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FIVE PETALS: THE MYSTERIOUS NUMBER "5" HIDDEN IN NATURE

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Abstract: This article examines why many flowers are five-petaled through the use of a five-petal model that draws insight from the location of cell clusters at a shoot apex, rather than from concepts such as the Fibonacci sequence or the Golden ratio which have been referred to in the past. The conclusions drawn are that flowers are most likely to be five-petaled, followed by six-petaled flowers, and that four petals are unstable and almost no flower can be seven-petaled.

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1. Many Five-Petaled Flowers

I am deeply interested in pentagonal forms in the natural world. A hexagon, as seen in bee's nests or snow crystals, is mathematically explained, but no clear explanation is made about a pentagon.

Echinodemata such as sea urchins, starfish, and sea cucumbers are five-actinomorphic with a bony plate on the skin and a unique water-vascular system. In other words, they are pentagonal and rotationally symmetrical. The arm of a starfish has strong regeneration power as indicated by the fact that one of its five arms can regenerate immediately. Even more surprising is the fact that one arm can regenerate the remaining four arms (Ichikawa, 1982, [4]).

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Is something that determines five arms strongly coded for in DNA?

Using a compass or ruler, a regular pentagon can not be drawn as easily as a regular triangle, tetragon or hexagon. Although we can draw a regular pentagon with a protractor, by way of a 360° central angle which is divided into five, starfish do not use a compass or ruler, nor have mathematical knowledge. How can such a primitive aquatic creature draw a regular pentagon so easily?

"Five," as seen in sea urchins and starfish is also observed in plants. Flicking through the "Illustrated Guide to Plants (Shokubutu no Zukan)" (Shogakukan), many five-petaled flowers are found: spring flowers such as cyclamen, pansy, gypsophila, ume (Japanese apricot), cherry, azalea, and peach; summer flowers such as morning glory, bell bind, and oleander; as well as autumn flowers such as cotton rose, balloon flower, gillyflower, and gentian produce five-petal flowers. Farm products such as watermelon, melon, pear, and apple also have five petals.

Having said that, there are also exceptions ? a calla with one petal; an iris with three petals; daphne, dogwood, and fragrant olive with four petals; and lily, narcissus, and orchid with six petals.

As for the one-petaled and four-petaled flowers among these exceptions, some theories say that what looks like a petal is actually a sepal. Firstly, a calla belongs to the arum family, and what appears to be a white petals is a bract with small flowers on a thick axis inside. Calla's bract is sometimes called a spathe, as it looks like the halo or flames often seen in the Buddha statues. Secondly, with respect to four-petaled flowers, a daphne, spring flower, has four sepals, not petals. The same thing applies to the autumn flower of fragrant olive with yellow blossoms.

2. Trimerous, Tetramerous, and Pentamerous Flowers

Makino(1989, [6]) neatly classifies all plants according to kingdom, division, class, order, family, genus, and species. The book specifies the number of sepals, petals, stamens, and pistils of all the families. A flower diagram shows the pattern of locations and distributions of these to aid people easily understand the structure of a flower.

The classification of trimerous, tetramerous, and pentamerous flowers is based on a flower's components. If flowers have three (or a multiple of that number) sepals, petals, stamens, and pistils, they are called trimerous flowers. Many monocotyledons such as lily, iris, and spiderwort belong to this category. Similarly, flowers with four (or a multiple of that number) components, such as Japanese laurel and evening primrose, are tetramerous, and those with five (or a multiple of that number), such as azalea and morning glory, are pentamerous.

Flowers that belong to the same family have the same number of petals. All you have to do in order to know the number of petals of plants is research at a family level. The result of such research is described below. The spermatophyte division, subject of the research, has 219 families. The division is divided into the gymnosperm subdivision (13 families) and the angiosperm subdivision (206 families). The gymnosperm subdivision has no sepal or petal, and is classified as zero-petaled. The angiosperm subdivision branches off the monocotyledon class (35 families) and the dicotyledon class (171 families). The monocotyledon class includes the iris family and the lily family, and many families in the class are three-petaled and six-petaled (trimerous). They have no sepal, and are counted by tepals. The dicotyledon class is split into the choripetalae subclass(125 families) and the gamopetalae subclass(46 families). Many families in the choripetalae subclass are five-petaled (pentamerous) like the rose family, mallow family, and violet family, or four-petaled (tetramerous) like the mustard family and dogwood family. The gamopetalae subclass that includes the heath family and morning-glory family has many five-petaled (pentamerous) families.

Table 1 categorized the 219 families in the spermatophyte division according to the number of petals.

Number of petals	Number of families	Percentage
0	38	17.4%
1	2	0.9%
2	6	2.7%
3	13	5.9%
4	38	17.4%
5	84	38.4%
6	24	11.0%
More	7	3.2%
Unknown	7	3.2%
Total	219	100.0%

Table 1. Classification by Number of Petals (All)

Table 2 picks up the part from three-petaled to six-petaled. Families with no petal are included as long as the number of sepals or bracts can be counted. As a result, it is found that the largest number of families is five-petaled and that five-petaled followers belong to the dicotyledon class, the angiosperm subdivision, which is an advanced plant community from the perspective of the evolution theory.

Number of petals	Number of families	Percentage
3	13	8.2%
4	38	23.9%
5	84	52.8%
6	24	15.1%
Total	159	100.0%

Table 2. Classification by Number of Petals (3-6 Petals)

3. Chrysanthemum Also Five-petaled

We are familiar with chrysanthemums. According to the "Illustrated Guide to Plants (Shokubutsu no Zukan)" (Shogakukan), the aster family accounts for the largest share of 9%, with its 135 species among the total of 1495 species. The representative species of the aster family are a dandelion in spring, sunflower in summer, and cosmos in autumn. I had mechanically classified the aster family as multipetaled (seven or more), but it actually is five-petaled.

A flower of the aster family consists of a ray flower and a tubular flower, the latter surrounded by the former. The ray flower is a coalescent five-petaled flower. That is, the four of the five original petals had been degenerated with one ray petal remaining. The tubular flower at the center is an aggregate flower with hundreds of condensed small flowers.

Cosmos looks like an eight-petal flower but these eight petals are actually eight flowers. You can find this by looking at cosmos' tubular flowers at a park through a magnifying glass with a power between 10 and 15. Each one of densely located small flowers splits into five at the edge. Its appearance is similar to the flower of a balloon flower, and obviously five-petaled.

The flower of a tare in the legume family is tubular at the bottom, and divided into five at the edge. One flag petal, two wing petals, and two keel petals of the five petals constitute the corolla (whole of petals). The flowers of the legume family, mint family, and violet family are symmetrical if horizontally observed. Such a flower is called a zygomorphic flower. In any case, they are five-petaled.

The okra, which is edible, has the shape of a regular pentagon. It is also called America neri in Japan, and belongs to the mallow family, the Hibiscus manihot genus (Abelmoscus). The mallow family belongs to the mallow order (Malvales), the choripetalae subclass, the dicotyledon class, the angiosperm subdivision, and it is pentamerous. Its sepals, petals, and stamens can be found from outside, but its ovary can not. This ovary has to do with the pentagonal shape of okra's fruit. The ovary located below the pistil is divided into five cells, which makes the fruit pentagonal. The cross section of the fruit of a pear shows pentagon-like okra a pear belongs to the rose family, which is pentamerous.

4. What is a Flower?

Let us get an idea about what is a flower in shape, in reference to Hrara (1994, [2]). A plant's organs are the root, stem, and leaf. The root is an underground part of a plant body, which exists to support the plant, and absorbs water and inorganic salts. Stems are an aboveground part of the plant body designed to support the plant, and disseminate matter. Leaves regularly line up around a stem, and carry out photosynthesis with their flat shape. A plant, through its leaves, actively provides oxygen in exchange for carbon dioxide from outside, and transpires.

A unit consisting of a stem and leaves regularly arranged around the stem is called a shoot in botany. One example of a shoot is a branch. An inflorescence, a bud and the growth that develops and extends from it are also considered as a shoot.

Many plant flowers consist of sepals, petals, stamens, and a pistil, which is formed by one to several carpels, equivalent to leaves, sticking to one another. A sepal, petal, or stamen is also regarded as a modified leaf. Therefore, a flower can be considered as a metamorphosed short stem and variations of leaves regularly surrounding it (Figure 1).

A flower, from the perspective of botany, is an organ that produces a fruit and seed. In other words, a flower is ultimately an organ for a descendent from a seed. In this sense, a flower is called a reproduction organ.

Some mechanism to determine a five-petaled characteristic may exist in a pollen. From such a standpoint, I looked at the picture of pollen taken with a scanning electron microscope in the "Compendium of Modern Botany, Vol. 7 (Gendai Seibutsu-gaku Taikei, Di 7 Kan)" (Nakayama Shoten). Although various forms of pollen are observed, some of which are spherical-like fertilized eggs and the morulae of a sea urchin or starfish, no sign of anything that might decide "five" is observed. The next candidate is a seed, in which the origins of only a cotyledon, plumule, and radicle, in other words, leaf, bud, and root are found with no sign of flower. The growth of a plant has two stages, and a flower is formed during the second phase growth, which indicates that the shape of a flower is not determined yet.

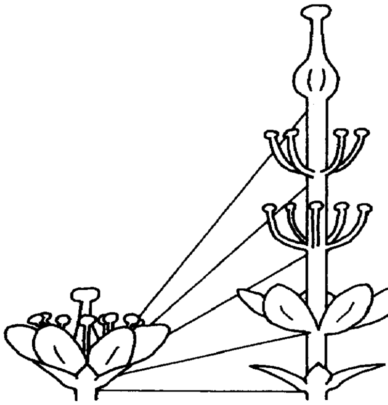


Figure 1: A flower as a shoot (1: sepal; 2: petal; 3 and 4: stamen; 5: pistil).This figure has adapted by the author from Hara (1994, [2])

5. Spiral Phyllotaxis and Fibonacci Sequence

A rafflesia of the tropical rain forest in Sumatra, the largest flower in the world, is five-petaled. I saw a documentary TV program on how its flower blooms; I had thought that one petal was followed by a petal next to it when the flower bloomed, but the picture showed that the next petal was skipped and every other petal opens in the first round and that the second round completed the bloom of the five-petaled flower. Such an order in flower blooming is confirmed by peeling and observing the petals of a blossom bud pulled off a plant of the tea family.

The order of petal opening is similar to that of leaf arrangement, which is explained later (spiral phyllotaxis). This suggests that a flower consists of metamorphosed leaves with a very shortened stem. By the way, the leaf primordium theory about a flower was originally advocated by Goethe in1970. this theory is correct on the whole, although it also contains an error in the sense that a leaf bud and a flower bud are different and that all the leaves do not become flowers.

The role of leaves is photosynthesis. Sunlight must be made most of in photosynthesis, and leaves are arranged so that the shade of a leaf on other leaves is avoided as possible as it can. There are four such arrangements (Figure 2). The first one is called opposite, with two leaves at the same height of a stem.

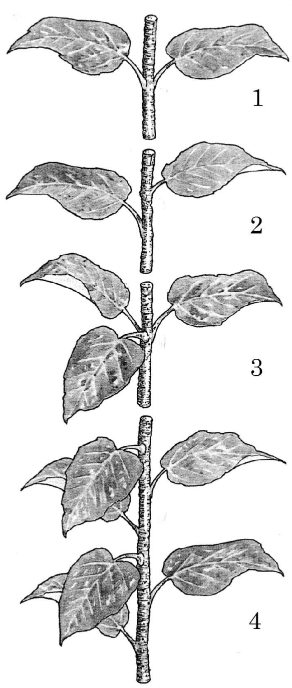


Figure 2: Leaf arrangement (1. opposite, 2. alternate, 3. verticillate, 4. spiral)

The second one is alternate, with leaves alternately arranged at different levels of a stem. The third arrangement is verticillate, with three leaves at the same height of a stem. The last one is called spiral, with three leaves at a different heights of a stem. When there are many leaves, a spiral arrangement can reduce shades most effectively. The arrangement of leaves is called phyllotaxis and a spiral leaf arrangement is called spiral phyllotaxis.

An alternate leaf arrangement and a spiral phyllotaxis are further classified by the divergence. The divergence is an angle formed by the two lines on the sectional view of a leaf arrangement, the one line to connect a stem and the center of a leaf and the other to connect a stem and the center of another leaf immediately above (or below). When the divergence is, for example, one of 144°, the leaf arrangement is called a two-fifths leaf arrangement because

144° is two-fifths of 360°. Many types of leaf arrangements exist, such as a half, one-third, two-fifths, and three-eighths ones. It can be considered that the numerator and denominator reflect the numbers of spirals and leaves in the spirals respectively. For example, three-eighths leaf arrangement implies that there are eight leaves in three spirals.

These leaf arrangements of plants can be explained by way of the Fibonacci sequence. The Fibonacci sequence, which is taught in mathematics at high schools, derives from botanical studies, and is a sequence formulated by the sums of two previous terms, as $1, 1, 2, 3, 5, 8, 13, \cdots$ If two every other figures are picked up with larger one as denominator and smaller one as numerator, they stand for a certain type of leaf arrangements. For example, 2 and 5 form the two-fifths of a two-fifths leaf arrangement, and 3 and 8 form the three-eighths of a three-eighths leaf arrangement.

Can the Fibonacci sequence explain all leaf arrangements? In response to this question, Hara (1984, [1]) makes an interesting suggestion as follows "such a mathematical relationship is found, it is natural that academic interests deepen in that direction. Arguments on leaf arrangements thus proceeded on the basis of mathematical figures rather than actual botanical observations of leaf production process on stem apex, and proceeded in connection with a theory that the Fibonacci sequence had a direct bearing on the golden section. The arguments developed as far as a theory that a leaf primordium on a stem apex came out ultimately at the locations related to the golden section. However, it is not appropriate to discuss this issue only with reference to mathematics, without actual and elaborate observations on how a leaf grows from a stem apex and how an embryotic leaf primordium changes its location secondarily."

I do not deny the Fibonacci sequence theory in every aspect, but it does not sufficiently explain why a flower is five-petaled. Actual observations are required.

6. Cells of Stem Apex

A growing point is a stem apex or a root apex where cell divisions occur, and this is certainly a key to solve the mystery. Specifically, the shape, size, and arrangement of cells at a growing point should be looked at.

Hara(1994, [2])(op. cit.) explains the basic structure of a stem apex, which is also called a shoot apex. A shoot is a unit consisting of a stem and leaves. A shoot apex directly generates a stem and leaves as well as the axil meristematic tissue of a leaf.

The apex meristem of an angiosperm shoot has the structure of a tunica, corpus, and cellular structure band (Figure 3). The shoot apex has one to several parallel layers of cells on its surface. This layer structure is a tunica, and a nonlayer structure inside is a corpus. A tunica constitutes of the cell layers that repeat vertical divisions, and a corpus comprises cells that make division planes in various directions.

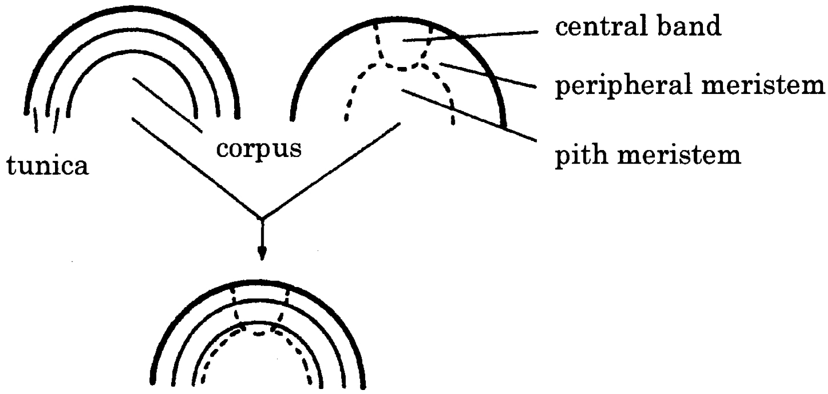


Figure 3: Tunica, corpus, and cellular structure band at shoot apex.Source: This figure has been adapted by the author from Hara (1984, [1])

In addition to a tunica and corpus structure, a shoot apex has a cellular structure band, which is divided into three zones according to the nature of component cells. The uttermost end of a shoot apex is called a central band. A zone that surrounds a central band is a peripheral meristem, and a zone, below a central band, which is surrounded by a peripheral meristem, is a pith meristem.

In a cellular structure band, a central band draws attention. Cells in a central band are large and near circular with low stain ability and many vacuoles. In particular, its less frequent cell division is unexpected, nevertheless it is located at the center of a meristem. Cells in a peripheral meristem surrounding a central band are active, and leaf primordiums are generated at the part of a peripheral meristem close to a central band.

Major changes take place at a shoot apex when it enters a reproductive phase from a vegetative phase. The cell arrangement of a tunica and corpus as well as the structure of a cellular structure band are gradually lost during this conversion phase, and several layers of cell are generated on the surface of a shoot apex. Cells in this part have a dense cell layer, and are the sites of active cell division.

In the formation of flowers, a reproductive shoot apex generates the primordiums of a floral leaf, usually in the order of the primordium of sepals, petals, and carpels. When carpel primordiums are made up, a reproductive shoot apex completes its division and becomes a part of carpels to end it (Kaku, 1982, [5]).

Figure 4 illustrates growing points as explained below. A stem edge, as seen in its cross-sectional view (Figure 4(A)), can be divided into dome-like cells and lateral parts where small leaves are being formed. More three-dimensional Figure 4(B) shows the state of vegetative reproduction, in which leaves are generated one after another. On the other hand, Figure 4(C) shows an apical part during an early flower bud formation period. The formation of a flower bud occurs at a centric part, where the growing part of leaf premordium change into flower premordium. Though by very small degree, the place of leaf formation and the place of flower formation are disjointed at the apical part.

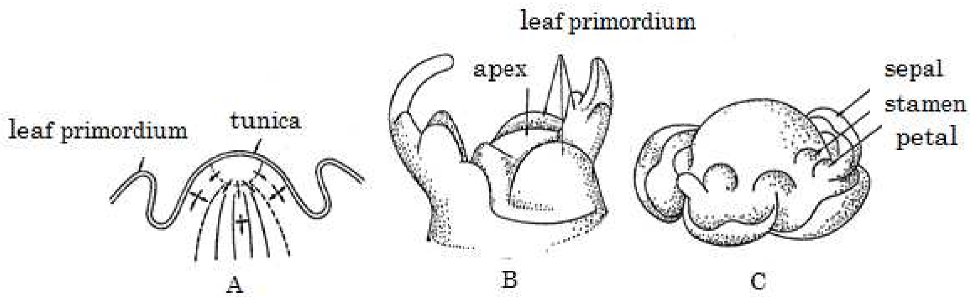


Figure 4: Growing point(A) Cross-sectional view; (B) three-dimensional view; (C) flower primordium.Source: This figure has been adapted by the author from Kaku (1982, [5])

7. Fullerrene C_{60}

The 1996 Nobel Prize in Chemistry was awarded to the discovery of a soccer-ball-shape molecule, C_{60}. the prize winner were Robert F. Curl Jr., Richard E. Smalley, and Sir Harold W. Kroto. Figure 5 shows the molecule model of Fullerene C_{60} and a soccer ball. Carbon atoms are placed at vertexes of the polyhedron. There are 12 pentagonal aspects and 20 hexagonal aspects, with one pentagon surrounded by 5 hexagons. This carbon-only structure, which is perfectly symmetrical, stands for a totally new concept in molecule structures. The name of Fullerene is an abbreviated name of Buckminster Fullerene, after the famous architect Richard Buckminster Fuller who invented the geodesic dome structure.

What is mathematically interesting is the fact that a graphite sheet with 60 atoms, according to the Euler's formula, cannot form a closed ball. Similarly, a familiar wire fabric with hexagonal elements cannot form a ball. This is easily proved by the Euler's polyhedron theorem. The theorem maintains, $V + F = E + 2$ (V: number of vertices, F: number of faces, E: number of edges). As studies on Fullerene C_{60} advances, soccer-ball-like or multiple-layered molecules with more than 60 carbon atoms as well as "Carbon Nanotube," the tube-shape cluster of carbons, have been found one after another. Carbon Nanotube was discovered by Sumio Iijima, a researcher at NEC Corporation, in 1991, and is also called Buckytube. If Fullerene with the same diameter as C_{60}@is extended, the length is about $61\mathring{A}$ in the case of C_{500}. The noteworthy discovery about Nanotubes is that some of them include seven-member rings. While five-member rings close Fullerene in a ball-shape, seven-member rings extend Fullerene. Seven-member rings occur on a saddle-like curved surface (negative curved surface ratio).

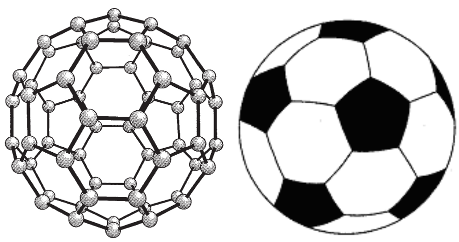


Figure 5: C_{60} molecule model and soccer ball

8. Cell Arrangement and Five Petals

The stem apex has a dome-like shape. How are cell clusters arranged in such a dome-like shape? There is a molecule called Carbon Nanotube, which is a variation of Fullerene. It has a long, slender tubular shape, and reminds one of a plant's stem. The edge of Nanotube is convex, and has five-member rings. I infer that a stem is similar to Nanotube.

A petal is formulated by thousands of cells, not by one cell. This cell cluster forming a petal is located at a stem apex, and its arrangement certainly has to do with the form of a flower. If the arrangement is important, questions such as how the arrangement develops and what is the optimal arrangement arise. From such a perspective, I have a feeling that it is not difficult for plants to choose five petals, and, rather, the choice of four petals is unstable and difficult.

Let us suppose that a cell cluster at a stem apex is a ball (or a hexagon) [3]. In the first place, it is common knowledge that the optimal shape to cover a level ground is a regular hexagon. The hexagonal shape filling by circular discs has less opening than the square shape filling. To arrange 10-yen coins as if a coin is a hexagon, as shown in Figure 6(1), makes opening the least, while to arrange them as if a coin is a square, as shown in Figure 6(2), has more opening, which allows coins to move. In these sense, the hexagonal shape filling is the optimal arrangement. It is well known that a regular hexagon in a bee's nest and a snow crystal is connected with this. On the other hand, flower petals choose five, not six, in many cases.

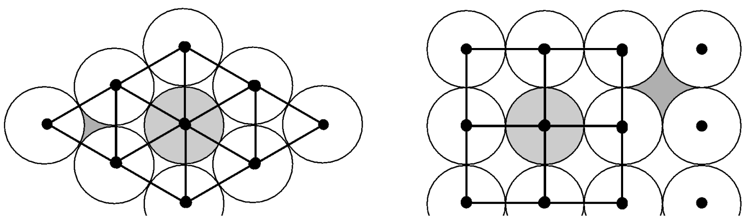
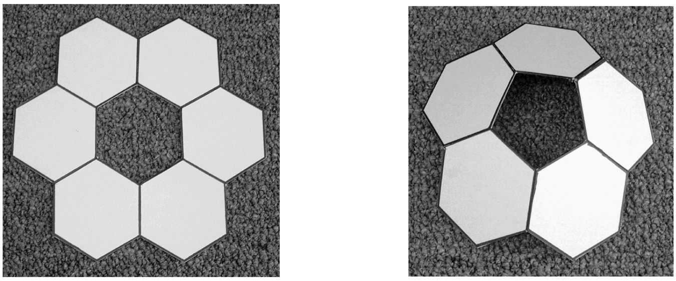


Figure 6: High density filling by circles

A guiding principle in a form or pattern generation in nature is an action or binding force of a space or spot. The leaf arrangement of plants can be explained by the Fibonacci sequence or the golden ratio, but plants do not know mathematics. Plants simply try to occupy the largest space, and the result of such a tendency happens to be described by those mathematical rules. All the beauty and mathematics is a natural byproduct of a simple growth system that is interactive with a surrounding space (Stevens, 1987, [7]).

Thus, I here present cell arrangement models as explained below. Figure 7(1) shows six regular hexagons arranged without opening to surround one regular hexagon at the center. As you can see from Figure 7(1), the arrangement is flat. If the number of hexagons is reduced from six to five, the five hexagons forms a shape like an upside-down bowl, convex toward top or concave toward bottom, as seen in Figure 7(2). On the other hand, if one hexagon is added, a saddle shape, uneven like waves, is formed by the seven hexagons.



(1) Six Regular Hexagons (2) Five Regular Hexagons

Figure 7: Models of cell cluster arrangements

These transformations to a bowl or a saddle depend on expanding velocity of a surrounding part in relation to a central part. These transformations are attributable to the nature of space. The relatively high growth velocity at a central part creates a dome-like shape, and that at a surrounding part a saddle-like shape. A shoot apex (stem apex) is a growing point, which means that cells at the center are most active, and its shape is convex toward the edge, like a dome.

Cells are, there fore, arranged as shown in Figure 7(2). Five hexagons, or five cell clusters, surround one pentagon, or a stem. These five cell clusters transform into sepals, petals, stamens, and ovaries. With this, the mystery of a five-petaled flower is solved. From the same reasoning, we can infer the extremely low possibility of a seven-petaled flower, since a saddle-like shape formed by an arrangement where seven cell clusters surround a stem is unstable and it is not natural for a surrounding part to have higher growth velocity.

A six-petaled flower can be generated from a flat stem apex. If a stem apex is flat, six cell clusters are arranged around a stem as shown in Figure 7(1), and they become sepals, petals, meaning the formation of a six-petaled flower. However, it is not natural for a stem apex to be flat, and the possibility of six petals is lower than that of five petals.

A four-petaled flower can be formed when the arrangement of cell clusters at a stem apex is like the square shape filling of Figure 6(2). The square shape filling has lower density than the hexagonal shape filling, but, given the lower variability of plant cells compared with animal cells, such an arrangement is not impossible to take place. A stem apex is formed as square shape filling and if cell clusters of stem apex are also arranged in this manner, if we assume that one of them is a stem, there are fore neighboring cell clusters around that stem. Accordingly, because these four cell clusters become sepals and petals, the four cell clusters generate a four-petaled flower. The possibility of four petals, however, is lower than that of five petals, since the square shape filling is less dense than hexagonal shape filling.

The Greek myths have an episode about the statue of Daphne. Daphne's arm changes to branches and leaves, and the whole body transforms into a tree. If this myths has scientific grounds, it may be possible to explain the reason of five fingers by relating human arm, hand, and finger to plant stem, branch, leaf, and flower. Fingers in both hands total 10. Ten fingers might contribute to the decimal system, and be reflected in culture, history, society, and ideology. I cannot help feelong that "five" fides across the universe.

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