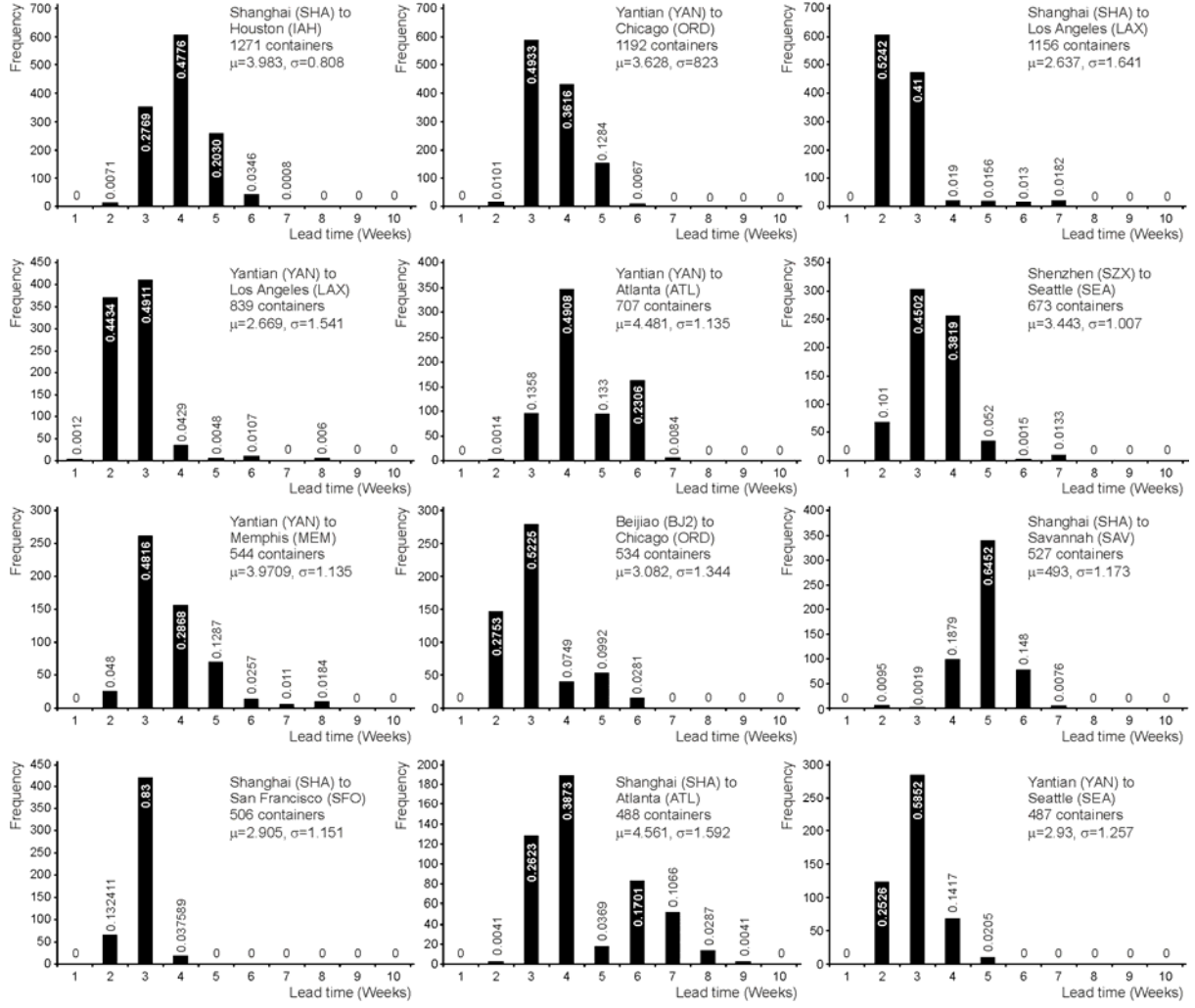


Figure 1 – Empirical port-to-port lead time distributions from Chain to the USA.

follows the linear proportional order-up-to (POUT) policy (Disney and Lambrecht, 2008).

The structure of our paper is as follows. We first define the POUT policy and then show how to capture the effect of the stochastic lead times analytically. Finally, in order to validate our theoretical results, we numerically analyze a stochastic lead time problem with linear inventory holding and backlog costs.

## The proportional order-up-to policy

We first briefly review the POUT policy before moving on to the case of stochastic lead times with order crossover. We assume throughout that a linear system exists and that demand,  $D_t$ , at time  $t$ , is an independently and identically distributed (i.i.d.) random variable drawn from a normal distribution with a mean of  $\mu$  and a standard deviation of  $\sigma$ .

The POUT policy,  $O_t$ , generates orders at time  $t$ , with the following difference equation (Disney and Towill 2003),

$$O_t = \mu + \beta(T + \mu\bar{k} - (I_t + W_t)). \quad (1)$$

Here, the variable  $T$ , is a safety stock – the mean inventory;  $\bar{k}$  is the average lead-time;  $I_t$  is the on-hand inventory at time  $t$ ; and  $W_t$  is the on-order inventory or work in progress, WIP. The variable  $0 \leq \beta < 2$  is a proportional feedback controller that regulates the speed at which deviations in the inventory position are recovered. When  $\beta = 1$ , the POUT policy degenerates into the OUT policy. The inventory balance equation defines the inventory

$$I_t = I_{t-1} + R_t - D_t, \quad (2)$$

where  $R_t = O_{t-k-1}$  is the receipts received, the incoming orders after the stochastic lead time  $k$  and the sequence of event delay (the  $-1$ ). The system is completed when we define the WIP,

$$W_t = \sum_{i=1}^k O_{t-i}. \quad (3)$$

### Modelling stochastic lead times with order crossover

As the OUT/POUT policy operates on discrete time the stochastic lead times must also be discretely distributed, which is why we presented the data in Figure 1 as a discrete distribution. Let  $p_k$  be the probability that the lead time of a particular order is  $k$  periods long,  $k$  is an integer greater than or equal to zero. The maximum lead time is  $k^+$ , the smallest lead time is  $k = 0$ , and the average lead time is  $\bar{k} = \sum_{k=0}^{k^+} kp_k$ .

Let  $\mathbf{M}$  be a binary matrix with  $j = 1$  to  $k^+$  columns and  $i = 1$  to  $2^{k^+}$  rows. Assign the  $(i, j)^{th}$  element of  $\mathbf{M}$  a value according to

$$m_{i,j} = \left[ 1 + (-1)^\nu \right] / 2, \quad (4)$$

where  $\nu = \left\lceil 2^{j-k^+} i \right\rceil$ . Here  $\lceil x \rceil$  is the ceiling function. Each row of the  $\mathbf{M}$  matrix represents a  $k^+$ -tuple of binary digits that describes the state of the WIP pipeline. A zero in element  $m_{i,j}$  of matrix  $\mathbf{M}$  indicates that for state  $i$ , the order placed  $j-1$  periods ago has been received (the order is closed), unity indicates that the order placed  $j-1$  periods ago has not yet been received (it is open). Note, the order placed  $k^+$  periods ago is always closed, thus  $j$  indexes from 1 to  $k^+$  to denote the lead times  $k = 0$  to  $k^+ - 1$ . There are  $2^{k^+}$  rows to  $\mathbf{M}$ , one for each possible state of the order pipeline. The probability that the WIP pipeline is in state  $i$  is given by

$$q_i = \frac{1}{2^{k^+}} \prod_{j=1}^{k^+} \left( 1 + (-1)^\nu \left( 2 \sum_{k=j}^{k^+} p_k - 1 \right) \right), \quad (5)$$

One can derive (5) by observing that in the  $i$ th pipeline state, the probability that an order placed  $j$  periods ago is open/closed can be expressed universally as

$$q_{i,j} = m_{i,j} \sum_{k=j}^{k^+} p_k + (1 - m_{i,j}) \sum_{k=1}^{j-1} p_k = \frac{1}{2} \left[ 1 + (-1)^v \left( 2 \sum_{k=j}^{k^+} p_k - 1 \right) \right], \quad (6)$$

and that  $q_i$  is the product of  $q_{i,j}$  over  $j$ .

We now require the variance of the inventory levels in each of the sub-processes. We obtain this by first determining the variance of the WIP in each sub-process and then each sub-process is combined with a scaled order to obtain something we call the *scaled shortfall distribution* sub-process, from which we can obtain the inventory distribution.

We can rearrange (1) to obtain

$$I_t = T + \mu(\bar{k} + 1/\beta) - (W_t + O_t/\beta). \quad (7)$$

For OUT policy (that is, when  $\beta = 1$ ) we can see that the inventory distribution is a reflected *shortfall distribution*,  $(W_t + O_t)$ , translated by  $T + \mu(\bar{k} + 1/\beta)$  (Zalkind, 1978; Robinson et al., 2001). When  $\beta \neq 1$  the  $O_t$  component has become scaled by  $O_t/\beta$ , in which case we call the distribution of  $(W_t + O_t/\beta)$  the *scaled shortfall distribution*. We now require the mean and the variance of the scaled shortfall distribution of each sub-process. The complicating factors are that  $O_t$  is auto-correlated and that the distributions of  $W_t$  and  $O_t/\beta$  are correlated with each other. As the system is linear the simplest way to proceed is to exploit the z-transform, which is defined by

$$F(z) = Z\{f[t]\} = \sum_{t=0}^{\infty} f[t] z^{-t}. \quad (8)$$

To determine the variance of the WIP in sub-process,  $i$ , we first note that the variance of the orders maintained by the POUT policy is independent of the lead-time, as

$$\frac{\sigma_o^2}{\sigma^2} = \sum_{t=0}^{\infty} \left( Z^{-1} \left\{ \frac{z\beta}{z + \beta - 1} \right\} \right)^2 = \sum_{t=0}^{\infty} \left( (1 - \beta)^t \beta \right)^2 = \frac{\beta}{2 - \beta}. \quad (9)$$

Here,  $z$  is the z-transform operator,  $Z^{-1}\{F(z)\} = \frac{1}{2\pi j} \oint_c F(z) z^{t-1} dz = f[t]$  is the inverse z-

transform of transfer function,  $F(z)$ .  $z\beta(z + \beta - 1)^{-1}$  is the transfer function of the orders maintained by the POUT policy under i.i.d. demand and minimum mean squared error forecasting (Disney and Towill 2003). The relationship between the variance ratio and the sum of the squared impulse response is known as Tsytkin's (1964) relationship.

The probability density function of the normal distribution with an argument of  $x$ , a mean of  $\mu$ , and a standard deviation of  $\sigma$ , is defined by

$$\phi[x | \mu, \sigma] = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\sigma}. \quad (10)$$

Using this notation, (9) leads to an order process described by the pdf,

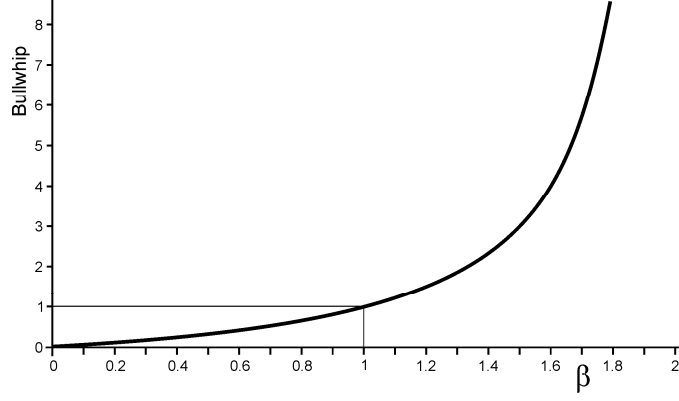


Figure 2. The bullwhip ratio for different  $\beta$

$$\phi_o = \phi \left[ x | \mu, \sqrt{\sigma^2 \beta / (2 - \beta)} \right]. \quad (11)$$

The bullwhip ratio,  $\sigma_o^2 / \sigma^2$ , is plotted in Figure 2. The ratio is unity at  $\beta = 1$ , zero at  $\beta = 0$ ,  $\infty$  at  $\beta = 2$ , strictly increasing, and convex in  $\beta$ . Note that the bullwhip ratio and order variance are not affected by the stochastic lead time.

The variance of WIP sub-process,  $i$ , is given by the variance of the sum of the impulse responses of the open orders,

$$\frac{\sigma_{W,i}^2}{\sigma^2} = \sum_{t=0}^{\infty} \sum_{j=1}^{k^+} \left( m_{i,j} Z^{-1} \left\{ \frac{\beta z^{1+j}}{z + \beta - 1} \right\} \right)^2. \quad (12)$$

where  $m_{i,j}$  is an element of the binary matrix  $\mathbf{M}$  that captures whether an order is open or closed. The distribution of the scaled orders,  $O_t / \beta$ , for all sub-processes, can be obtained using

$$\frac{\sigma_{O/\beta}^2}{\sigma^2} = \sum_{t=0}^{\infty} \left( Z^{-1} \left\{ z / (z + \beta - 1) \right\} \right)^2 = \sum_{t=0}^{\infty} \left( (1 - \beta)^t \right)^2 = (2\beta - \beta^2)^{-1} = \frac{\sigma_o^2}{\sigma^2 \beta^2}, \quad (13)$$

which leads to the following expressions for its pdf,

$$\phi_{O/\beta} = \phi \left[ x | \mu / \beta, \sqrt{\sigma^2 / (2\beta - \beta^2)} \right]. \quad (14)$$

The covariance between the WIP sub-process and the scaled orders sub-process is then

$$\text{cov}(W_i, O/\beta) = \sum_{t=0}^{\infty} \left( Z^{-1} \left\{ \sum_{j=1}^{k^+} m_{i,j} \frac{\beta z^{1-j}}{z + \beta - 1} \right\} Z^{-1} \left\{ \frac{z}{z + \beta - 1} \right\} \right) = \text{cov}(W_i, O) / \beta. \quad (15)$$

$\sigma_{NS,i}^2$ , the variance of sub-process  $i$  in the inventory distribution, is equal to the variance of the shortfall distribution, which is given by

$$\sigma_{NS,i}^2 = \sigma_{S,i}^2 = \sigma_{W,i}^2 + \sigma_o^2 / \beta^2 + 2 \text{cov}(W_i, O) / \beta. \quad (16)$$

The mean of the each of the sub-processes of the inventory distribution can be shown to be

$$\mu_{NS,i} = T + \mu \left( \bar{k} - \sum_{j=1}^{k^+} m_{i,j} \right). \quad (17)$$

The complete pdf inventory distribution is given by

$$\phi_{NS} = \sum_{i=1}^{2^{k^+}} \phi \left[ x \mid \mu_{NS,i}, \sqrt{\sigma^2 \sigma_{NS,i}^2} \right]. \quad (18)$$

The variance of the complete, multi-modal, inventory is given by

$$\sigma_{NS}^2 = \int_{-\infty}^{\infty} \phi_{NS} (T - x)^2 dx = \sum_{i=1}^{2^{k^+}} q_i \left( \left( \sum_{j=1}^{k^+} m_{i,j} \right)^2 \mu^2 + \sigma^2 \sigma_{NS,i}^2 \right), \quad (19)$$

Equation (19) shows that the mean demand has an influence on the variance of the inventory levels, something that does not happen with constant lead times. Furthermore, we can see that the inventory variance also contains a weighted sum of individual sub-process's variances.

## A numerical example when $k^+ = 4$

Consider the situation when  $k^+ = 4$ . Table 1 details the pipeline states ( $\mathbf{M}$ ), the variance of the net stock, and the mean of each of the  $2^{k^+} = 16$  individual sub-processes to the inventory distribution. It can be easily shown that each of the expressions for the variance (and the standard deviations) of the inventory sub-processes is infinite at  $\beta = \{0, 2\}$ . Furthermore, each sub-process has a single unique minimum,  $\beta_i^*$ , which is also detailed in Table 1. We can see that  $\beta_i^* = 1$  exists only in the sub-processes that do not contain order cross-overs. All of the sub-processes that contain order-crossover have  $\beta_i^* < 1$ . This leads us to speculate that in cases where order crossover exists that the POUT will produce inventory pdfs with less variance than the OUT policy and that the optimal  $\beta$  lies in the region,  $0 < \beta^* < 1$ .

To make the results in Table 1 specific, we first need to specify the lead time probabilities – assume  $\{p_0 = \frac{1}{2}, p_1 = p_2 = p_3 = 0, p_4 = \frac{1}{2}\}$ . The maximum lead time is  $k^+ = 4$  and the average lead time is  $\bar{k} = 2$ . Using (5) we are then able to determine the probability that the pipeline is in state  $i$ , is  $\forall i, q_i = 0.0625$ . Note that in general, the probability that the pipeline is in a particular state need not be, and almost never is, the same as the probability that the pipeline is another state.

Consider now that the following inventory cost function  $J$ , exists

$$J = E \left[ h(ns_t)^+ + b(-ns_t)^+ \right], \quad (20)$$

where  $h$  is the unit inventory holding cost, and  $b$  is the unit backlog cost. It is possible to show that

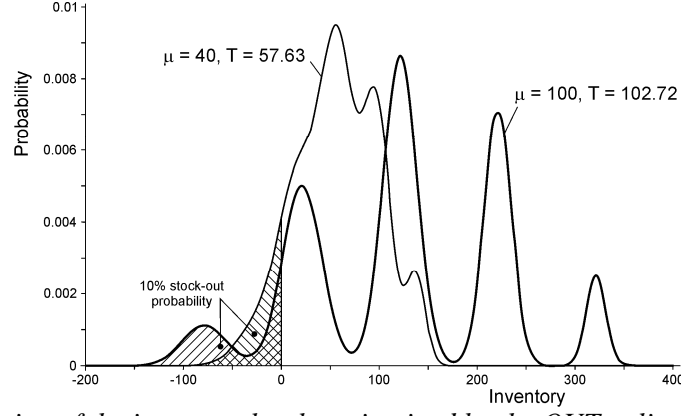


Figure 3 – Distribution of the inventory levels maintained by the OUT policy for 90% availability when  $\{p_0 = \frac{1}{2}, p_1 = p_2 = p_3 = 0, p_4 = \frac{1}{2}, \beta = 1, \sigma = 10\}$

Table 1. Generic characteristics of the sub-processes when  $k^+ = 4$

|     | <b>M</b> |   |   |   | $\frac{\sigma_{NS,i}^2}{\sigma^2}$  | $\mu_{NS,i}$           | $\beta_i^*$ |
|-----|----------|---|---|---|---|------------------------|-------------|
|     | $j$      |   |   |   |   |                        |             |
| $i$ | 1        | 2 | 3 | 4 |   |                        |             |
| 1   | 0        | 0 | 0 | 0 | $(2\beta - \beta^2)^{-1}$   | $T + \mu\bar{k}$       | 1           |
| 2   | 0        | 0 | 0 | 1 | $\frac{1 + \beta(2 - \beta(7 - 2\beta(6 + \beta(\beta - 4))))}{\beta(\beta - 2)}$ | $T + \mu(\bar{k} - 1)$ | 0.656633    |
| 3   | 0        | 0 | 1 | 0 | $(\beta(\beta - 2))^{-1} - (1 + 2\beta(\beta - 1))$                               | $T + \mu(\bar{k} - 1)$ | 0.689845    |
| 4   | 0        | 0 | 1 | 1 | $2\beta - 2(\beta - 1)^3 + (\beta(2 - \beta))^{-1}$                               | $T + \mu(\bar{k} - 2)$ | 0.60974     |
| 5   | 0        | 1 | 0 | 0 | $(2\beta)^{-1} - 2\beta - 1 - 9(2(\beta - 2))^{-1}$                               | $T + \mu(\bar{k} - 1)$ | 0.751274    |
| 6   | 0        | 1 | 0 | 1 | $\frac{1 + 2\beta(2 + \beta(\beta - 2))(1 + \beta(\beta - 1))}{\beta(2 - \beta)}$ | $T + \mu(\bar{k} - 2)$ | 0.676129    |
| 7   | 0        | 1 | 1 | 0 | $2 + 2(\beta - 1)\beta + (\beta(2 - \beta))^{-1}$                                 | $T + \mu(\bar{k} - 2)$ | 0.689845    |
| 8   | 0        | 1 | 1 | 1 | $\frac{1 + \beta(6 - \beta(9 - 2\beta(6 + \beta(\beta - 4))))}{\beta(\beta - 2)}$ | $T + \mu(\bar{k} - 3)$ | 0.656633    |
| 9   | 1        | 0 | 0 | 0 | $(1 - \beta(\beta - 2))(\beta(\beta - 2))^{-1}$                                   | $T + \mu(\bar{k} - 1)$ | 1           |
| 10  | 1        | 0 | 0 | 1 | $2(1 + \beta(\beta - 1)) + (\beta(2 - \beta))^{-1}$                               | $T + \mu(\bar{k} - 2)$ | 0.689845    |
| 11  | 1        | 0 | 1 | 0 | $(2\beta)^{-1} - 9(2(\beta - 2))^{-1} - 2\beta$                                   | $T + \mu(\bar{k} - 2)$ | 0.751274    |
| 12  | 1        | 0 | 1 | 1 | $(\beta(\beta - 2))^{-1} - (3 + 2\beta(\beta - 1))$                               | $T + \mu(\bar{k} - 3)$ | 0.689845    |
| 13  | 1        | 1 | 0 | 0 | $(1 - 2\beta(2 - \beta))(\beta(\beta - 2))^{-1}$                                  | $T + \mu(\bar{k} - 2)$ | 1           |
| 14  | 1        | 1 | 0 | 1 | $1 - 2\beta + 9(2(2 - \beta))^{-1} + (2\beta)^{-1}$                               | $T + \mu(\bar{k} - 3)$ | 0.751274    |
| 15  | 1        | 1 | 1 | 0 | $(1 - 3\beta(\beta - 2))(\beta(2 - \beta))^{-1}$                                  | $T + \mu(\bar{k} - 3)$ | 1           |
| 16  | 1        | 1 | 1 | 1 | $(1 - 4\beta(\beta - 2))(\beta(2 - \beta))^{-1}$                                  | $T + \mu(\bar{k} - 4)$ | 1           |

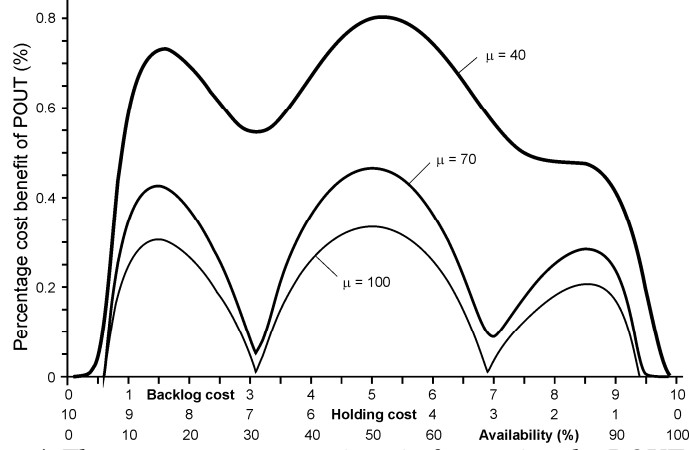


Figure 4. The percentage economic gain from using the POUT policy

$$\left. \frac{dJ}{d\beta} \right|_{\beta=1} = \frac{\sqrt{\sigma^2} (b+h) e^{-(T+\mu)^2/4\sigma^2} \left( 6 + 3\sqrt{2} e^{(T^2+6T\mu+\mu^2)/8\sigma^2} + 4\sqrt{6} e^{(T^2+6T\mu+3\mu^2)/12\sigma^2} \right)}{64\sqrt{\pi}} > 0, \quad (21)$$

implying that for all cost combinations the OUT policy is never optimal as there always exists a  $\beta < 1$  which is more economical.

Together with the variances of the individual sub-processes detailed in Table 1, (4) and (18) we are now able to obtain an expression of the pdf of the inventory levels, which we plot in Figure 3. Figure 3 plots two cases of the OUT policy, and in both we have set the safety stock,  $T$ , to minimize  $J$  when  $h = 1$ ,  $b = 9$ . In the first case  $\mu = 100$ , and we can clearly see that there are five modes in the inventory pdf. In the other case,  $\mu = 40$  and the five modes overlap somewhat. Furthermore, the complete pdf of the  $\mu = 40$  case, has less variance, and requires less safety stock, than the  $\mu = 100$  case.

When  $\mu = 100$ , the inventory levels have a variance of 10,300 for the OUT policy. Numerical experiments show us that there is a single minimum inventory variance (or standard deviation) at  $\beta_\sigma^* = 0.73$  and the net stock variance is 10,280—0.2% less than the OUT variance.

Using numerical techniques we can find the optimal proportional feedback controller  $\beta^*$ , and safety stock  $T$ , that minimizes the inventory cost. When we have set  $\{\beta^*, T\}$  optimally, Figure 4 describes the percentage economic gain  $((J_{OUT} - J_{POUT}) J_{OUT}^{-1} \times 100\%)$ , from using the POUT policy. While the improvement is rather small (note that 0.8 means 0.8% not 80%), the POUT is always more economical than the OUT policy. The economic benefit increases as  $\mu$  decreases. When the cost ratio is such that near 100% availability is desirable,  $\beta^* \rightarrow 1$ .

Figure 5 plots  $\beta^*$  for different cost ratios and different mean demands. We see that  $\beta^*$  is near unity when the availability target is (very) near 0% or 100%, but for most availability targets  $\beta^* \approx 0.725$ . Interestingly, almost always,  $\beta^* \neq \beta_\sigma^*$  implying that the tightest inventory control does not always lead to the minimal cost.

Figure 6 shows the safety stock requirements when  $\beta^*$  is used for different cost ratios. The multi-model nature of the  $\mu = 100$  case results in rapid increases in the safety stock requirement at predictable points on the availability scales. These are related to the multi-model pdf of the inventory levels. Furthermore, between 40-60% availability, the two demand settings require very similar amounts of safety stock. As it is not possible to visually distinguish



between the optimal POUT safety stock,  $T_{POUT}$ , and the optimal safety stock for the OUT policy,  $T_{OUT}$ , we have plotted the difference,  $T_{POUT} - T_{OUT}$ , in Figure 7. Here we can see that, although the POUT policy is economically superior, minimum safety stock do not always coincide with least costs. This is an insight that is contrary to the constant lead time case where the least cost solution always has the smallest safety stocks.

Figure 8 highlights the bullwhip ratio achieved when  $\beta^*$  is used in the POUT policy. We can see that a 40% reduction in bullwhip is possible between 8% to 92% availability. This is interesting as the objective function consists only of inventory related costs. If costs are associated with bullwhip are also present, these will also be reduced. This is an important result as bullwhip costs are somewhat harder to quantify, even though bullwhip is widely recognized as having a negative effect on supply chain performance (Lee et al. 2000).

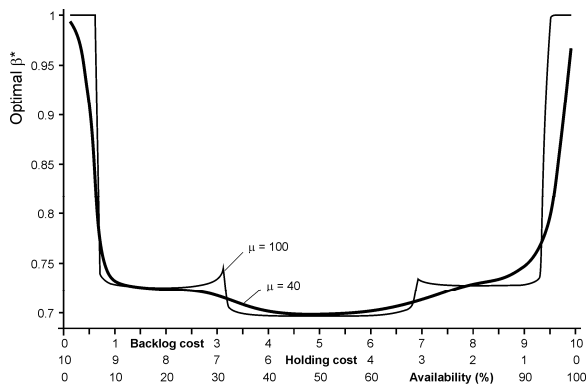


Figure 5. The inventory cost optimal  $\beta^*$  in the POUT policy

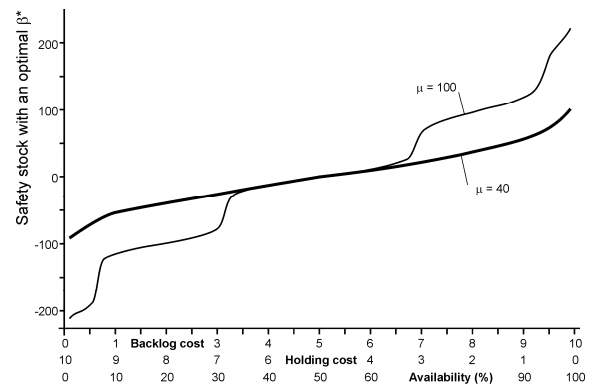


Figure 6. Safety stock with the inventory cost optimal POUT policy

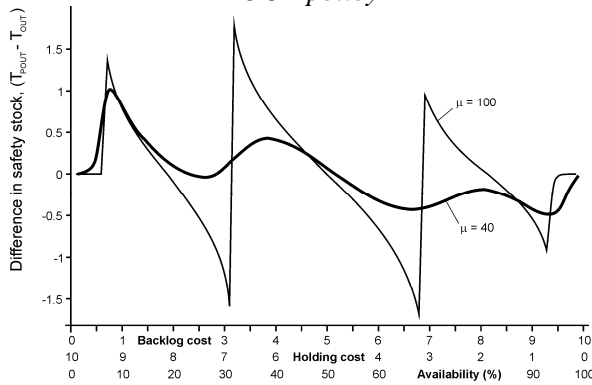


Figure 7. Safety stock difference between the OUT policy and the inventory cost optimal POUT policy ( $T_{POUT} - T_{OUT}$ )

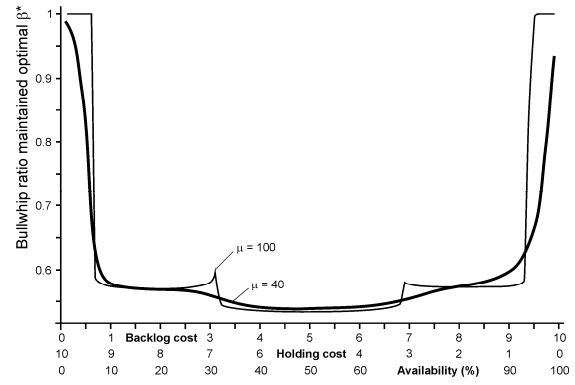


Figure 8. Bullwhip with the inventory cost optimal  $\beta^*$

## Conclusions

We have studied the impact of a stochastic lead time on a linear periodic review OUT policy. Our novel contribution is a new method to obtain the distribution of the inventory levels in the presence of correlation between the WIP and orders, via the so-called scaled shortfall distribution. This builds upon another unique contribution – the M-matrix and the associated method to determine the probability of the pipeline being in each of its possible  $2^{k^+}$  states.

In the constant lead time case,  $\beta = 1$  will minimize the variance (or equivalently the standard deviation) of the inventory levels and result in the minimum inventory costs when the

safety stock is set to the critical fractile (Brown 1963). However, in the stochastic lead time case, minimizing the variance of the inventory levels, by tuning  $\beta$ , will not always result in minimal costs. While the optimal,  $\beta^*$ , is never unity, it may be near unity, and changes significantly with the availability target, see Figure 5.

The stochastic lead time case with order crossover results in a surprising paradox. Minimizing inventory costs does not always lead to minimum safety stocks. However, the relationship between holding and backlog costs and the availability achieved at the most economical solution does still hold. This leads to an important insight: Costs should be used to design the system because focusing on minimizing inventory variance, or safety stocks, can lead to an incorrectly specified system.

We have demonstrated that the OUT policy is not the optimal policy when order crossover exists, as the linear POUT economically outperforms it. We have not proven the optimality of the POUT policy itself because we do not know whether there exists a better performing policy, linear or non-linear. Indeed, it is known that the optimal policy is non-linear (Srinivasan et al., 2011). However, the POUT policy has a long history and has been successfully implemented in practice. See Potter and Disney (2010) for details of an implementation at the UK grocery retailer, Tesco and Disney et al., (2013) for an implementation in the global printer manufacturer, Lexmark.

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