NJ ASK

**New Jersey
Assessment of Skills
and Knowledge**

New Jersey Assessment of Skills and Knowledge

Mathematics

Grade 3 and Grade 4

Assessment Samples

PTM #1505.57

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New Jersey Assessment of Skills and Knowledge

Mathematics

Grade 3 and Grade 4

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Introduction

This document contains samples of Mathematics items from the New Jersey Assessment of Skills and Knowledge (NJ ASK). These materials, which appeared on actual grade 3 and grade 4 tests, are released samples and, therefore, are no longer considered secure assessment documents. The materials are illustrative of the kinds of test questions that students encounter with the NJ ASK. Pages from this document may be duplicated and used for instructional purposes in the classroom.

Pages 2-25 provide the grade 3 samples of multiple-choice and open-ended items.

Pages 48-83 provide the grade 4 samples of multiple-choice and open-ended items.

Appendix A provides the answer key for the multiple-choice questions for grade 3.

Appendix B provides the item-specific rubric for the open-ended question in grade 3.

Appendix C provides the test specifications for grade 3.

Appendix D provides the answer key for the multiple-choice questions for grade 4.

Appendix E provides the item-specific rubric for each open-ended question in grade 4.

Appendix F provides the test specifications for grade 4.

Directions to the Student

When you are taking this test, remember these important things:

1. Read each question carefully and think about the answer.

2. If you do not know the answer to a question, go on to the next question. You may come back to the skipped question later if you have time.

3. When you see a STOP sign, do **not** turn the page until you are told to do so.

DIRECTIONS:

The next section of the test has 4 multiple-choice questions. You will fill in the circle next to the answer you choose. You may NOT use a calculator.

Sample Multiple-Choice Questions

The sample questions below show you what the questions are like and how to mark your answers.

Example:

1. Find the exact answer: 110 + 70

    Ⓐ **18**

    Ⓑ **81**

    ● **180**

    Ⓓ **810**

The correct answer is C. The circle with the C in it has been filled in to show that C is the correct answer.

2. Estimate 123 + 685. The sum is between which numbers?

    Ⓐ   **400 and 600**

    ●   **700 and 900**

    © **1,000 and 1,200**

    Ⓓ **1,300 and 1,500**

The correct answer is B. The circle with the B in it has been filled in to show that B is the correct answer.

DIRECTIONS:

Choose the best of the answer choices given for each of the following problems. Fill in the circle next to your choice. You may NOT use a calculator.

1. **Find the exact answer: 145 + 281 + 62**

    Ⓐ   **426**

    Ⓑ   **488**

    Ⓒ   **946**

    Ⓓ **1,046**

2. **Find the exact answer: 365 − 56**

    Ⓐ **300**

    Ⓑ **309**

    Ⓒ **311**

    Ⓓ **421**

3.   **Estimate 153 + 44. The sum is between which numbers?**

    Ⓐ **100 and 299**

    Ⓑ **300 and 499**

    Ⓒ **500 and 699**

    Ⓓ **700 and 899**

4.   **Estimate 999 − 103. The difference is between which numbers?**

    Ⓐ **1,300 and 1,500**

    Ⓑ **1,000 and 1,200**

    Ⓒ **700 and 900**

    Ⓓ **400 and 600**

If you have time, you may review your work in this section only.

STOP

**DO NOT GO ON
UNTIL YOU ARE
TOLD TO DO SO.**

Directions for Students

You are allowed to use your calculator for the following multiple-choice and open-ended items. You may also use the ruler and colored shapes.

Sample Multiple-Choice Question

The sample question below will show you what the multiple-choice questions are like and how to mark your answers. For each multiple-choice question, select the best answer and fill in the circle next to your choice. Make sure you fill in the correct circle.

Example:

1. **Mark has a stamp collection. He has 22 stamps from Japan, 34 from Canada, and 17 from Mexico. How many stamps does he have in all?**

   Ⓐ **53**

   Ⓑ **63**

   ● **73**

   Ⓓ **83**

The correct answer is C. The circle with the C in it has been filled in to show that C is the correct answer.

Sample Open-Ended Question

The sample question below will show you what the open-ended question is like and how to write your answer. You will write or draw your answer in the work area provided. When asked to explain an answer, you may use words, tables, diagrams, or pictures.

Example:

1. **A juice machine charges 65¢ for a can of juice and accepts only nickels, dimes, and quarters. The machine requires exact change.**

   • **What combination of coins could you put in the juice machine to get a can of juice?**

   Show your work or explain your answer.

Work area for question 1.

OR

I used two quarters, one dime, and one nickel.

These are just two examples of the many possible ways to answer open-ended questions. Be sure to answer all parts of the question, show your work, or explain your answer. You may use your calculator, ruler, and colored shapes.
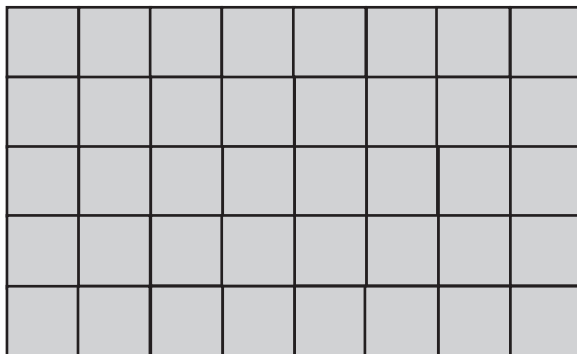
DO NOT WRITE IN THIS AREA.

5. **Compare the shaded regions. Which symbol belongs in the square?**

Ⓐ <

Ⓑ >

Ⓒ =
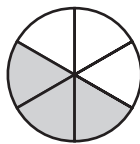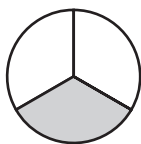
Ⓓ **None of the above**

6. **Amanda wants to cover the top of her doll's table with colored paper. The top of the table is shown below.**
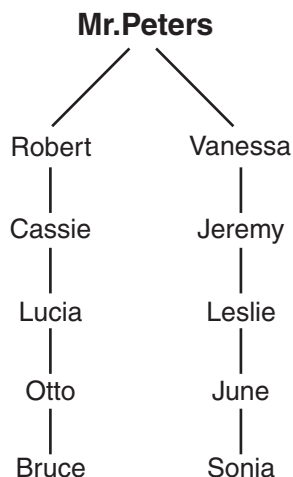

How many square centimeters of paper does Amanda need if each square equals 1 square centimeter?

Ⓐ   **5**

Ⓑ   **8**

Ⓒ **26**

Ⓓ **40**

7. The mathematics club uses this phone tree to remind members about club activities.

Mr. Peters calls Robert and Vanessa, and then each student calls the person whose name is listed under their name. This continues until every student is called.

<pre>
                    Mr.Peters
                   /   \
            Robert   Vanessa
               | |
            Cassie   Jeremy
               | |
            Lucia   Leslie
               | |
            Otto   June
               | |
            Bruce   Sonia
</pre>

Which student will Leslie call?

Ⓐ **Jeremy**

Ⓑ **June**

Ⓒ **Lucia**

Ⓓ **Otto**

8. Which of these letters has a line of symmetry?

P F T L

Ⓐ **P**

Ⓑ **F**

Ⓒ **T**
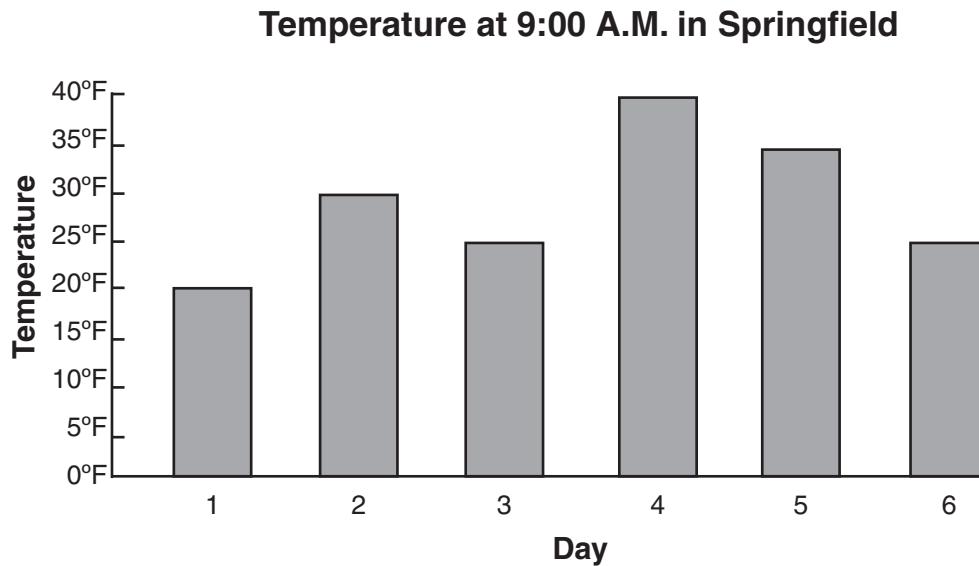
Ⓓ **L**

TURN TO THE NEXT PAGE. ➡

9. José had 25 baseball cards. His mother gave him some more cards. He now has 40 baseball cards. Which number sentence could you use to find how many baseball cards his mother gave him?

Ⓐ $\square - 40 = 25$

Ⓑ $25 + \square = 40$

Ⓒ $25 \times \square = 40$

Ⓓ $25 + 40 = \square$

10. What was the temperature at 9:00 A.M. in Springfield on Day 3?

Temperature at 9:00 A.M. in Springfield

Ⓐ **20°F**

Ⓑ **25°F**

Ⓒ **30°F**

Ⓓ **35°F**

11. Look at the pattern below.

If the pattern continues, how many dots will be in the next triangle?

Ⓐ   **5**

Ⓑ **10**

Ⓒ **15**

Ⓓ **20**

If you have time, you may review your work in this section only.

STOP

**DO NOT GO ON
UNTIL YOU ARE
TOLD TO DO SO.**

DIRECTIONS:

Choose the best of the answer choices given for each of the following problems. Fill in the circle next to your choice. You may use your calculator, ruler, and colored shapes.

12. There are 10 red chips and 5 blue chips in a bag. What are the chances of picking a blue chip if you reach into the bag and pick one without looking?

   Ⓐ **1 out of 5**

   Ⓑ **2 out of 15**

   Ⓒ **5 out of 10**

   Ⓓ **5 out of 15**

13. Use your ruler to answer this question. To the nearest centimeter, what is the length of line segment *AB*?

A   *B*

Ⓐ   **34 cm**

Ⓑ   **33 cm**

Ⓒ   **4 cm**

Ⓓ   **3 cm**

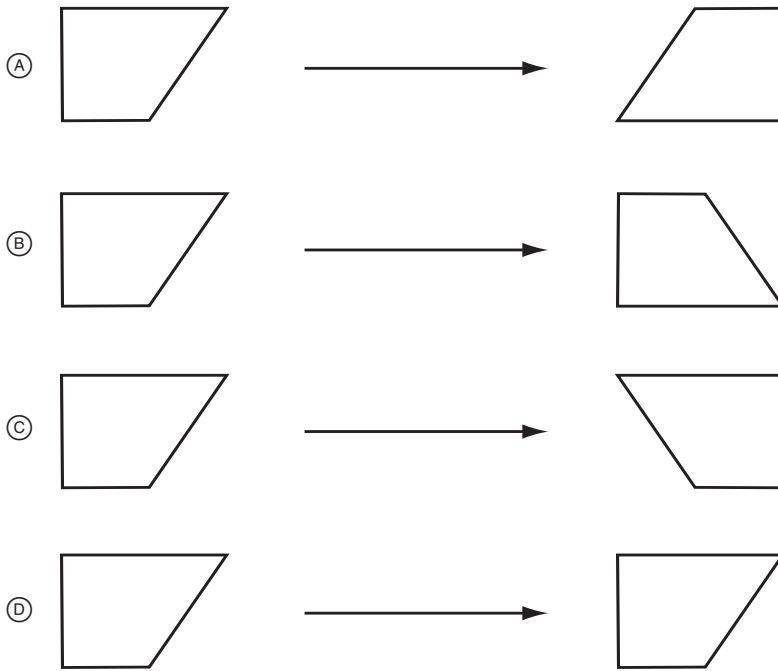**14. The population of the city where Michelle was born is 145,826. What is the value of the 5 in the number 145,826?**

Ⓐ   **5 thousands**

Ⓑ   **5 hundreds**

Ⓒ   **5 tens**

Ⓓ   **5 ones**

15. Which picture shows ONLY a slide?

Ⓐ

Ⓑ

Ⓒ

Ⓓ

16. What does the *p* equal in 3 + *p* = 15?

Ⓐ **3**

Ⓑ **5**

Ⓒ **12**

Ⓓ **18**

TURN TO THE NEXT PAGE.

17. Sue is having some friends over for pizza. She surveyed what toppings they would like on their pizza.

Toppings	Number of Votes
Cheese	|||
Pepperoni	⦀⦀	
Sausage	||||
Mushrooms	
Onions		

What can Sue most likely conclude from her survey?

Ⓐ **More of Sue's friends like cheese than pepperoni pizza.**

Ⓑ **Sausage is the group's second-favorite type of pizza.**

Ⓒ **A pizza with both pepperoni and mushrooms should be ordered.**

Ⓓ **Sue needs to order only one onion pizza.**

18. When 10 is dropped into this machine, it comes out as 5.

10

?

5
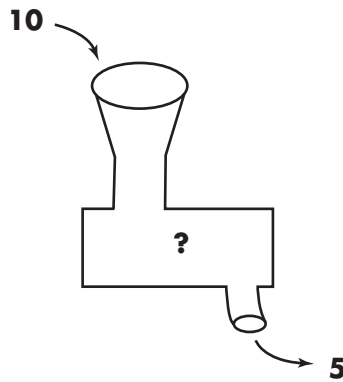
When 16 is dropped in, it comes out as 8.

When 4 is dropped in, it comes out as 2.

If 8 is dropped into the machine, what number will it come out as?

Ⓐ **3**

Ⓑ **4**

Ⓒ **5**

Ⓓ **6**

If you have time, you may review your work in this section only.

STOP

DO NOT GO ON UNTIL YOU ARE TOLD TO DO SO.

## Directions for the Open-Ended Question

The following question is an open-ended question. Remember to:

- Read the question carefully and think about the answer.
- Answer all the parts of the question.
- Show your work or explain your answer.

You can answer the question by using words, tables, diagrams, OR pictures. You may use your calculator, ruler, and colored shapes.

19. Look at the figures below.
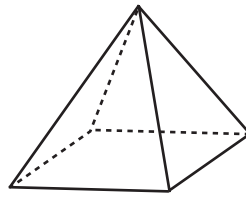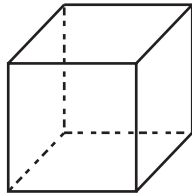
• **Name each figure.**

• **How many faces does each figure have?**

• **Write one way the figures are the same.**

• **Write one way the figures are different.**

Work area for question 19

More work area for question 19

If you have time, you may review your work in this section only.

STOP

**CLOSE YOUR
TEST BOOKLET.**

DO NOT WRITE IN THIS AREA.

APPENDIX A:

SCORING KEYS

NJ ASK
Grade 3 Mathematics
Scoring Keys

Item #	Correct Answer	Standard, Grade level	Strand	CPI	Mathematical Processes
1	B	1, 3rd	B	4	Problem solving
2	B	1, 3rd	B	4	Problem solving
3	A	1, 3rd	C	2	Problem solving, reasoning
4	C	1, 3rd	C	2	Problem solving, reasoning
5	A	1, 3rd	A	6	Reasoning, representation
6	D	2, 3rd	E	1	Problem solving, reasoning, representation
7	B	4, 3rd	D	2	Connections, reasoning, representation
8	C	2, 3rd	A	3	Reasoning
9	B	3, 3rd	C	2	Problem solving, reasoning
10	B	4, 3rd	A	2	Reasoning, representation
11	C	3, 3rd	A	1	Reasoning
12	D	4, 3rd	B	2	Problem solving
13	D	2, 3rd	D	2	Reasoning
14	A	1, 3rd	A	2	Reasoning
15	D	2, 3rd	B	1	Reasoning
16	C	3, 3rd	C	2	Problem solving
17	B	4, 3rd	A	2	Reasoning, connections
18	B	3, 3rd	B	1	Problem solving, reasoning
19	rubric	2, 3rd	A	2	Communication, reasoning, representation

APPENDIX B:

ITEM-SPECIFIC SCORING RUBRIC FOR THE OPEN-ENDED QUESTION

3 points

The student

- identifies the shapes
- indicates the correct number of faces for each shape
- indicates one similarity (size, flat surface, straight lines, etc.)
- indicates one difference (number of faces, name, kind of shape, etc.)

Sample 3 point response:

- cube and pyramid
- the cube has 6 faces, the pyramid has 5 faces
- both shapes have straight lines (many acceptable answers)
- both shapes have a different number of faces (many acceptable answers)

2 points

The student

- identifies the shapes (Cube and Pyramid)
- indicates the Cube has 6 faces **and/or** the Pyramid has 5 faces
- indicates one similarity **or** indicates one difference

<div align="center">**OR**</div>

- identifies the shapes (Cube and Pyramid)
- indicates the Cube has 6 faces **and** the Pyramid has 5 faces
- has an unclear explanation for similarities and differences, or has not given an answer for these two bullets

<div align="center">**OR**</div>

- identifies the shapes (Cube and Pyramid)
- indicates the Cube has 6 faces **or** the Pyramid has 5 faces
- indicates one similarity **and** indicates one difference

<div align="center">**OR**</div>

- identifies the shapes (Cube and Pyramid)
- does not indicate the correct number of faces for either shape
- indicates one similarity
- indicates one difference

<div align="center">**OR**</div>

- identifies 1 of the shapes correctly (Cube or Pyramid)
- indicates the Cube has 6 faces **and/or** the Pyramid has 5 faces
- indicates one similarity
- indicates one difference

<div align="center">**OR**</div>

- identifies 1 of the shapes correctly (Cube or Pyramid)
- indicates the Cube has 6 faces **and** the Pyramid has 5 faces
- indicates one similarity **or** one difference

- does not correctly identify the shapes as a Cube or a Pyramid
- indicates the cube figure has 6 faces **and** the pyramid figure has 5 faces
- indicates one similarity **and** one difference

1 point
The student

- identifies the shapes (Cube and Pyramid)
- indicates the Cube has 6 faces **or** the Pyramid has 5 faces
- does not indicate a similarity or difference
<div align="center">**OR**</div>
- identifies 1 of the figures (Cube or Pyramid)
- indicates the Cube has 6 faces **or** the Pyramid has 5 faces
- indicates a similarity **or** difference
<div align="center">**OR**</div>
- identifies the shapes (Cube and Pyramid)
- does not indicate the correct number of faces for either figure
- indicates one similarity **or** one difference
<div align="center">**OR**</div>
- identifies 1 of the shapes (Cube or Pyramid)
- indicates the Cube has 6 faces **and/or** the Pyramid has 5 faces
- does not indicate one similarity or one difference
<div align="center">**OR**</div>
- identifies 1 of the shapes (Cube or Pyramid)
- does not indicate a correct number of faces for either shape
- indicates one similarity **and/or** one difference
<div align="center">**OR**</div>
- does not identify the shapes correctly
- indicates one figure has 6 faces **and/or** the other figure has 5 faces
- indicates one similarity **or** one difference
<div align="center">**OR**</div>
- does not identify the shapes
- indicates one figure has 6 faces **or** the other figure has 5 faces
- indicates one similarity **and** one difference
<div align="center">**OR**</div>
- identifies the shapes (Cube and Pyramid)
<div align="center">**OR**</div>
- indicates the Cube has 6 faces **and** the Pyramid has 5 faces
<div align="center">**OR**</div>
- indicates one similarity **and** one difference

0 points

- The response shows insufficient understanding of the problem's mathematical concepts.

APPENDIX C:

DIRECTORY OF TEST SPECIFICATIONS FOR GRADE 3

Standards and Strands

4.1. Number and Numerical Operations
A. Number Sense
B. Numerical Operations
C. Estimation

4.2. Geometry and Measurement
A. Geometric Properties
B. Transforming Shapes
C. Coordinate Geometry
D. Units of Measurement
E. Measuring Geometric Objects

4.3. Patterns and Algebra
A. Patterns and Relationships
B. Functions
C. Modeling
D. Procedures

4.4. Data Analysis, Probability, and Discrete Mathematics
A. Data Analysis (Statistics)
B. Probability
C. Discrete Mathematics--Systematic Listing and Counting
D. Discrete Mathematics--Vertex-Edge Graphs and Algorithms

4.5 Mathematical Processes
A. Problem Solving
B. Communication
C. Connections
D. Reasoning
E. Representation
F. Technology

MATHEMATICS TEST SPECIFICATIONS

STANDARD 4.1   (NUMBER AND NUMERICAL OPERATIONS)   ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.

Descriptive Statement: Numbers and arithmetic operations are what most of the general public think about when they think of mathematics; and, even though other areas like geometry, algebra, and data analysis have become increasingly important in recent years, numbers and operations remain at the heart of mathematical teaching and learning. Facility with numbers, the ability to choose the appropriate types of numbers and the appropriate operations for a given situation, and the ability to perform those operations as well as to estimate their results, are all skills that are essential for modern day life.

Number Sense. Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.

Numerical Operations.   Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

Estimation. Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

Number and operation skills continue to be a critical piece of the school mathematics curriculum and, indeed, a very important part of mathematics. But, there is perhaps a greater need for us to rethink our approach here than to do so for any other curriculum component. An enlightened mathematics program for today's children will empower them to use all of today's tools rather than require them to meet yesterday's expectations.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 3,** students will:

A. Number Sense (4.1.3.A.1-6)
   1. Use real-life experiences, physical materials, and technology to construct meanings for numbers **(unless otherwise noted, all indicators for grade 3 pertain to these sets of numbers as well)**.
      - Whole numbers through hundred thousands
      - Commonly used fractions (denominators of 2, 3, 4, 5, 6, 8, 10) as part of a whole, as a subset of a set, and as a location on a number line
   2. Demonstrate an understanding of whole number place value concepts.
   3. Identify whether any whole number is odd or even.
   4. Explore the extension of the place value system to decimals through hundredths.
   5. Understand the various uses of numbers.
      - Counting, measuring, labeling (e.g., numbers on baseball uniforms)
   6. Compare and order numbers.

B. Numerical Operations (4.1.3.B.1-7)
   1. Develop the meanings of the four basic arithmetic operations by modeling and discussing a large variety of problems.
      - Addition and subtraction: joining, separating, comparing
      - Multiplication: repeated addition, area/array
      - Division: repeated subtraction, sharing
   2. Develop proficiency with basic multiplication and division number facts using a variety of fact strategies (such as "skip counting" and "repeated subtraction").
   3. Construct, use, and explain procedures for performing whole number calculations with:
      - Pencil-and-paper
      - Mental math
      - Calculator
   4. Use efficient and accurate pencil-and-paper procedures for computation with whole numbers.
      - Addition of 3-digit numbers
      - Subtraction of 3-digit numbers
      - Multiplication of 2-digit numbers by 1-digit numbers
   5. Count and perform simple computations with money.
      - Cents notation (¢)
   6. Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.
   7. Check the reasonableness of results of computations.

C. Estimation (4.1.3.C.1-4)
   1. Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set**.**
   2. Construct and use a variety of estimation strategies (e.g., rounding and mental math) for estimating both quantities and the result of computations.
   3. Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.
   4. Use estimation to determine whether the result of a computation (either by calculator or by hand) is reasonable.

STANDARD 4.2   (GEOMETRY AND MEASUREMENT)   ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.

Descriptive Statement: Spatial sense is an intuitive feel for shape and space. Geometry and measurement both involve describing the shapes we see all around us in art, nature, and the things we make. Spatial sense, geometric modeling, and measurement can help us to describe and interpret our physical environment and to solve problems.

Geometric Properties. This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.

Transforming Shapes. Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.

Coordinate Geometry. Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

Units of Measurement. Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

Measuring Geometric Objects. This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important that they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.

Students of all ages should realize that geometry and measurement is all around them. Through study of these areas and their applications, they should come to better understand and appreciate the role of mathematics in their lives.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 3,** students will:

A. Geometric Properties (4.2.3.A.1-5)
1. Identify and describe spatial relationships of two or more objects in space.
   - Direction, orientation, and perspectives (e.g., which object is on your left when you are standing here?)
   - Relative shapes and sizes
2. Use properties of standard three-dimensional and two-dimensional shapes to identify, classify, and describe them.
   - Vertex, edge, face, side, angle
   - 3D figures – cube, rectangular prism, sphere, cone, cylinder, and pyramid
   - 2D figures – square, rectangle, circle, triangle, pentagon, hexagon, octagon
3. Identify and describe relationships among two-dimensional shapes.
   - Same size, same shape
   - Lines of symmetry
4. Understand and apply concepts involving lines, angles, and circles.
   - Line, line segment, endpoint
5. Recognize, describe, extend, and create space-filling patterns.

B. Transforming Shapes (4.2.3.B.1-2)
1. Describe and use geometric transformations (slide, flip, turn).
2. Investigate the occurrence of geometry in nature and art.

C. Coordinate Geometry (4.2.3.C.1)
1. Locate and name points in the first quadrant on a coordinate grid.

D. Units of Measurement (4.2.3.D.1-3)
1. Understand that everyday objects have a variety of attributes, each of which can be measured in many ways.
2. Select and use appropriate standard units of measure and measurement tools to solve real-life problems.
   - Length – fractions of an inch (1/4, 1/2), mile, decimeter, kilometer
   - Area – square inch, square centimeter
   - Weight – ounce
   - Capacity – fluid ounce, cup, gallon, milliliter
3. Incorporate estimation in measurement activities (e.g., estimate before measuring).

E. Measuring Geometric Objects (4.2.3.E.1-3)
1. Determine the area of simple two-dimensional shapes on a square grid.
2. Determine the perimeter of simple shapes by measuring all of the sides.
3. Measure and compare the volume of three–dimensional objects using materials such as rice or cubes.

> **STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.**

Descriptive Statement: Algebra is a symbolic language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

Patterns. Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.

Functions and Relationships. The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

Modeling. Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules.   Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

Procedures. Techniques for manipulating algebraic expressions – procedures – remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

Algebra is a gatekeeper for the future study of mathematics, science, the social sciences, business, and a host of other areas. In the past, algebra has served as a filter, screening people out of these opportunities. For New Jersey to be part of the global society, it is important that algebra play a major role in a mathematics program that opens the gates for all students.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 3,** students will:

A. Patterns (4.3.3.A.1)
    1.   Recognize, describe, extend, and create patterns.
- Descriptions using words and number sentences/expressions
- Whole number patterns that grow or shrink as a result of repeatedly adding, subtracting, multiplying by, or dividing by a fixed number (e.g., 5, 8, 11, . . . or 800, 400, 200, . . .)

B. Functions and Relationships (4.3.3.B.1)
    1.   Use concrete and pictorial models to explore the basic concept of a function.
- Input/output tables, T-charts

C. Modeling (4.3.3.C.1-2)
    1.   Recognize and describe change in quantities.
- Graphs representing change over time (e.g., temperature, height)

    2.   Construct and solve simple open sentences involving addition or subtraction (e.g., $3 + 6 =$ __, $n = 15 - 3$, $3 +$ __ $= 3$, $16 - c = 7$).

D. Procedures (4.3.3.D.1-2)
    1.   Understand and apply the properties of operations and numbers.
- Commutative (e.g., $3 \times 7 = 7 \times 3$)
- Identity element for multiplication is 1 (e.g., $1 \times 8 = 8$)
- Any number multiplied by zero is zero

    2.   Understand and use the concepts of equals, less than, and greater than to describe relations between numbers.
- Symbols ($=, <, >$)

**STANDARD 4.4   (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)
ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND
TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS,
AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE
AND DRAW APPROPRIATE INFERENCES FROM DATA.**

Descriptive Statement: Data analysis, probability, and discrete mathematics are important
interrelated areas of applied mathematics. Each provides students with powerful mathematical
perspectives on everyday phenomena and with important examples of how mathematics is used
in the modern world.   Two important areas of discrete mathematics are addressed in this
standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

Data Analysis (or Statistics). In today's information-based world, students need to be able to
read, understand, and interpret data in order to make informed decisions. In the early grades,
students should be involved in collecting and organizing data, and in presenting it using tables,
charts, and graphs. As they progress, they should gather data using sampling, and should
increasingly be expected to analyze and make inferences from data, as well as to analyze data
and inferences made by others.

Probability. Students need to understand the fundamental concepts of probability so that they
can interpret weather forecasts, avoid unfair games of chance, and make informed decisions
about medical treatments whose success rate is provided in terms of percentages. They should
regularly be engaged in predicting and determining probabilities, often based on experiments
(like flipping a coin 100 times), but eventually based on theoretical discussions of probability
that make use of systematic counting strategies. High school students should use probability
models and solve problems involving compound events and sampling.

Discrete Mathematics—Systematic Listing and Counting.   Development of strategies for
listing and counting can progress through all grade levels, with middle and high school students
using the strategies to solve problems in probability.   Primary students, for example, might find
all outfits that can be worn using two coats and three hats; middle school students might
systematically list and count the number of routes from one site on a map to another; and high
school students might determine the number of three-person delegations that can be selected
from their class to visit the mayor.

Discrete Mathematics—Vertex-Edge Graphs and Algorithms.   Vertex-edge graphs,
consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve
problems based on real-world situations. Students should learn to follow and devise lists of
instructions, called "algorithms," and use algorithmic thinking to find the best solution to
problems like those involving vertex-edge graphs, but also to solve other problems.

These topics provide students with insight into how mathematics is used by decision-makers in
our society, and with important tools for modeling a variety of real-world situations. Students
will better understand and interpret the vast amounts of quantitative data that they are exposed to
daily, and they will be able to judge the validity of data-supported arguments.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 3,** students will:

A. Data Analysis (4.4.3.A.1-2)
1. Collect, generate, organize, and display data in response to questions, claims, or curiosity.
   - Data collected from the classroom environment
2. Read, interpret, construct, analyze, generate questions about, and draw inferences from displays of data.
   - Pictograph, bar graph, table

B. Probability (4.4.3.B.1-2)
1. Use everyday events and chance devices, such as dice, coins, and unevenly divided spinners, to explore concepts of probability.
   - Likely, unlikely, certain, impossible
   - More likely, less likely, equally likely
2. Predict probabilities in a variety of situations (e.g., given the number of items of each color in a bag, what is the probability that an item picked will have a particular color).
   - What students think will happen (intuitive)
   - Collect data and use that data to predict the probability (experimental)

C. Discrete Mathematics—Systematic Listing and Counting (4.4.3.C.1-2)
1. Represent and classify data according to attributes, such as shape or color, and relationships.
   - Venn diagrams
   - Numerical and alphabetical order
2. Represent all possibilities for a simple counting situation in an organized way and draw conclusions from this representation.
   - Organized lists, charts

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms (4.4.3.D.1-3)
1. Follow, devise, and describe practical sets of directions (e.g., to add two 2-digit numbers).
2. Explore vertex-edge graphs
   - Vertex, edge
   - Path
3. Find the smallest number of colors needed to color a map.

MATHEMATICAL PROCESSES

STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.

Descriptive Statement: The mathematical processes described here highlight ways of acquiring and using the content knowledge and skills delineated in the first four mathematics standards. These mathematical processes will be embedded within specific items contained on the assessment.

Problem Solving. Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

Communication. Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.

Connections. Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

Reasoning. Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

Representations. Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

Technology. Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar

concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

Strands (A, B, C, D, E, F) and associated Cumulative Progress Indicators

At each grade level, with respect to content appropriate for that grade level, students will:

A. Problem Solving (4.5.A.1-5)
1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
   - Open-ended problems
   - Non-routine problems
   - Problems with multiple solutions
   - Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication (4.5.B.1-4)
1. Use communication to organize and clarify their mathematical thinking.
   - Reading and writing
   - Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections (4.5.C.1-6)
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning (4.5.D.1-6)
1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
    *   Counterexamples as a means of disproving conjectures
    *   Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations (4.5.E.1-3)
1. Create and use representations to organize, record, and communicate mathematical ideas.
    *   Concrete representations (e.g., base-ten blocks or algebra tiles)
    *   Pictorial representations (e.g., diagrams, charts, or tables)
    *   Symbolic representations (e.g., a formula)
    *   Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology (4.5.F.1-6)
1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness standard 8.4-D).
3. Use graphing calculators and computer software to investigate properties of function and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the science (cf. science standards).

Directions to the Student

When you are taking this test, remember these important things:

1. Read each question carefully and think about the answer.

2. If you do not know the answer to a question, go on to the next question. You may come back to the skipped question later if you have time.

3. When you see a STOP sign, do **not** turn the page until you are told to do so.

DIRECTIONS:

The next section of the test has 4 multiple-choice questions. You will fill in the circle next to the answer you choose. You may NOT use a calculator.

Sample Multiple-Choice Questions

The sample questions below show you what the questions are like and how to mark your answers.

Example:

1. **Find the exact answer: 110 + 70**

    (A) **18**

    (B) **81**

    ● **180**

    (D) **810**

The correct answer is C. The circle with the C in it has been filled in to show that C is the correct answer.

2. **Estimate 123 + 685. The sum is between which numbers?**

    (A) **400 and 600**

    ● **700 and 900**

    (C) **1,000 and 1,200**

    (D) **1,300 and 1,500**

The correct answer is B. The circle with the B in it has been filled in to show that B is the correct answer.

DIRECTIONS:

Choose the best of the answer choices given for each of the following problems. Fill in the circle next to your choice. You may NOT use a calculator.

1. **Find the exact answer: 800 − 301**

   Ⓐ **599**

   Ⓑ **500**

   Ⓒ **499**

   Ⓓ **401**

2. **Find the exact answer: 942 ÷ 3**

   Ⓐ **214**

   Ⓑ **304**

   Ⓒ **314**

   Ⓓ **642**

3. **Estimate 711 + 497. The sum is between which numbers?**

    Ⓐ   **50 and 400**

    Ⓑ   **450 and 700**

    Ⓒ   **750 and 1,000**

    Ⓓ **1,050 and 1,300**

4. **Estimate 32 × 68. The product is between what numbers?**

    Ⓐ   **18 and 28**

    Ⓑ   **180 and 280**

    Ⓒ   **1,800 and 2,800**

    Ⓓ **18,000 and 28,000**

If you have time, you may review your work in this section only.

STOP

DO NOT GO ON UNTIL YOU ARE TOLD TO DO SO.

Directions for Students

You are allowed to use your calculator for the following multiple-choice and open-ended items. You may also use the ruler and colored shapes.

Sample Multiple-Choice Question

The sample question below will show you what the multiple-choice questions are like and how to mark your answers. For each multiple-choice question, select the best answer and fill in the circle next to your choice. Make sure you fill in the correct circle.

Example:

1. **Mark has a stamp collection. He has 22 stamps from Japan, 34 from Canada, and 17 from Mexico. How many stamps does he have in all?**

   Ⓐ **53**

   Ⓑ **63**

   ● **73**

   Ⓓ **83**

The correct answer is C. The circle with the C in it has been filled in to show that C is the correct answer.

Sample Open-Ended Question

The sample question below will show you what the open-ended questions are like and how to write your answer. You will write or draw your answer in the work area provided. When asked to explain an answer, you may use words, tables, diagrams, or pictures.

Example:

1. **A juice machine charges $0.65 for a can of juice and accepts only nickels, dimes, and quarters. The machine requires exact change.**

   • **What combination of coins could you put in the juice machine to get a can of juice?**

   Show your work or explain your answer.

   Work area for question 1.

   (25¢) (25¢) (10¢) (5¢)

   ## OR

   I used two quarters, one dime, and one nickel.

These are just two examples of the many possible ways to answer open-ended questions. Be sure to answer all parts of the question, show your work, or explain your answer. You may use your calculator, ruler, and colored shapes.
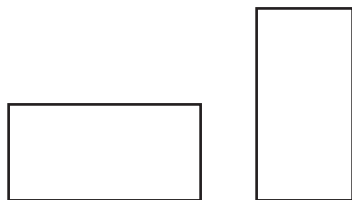
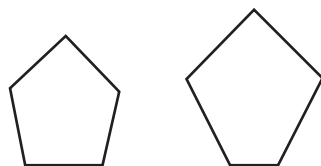5. Which of the following shows a pair of congruent figures?

Ⓐ

Ⓑ

Ⓒ

Ⓓ
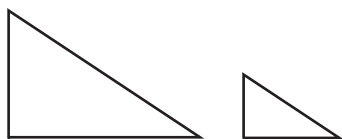
6. **The table shows the average (mean) temperature for several years in Thomas' hometown.**

Year	Average July Temperature
1994	81°F
1995	83°F
1996	83°F
1997	84°F
1998	85°F

By how much did the average (mean) temperature increase from 1995 to 1998?

Ⓐ **2°F**

Ⓑ **3°F**

Ⓒ **4°F**

Ⓓ **5°F**

TURN TO THE NEXT PAGE. ➡

7. **Tisha wrote the following riddle to her friend:**

I have 2 faces, no vertices, and I can roll.

What am I?

What is the answer to the riddle?

Ⓐ **cone**

Ⓑ **cylinder**

Ⓒ **sphere**

Ⓓ **prism**

8. At West Elementary School, there are 20 more girls than boys. If there are 180 girls, how can you find the number of boys?

Ⓐ add 20 to 180

Ⓑ subtract 20 from 180

Ⓒ multiply 180 by 20

Ⓓ divide 180 by 20

TURN TO THE NEXT PAGE. ➡

9.   If 84 ÷ □ = 7, then what is the value of □?

Ⓐ **91**

Ⓑ **77**

Ⓒ **12**

Ⓓ   **7**

10. **Mrs. Barber's class kept track of how many pages they read during the school year. Each star on the pictograph stands for 50 pages.**

Lucas	★ ★ ★ ★ ★ ★ ★ ★ ⬦
Yolanda	★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★
Katie	★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★
Sam	★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★
	★ = 50 pages

How many pages did Lucas read?

Ⓐ **425**

Ⓑ **450**

Ⓒ **475**

Ⓓ **500**

If you have time, you may review your work in this section only.

STOP

**DO NOT GO ON
UNTIL YOU ARE
TOLD TO DO SO.**

Directions for the Open-Ended Question

The following question is an open-ended question. Remember to:

- Read the question carefully and think about the answer.

- Answer all the parts of the question.

- Show your work or explain your answer.

You can answer the question by using words, tables, diagrams, OR pictures. You may use your calculator, ruler, and colored shapes.

TURN TO THE NEXT PAGE.

11. You are trying to save money to buy a present for your friend. You record the total amount of money you have at the end of each week in the chart below.

Week	1	2	3	4	5
Total amount of money saved	$1.75	$3.50	$5.25	$7.00	

- If you continue saving money following this pattern, how much money will you have at the end of Week 5? Explain the pattern you used to get your answer.

- The gift you would like to buy costs $12.00. How many weeks will it take you to save at least that much money? Show your work and explain your answer.

Work area for question 11

More work area for question 11

If you have time, you may review your work in this section only.

STOP

**DO NOT GO ON
UNTIL YOU ARE
TOLD TO DO SO.**

DIRECTIONS:

Choose the best of the answer choices given for each of the following problems. Fill in the circle next to your choice. You may use your calculator, ruler, and colored shapes.

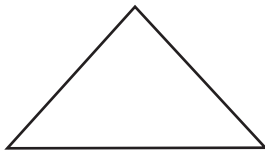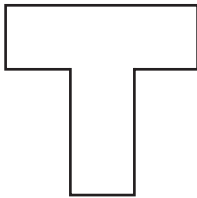**12. Neil made these cutouts for a mobile. Which cutout has more than one line of symmetry?**

Ⓐ

Ⓑ

Ⓒ

Ⓓ

13. **Mrs. Kinney bought batteries in packs of 4 for the students' science experiments. Which of these could be the total number of batteries that she bought?**

(A) **22**

(B) **26**

(C) **28**

(D) **30**

14. **Hiroshi practices a different basketball skill each day, according to this pattern:**

Day 1: Shooting goals   **Day 5:** Shooting goals

Day 2: Dribbling ball   **Day 6:** Dribbling ball

Day 3: Catching rebounds   **Day 7:** Catching rebounds

Day 4: Taking foul shots   **Day 8:** Taking foul shots

   and so on.

What skill will he practice on day 21?

Ⓐ **Shooting goals**

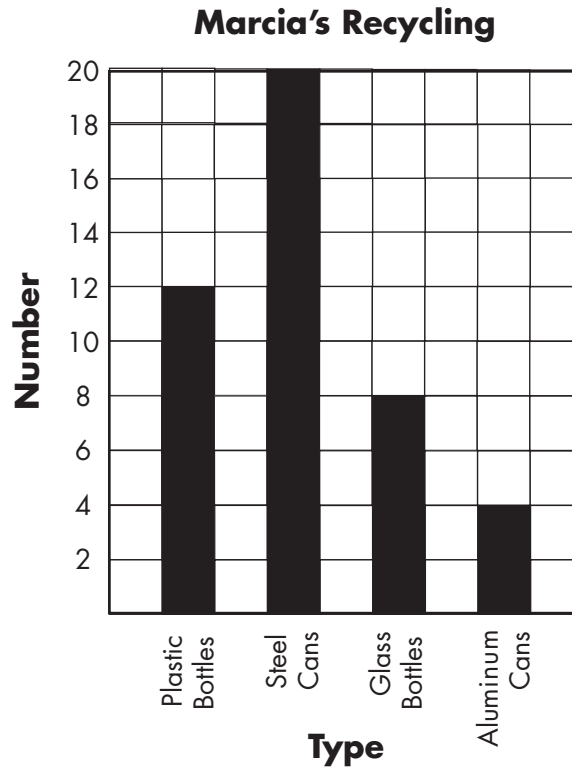Ⓑ **Dribbling ball**

Ⓒ **Catching rebounds**

Ⓓ **Taking foul shots**

15. Which group of numbers is in order from least to greatest?

Ⓐ **0.25** **1.6** **1.0**

Ⓑ **1.0** **0.25** **1.6**

Ⓒ **0.25** **1.0** **1.6**

Ⓓ **1.6** **1.0** **0.25**

16. This graph shows the number of each kind of item Marcia recycled last week.

Marcia's Recycling

How many more plastic bottles than glass bottles did Marcia recycle?

Ⓐ **2**

Ⓑ **4**

Ⓒ **8**

Ⓓ **12**

# Directions for the Open-Ended Questions

The following questions are open-ended questions. Remember to:

- Read each question carefully and think about the answer.

- Answer all the parts of the questions.

- Show your work or explain your answers.

You can answer the questions by using words, tables, diagrams, OR pictures. You may use your calculator, ruler, and colored shapes.

17. **Tonya built this rectangular model using 39 tiles.**
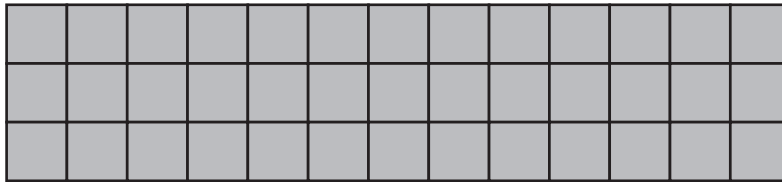
- **List two number sentences this model represents.**

- **Tonya found one more tile. Draw a new rectangular model using all of Tonya's tiles.**

- **List two multiplication number sentences this new model represents.**

Work area for question 17

More work area for question 17

18. **Jennifer has a new kitten. His name is Buddy.**

Buddy

Buddy needs a collar and a bell. Jennifer is looking at collars that come in blue, red, yellow, or green and bells that come in gold or silver.

- **Show all the different combinations of collars and bells that Jennifer can make for Buddy.**

Jennifer also wants a name tag for Buddy. The name tag can be large or small.

- **How many total combinations of collars, bells, and name tags are possible? Show your work or explain your answer.**

Work area for question 18

More work area for question 18

If you have time, you may review your work in this section only.

STOP

DO NOT GO ON UNTIL YOU ARE TOLD TO DO SO.
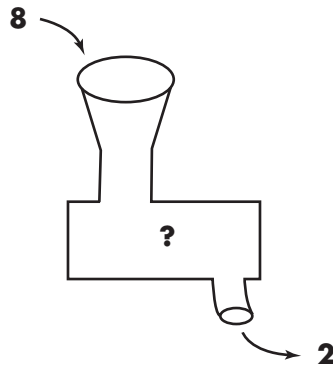
DIRECTIONS:

Choose the best of the answer choices given for each of the following problems. Fill in the circle next to your choice. You may use your calculator, ruler, and colored shapes.

19. What is the most reasonable estimate of the length of a city's swimming pool?

Ⓐ   **1 meter**

Ⓑ **25 meters**

Ⓒ   **1 kilometer**

Ⓓ **25 kilometers**

20. When 8 is dropped into this machine, it comes out as 2.

8

?

2

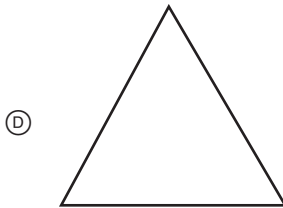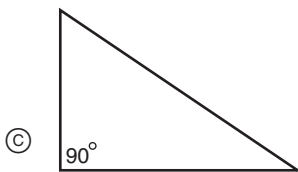The table shows some other input and output data for the machine.
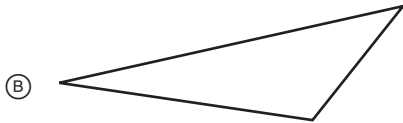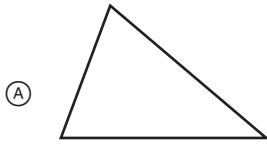
Input	8	10	15	12
Output	2	4	9	

What is the missing number in the table?

Ⓐ **11**

Ⓑ **10**

Ⓒ   **6**

Ⓓ   **5**

21. Which triangle has an angle larger than a right angle?

Ⓐ

Ⓑ

Ⓒ 90°

Ⓓ

22. Students pick cards out of a hat to see what kind of book they need to read for a book report. The hat contains 5 cards with *fantasy* written on them, 2 with *biography* written on them, and 3 with *adventure* written on them. What is the probability a student will pick a card with *fantasy* written on it?

Ⓐ $\dfrac{1}{10}$

Ⓑ $\dfrac{2}{10}$

Ⓒ $\dfrac{3}{10}$

Ⓓ $\dfrac{5}{10}$

23. There are 7 desks arranged in a row in Mr. Thompson's classroom. Hector sits 2 seats to the right of Kim. Tonya sits 3 seats to the right of Hector.

How many seats to the left of Tonya does Kim sit?

Ⓐ **2**

Ⓑ **3**

Ⓒ **5**

Ⓓ **12**

TURN TO THE NEXT PAGE. ➡

DO NOT WRITE IN THIS AREA.

Directions for the Open-Ended Questions

The following questions are open-ended questions. Remember to:

- Read each question carefully and think about the answer.

- Answer all the parts of the questions.

- Show your work or explain your answers.

You can answer the questions by using words, tables, diagrams, OR pictures. You may use your calculator, ruler, and colored shapes.

24. Maria is making apple pies for a party. She bought 3 bags of apples. Each bag has 12 apples. She needs 8 apples to make each pie.

   • What is the greatest number of pies Maria can make? Show your work or explain your answer.

   • How many more bags of apples does Maria need to buy in order to make a total of 6 pies? Show your work or explain your answer.

Work area for question 24

More work area for question 24

　　　TURN TO THE NEXT PAGE.

25. Look at the map below.

Abby left her house and followed this list of directions:

    1. Walk two blocks west.

    2. Walk three blocks north.

    3. Walk four blocks east.

    4. Walk one block north.

• **Use the map to help you list all of the places Abby passed on her walk, including the place where she ended her walk.**

After school, Bryan is going to Abby's house to trade cards. He wants to stop at home first to get his trading cards.

• **Make a list of directions that Bryan can follow to walk from school to his house and then to Abby's house.**

Work area for question 25

If you have time, you may review your work in this section only.

STOP

CLOSE YOUR TEST BOOKLET.

Do not write in this area.

APPENDIX D:

SCORING KEYS

Item #	Correct Answer	Standard, Grade level	Strand	CPI	Mathematical Processes
1	C	1, 4th	B	4	Problem solving
2	C	1, 4th	B	4	Problem solving
3	D	1, 4th	C	2	Problem solving, reasoning
4	C	1, 4th	C	2	Problem solving, reasoning
5	B	2, 4th	A	3	Reasoning, representation
6	A	4, 4th	A	2	Connections, reasoning, representation
7	B	2, 4th	A	2	Reasoning
8	B	1, 4th	B	1	Problem solving, reasoning
9	C	3, 4th	C	2	Problem solving
10	C	4, 4th	A	2	Reasoning, representation
11	rubric	3, 4th	B	1	Problem solving, communication, connection, reasoning, representation
12	A	2, 4th	A	3	Reasoning
13	C	1, 3rd	B	2	Problem solving, reasoning
14	A	3, 4th	A	1	Problem solving, reasoning
15	C	1, 4th	A	6	Reasoning
16	B	4, 3rd	A	2	Connections, reasoning, representation
17	rubric	1, 4th	B	1	Problem solving, communication, reasoning, representation
18	rubric	4, 4th	C	2	Problem solving, communication, connections, reasoning
19	B	2, 4th	D	2	Connections, reasoning
20	C	3, 3rd	B	1	Problem solving, reasoning
21	B	2, 4th	A	4	Reasoning
22	D	4, 4th	B	3	Problem solving, connections
23	C	2, 4th	A	1	Reasoning
24	rubric	1, 4th	B	1	Problem solving, communication, connections, reasoning, representation.
25	rubric	4, 4th	D	1	Problem solving, communication, connections, reasoning, representation

APPENDIX E:

ITEM-SPECIFIC SCORING RUBRICS FOR OPEN-ENDED QUESTIONS

NJ ASK Math Grade 4
Saving Money
Scoring Rubric

<u>3 points</u>
- The student determines the correct amount of money at the end of week five.
- The student explains the pattern used to get his/her answer.
- The student indicates how many weeks to save at least $12.00 (7 weeks).
- The student shows his/her work for finding how many weeks to save at least $12.00.

Sample 3–point response:
- $8.75
- I added $1.75 to $7.00
- 7 weeks
- I continued the chart until I hit $12.00. Week 6 – $10.50, Week 7 – $12.25

<u>2 points</u>
- The student determines the correct amount of money at the end of week five.
- The student does not explain the pattern used to get his/her answer.
- The student indicates how many weeks to save at least $12.00.
- The student shows his/her work for finding how many weeks to save at least $12.00.
OR
- The student determines the correct amount of money at the end of week five.
- The student explains the pattern used to get his/her answer.
- The student indicates how many weeks to save at least $12.00.
- The student does not show his/her work for finding how many weeks to save at least $12.00.
OR
- The student does not determine the correct amount of money at the end of week five.
- The student explains the pattern used for saving money.
- The student indicates how many weeks to save at least $12.00.
- The student shows his/her work for finding how many weeks to save at least $12.00.

<u>1 point</u>
- The student determines the correct amount of money at the end of week five.
- The student explains the pattern used to get his/her answer.
- The student does not correctly answer or fails to answer the second bullet.

NJ ASK Math Grade 4
Saving Money
Scoring Rubric
Continued

1 point–cont'd

OR

- The student determines the correct amount of money at the end of week five.
- The student does not explain the pattern used to get his/her answer.
- The student indicates how many weeks to save at least $12.00.
- The student does not show his/her work for finding how many weeks to save at least $12.00.

OR

- The student does not determine the correct amount of money at the end of week five.
- The student explains the pattern of saving.
- The student indicates how many weeks to save at least $12.00.
- The student does not show his/her work for finding how many weeks to save at least $12.00.

0 points

- The response shows insufficient understanding of the problem's mathematical concepts.

NJ ASK MATH Grade 4
Tonya's Rectangular Model
Scoring Rubric

3 points

The student:
- Lists any two correct number sentences that represent the given model.

  3 x 13 = 39   13 x 3 = 39
  13 + 13 + 13 = 39   3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3+ 3 + 3 + 3 + 3 = 39
- Draws a model when one more block is added to make a new rectangular model of 40.
- Lists two correct multiplication number sentences to represent the new rectangular model drawn.

  1 x 40 = 40   4 x 10 = 40   8 x 5 = 40   20 x 2 = 40
  2 x 20 = 40   5 x 8 = 40   10 x 4 = 40   40 x 1 = 40

Sample 3–point response:
- 3 x 13 = 39   13 x 3 = 39
- Draws a model of four rows of 10 blocks.
- 4 x 10 = 4   10 x 4 = 40

2 points

The student:
- Lists any two correct number sentences for the given model.
- Draws a new rectangular model of 40.
- Lists two correct number sentences with the same product, but one of those is not based on the new model shown.

OR

- Lists one correct number sentence for the given model.
- Draws a new rectangular model of 40.
- Lists two correct multiplication number sentences for the new model.

OR

- The two number sentences of the given model are incorrect or missing.
- Draws a new rectangular model of 40.
- Lists two correct multiplication number sentences for the new model.

OR

- Lists any two correct number sentences for the given model.
- Draws the model to represent a new rectangular model of 40.
- Lists one correct multiplication number sentence for the new model **OR** the two number sentences of the new model are incorrect or missing.

2 points–cont'd

The student:

- Lists one correct number sentence for the given model.
- Draws the model to represent a new rectangular model of 40.
- Lists one correct multiplication number sentence for the new model.

1 point

The student:

- Lists one correct number sentence for the given model.
- Draws the model to represent a new rectangular model of 40.
- The two number sentences of the new model are incorrect or missing.

OR

- The two number sentences of the given model are incorrect or missing.
- Draws the model to represent a new rectangular model of 40.
- Lists one correct number sentence to represent the new model.

OR

- Lists any two correct number sentences for the given model.

0 points

- The student shows insufficient understanding of the problem's essential mathematical concepts by having only one, a partial, or no elements correct.

NJ ASK MATH Grade 4
Buddy's Collar
Scoring Rubric

3 points

The student:
- Correctly shows a tree diagram or an organized list with a total of 8 correct combinations for the two items (collar and bell).
- Correctly states that there are 16 total combinations for the three items (collar, bell, and name tag).
- Correctly shows work (a tree diagram, mathematical solution, or an organized list) or explanation for the 16 combinations.

Sample 3–point response:
- There are 8 (4 collars ✗ 2 bells = 8 combinations) possible combinations of collars and bells:
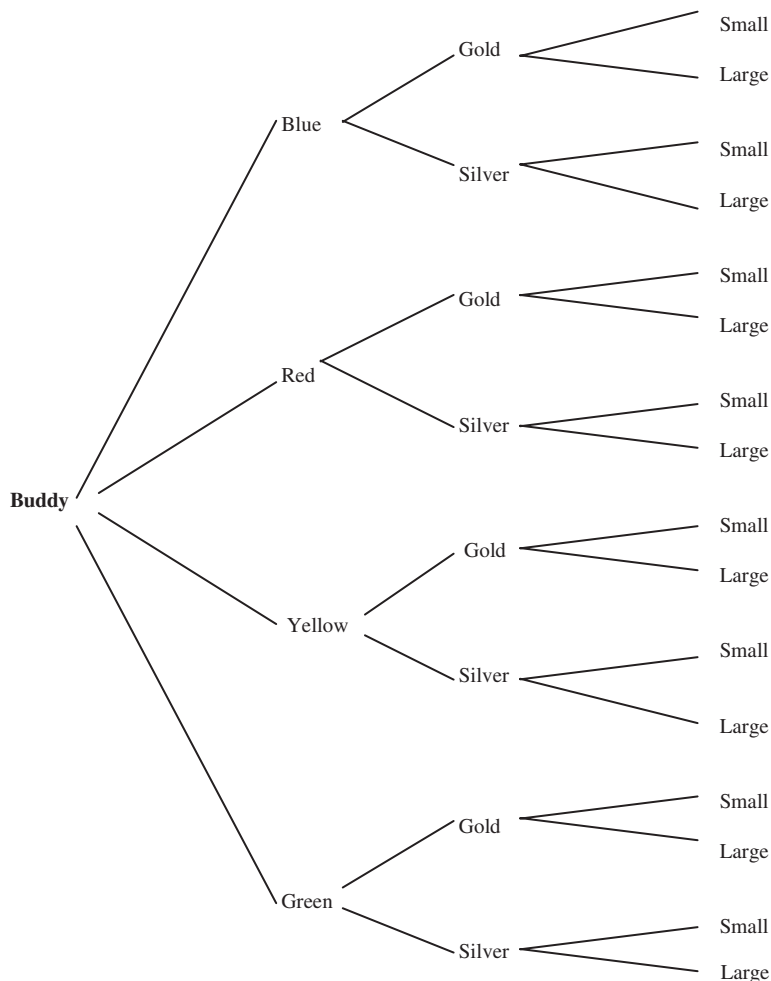
Blue & Gold	Red & Gold	Yellow & Gold	Green & Gold
Blue & Silver	Red & Silver	Yellow & Silver	Green & Silver

- There are 16 different combinations of the 3 items.
- 4 collars ✗ 2 bells ✗ 2 name tags = 16 combinations.

NJ ASK MATH Grade 4
Buddy's Collar
Scoring Rubric
Continued

2 points

The student:

- Shows a diagram or an organized list with a total of 8 correct combinations of collars and bells.
- Correctly notes that there are 16 different combinations for the collars, bells, and name tags.
- Does not provide an appropriate explanation or work.

OR

- Shows a diagram or an organized list that includes a minor error in the total number of combinations (7 combinations).
- States the number of combinations for the collars, bells, and name tags based on that error.
- Correctly shows or explains the number of combinations for three items that is consistent with the previous error made.

OR

- Shows a diagram or an organized list with the total number of 8 correct combinations.
- Does not state that there are 16 combinations for the collars, bells, and name tags, **but**
- Correctly shows or explains the 16 combinations for the three items.

OR

- Correctly notes that there are 16 different combinations for the collars, bells, and name tags.
- Correctly shows or explains the 16 combinations for the three items.

1 point

The student:

- States the total number of 8 correct combinations for bullet 1.
- Incorrectly states the number of combinations for the collars, bells, and name tags.
- Provides a diagram or an organized list that leads to an incorrect total number of combinations for the three items.

OR

- Shows only a tree diagram or organized list with a total of 8 correct combinations for bullet 1.

OR

- States the total number of 8 combinations for bullet 1 (no work).
- States the total number of 16 combinations for bullet 2 (no work).

0 points

- The student shows insufficient understanding of the problem's essential mathematical concepts.

NJ ASK MATH Grade 4
Maria's Apple Pies
Scoring Rubric

3 points
The student:
- Shows or explains that Maria can make a total of four pies with 36 apples with four apples left over; **and**
- Explains or shows work ($3 \times 12 = 36$, $36 \div 8 = 4$ R4).
- Shows or explains that Maria will need to buy one more bag of apples to make a total of six pies; **and**
- Gives an appropriate explanation or shows work (must use12); for example, $6 \times 8 = 48$, $48 \div 12 = 4$ bags. She has 3 bags and 4 apples left over. $12 + 4 = 16$, which is how many apples she needs to make two more pies which will then equal a total of six pies, so she needs one more bag.

Sample 3–point response:
- 4 pies with 4 apples left over.
- $3 \times 12 = 36$, $36 \div 8 = 4$ R4
- She will need to buy one more bag **because**
- If she buys one more bag, she will have a total of 16 apples ($12 + 4 = 16$) and $16 \div 8 = 2$ pies. Two pies added with four pies make a total of six pies. Maria needs to buy only one more bag to make a total of six pies.

2 points
The student:
- Shows or explains that Maria can make a total of four pies with 36 apples; **and**
- Explains or shows work ($3 \times 12 = 36$, $36 \div 8 = 4$ R4).
- Indicates that Maria will need to buy one more bag of apples to make a total of six pies, **but**
- Has a limited or no explanation.

<div align="center">

OR

</div>

- Shows or explains that Maria can make a total of four pies with 36 apples; **and**
- Explains or shows work ($3 \times 12 = 36$, $36 \div 8 = 4$ R4).
- Does not specifically state that Maria must buy one more bag, **but**
- Clearly shows with computations that Maria will need to buy one more bag of apples to make a total of six pies.

2 points – cont'd

OR

- Shows or explains that Maria can make a total of four pies with 36 apples; **but**
- Does not explain or show work (3 x 12 = 36, 36 ÷ 8 = 4 R4).
- Shows or explains that Maria will need to buy one more bag to make a total of six pies, **and**
- Gives an appropriate explanation or shows work (must use 12).

1 point
The student:
- Shows or explains that Maria can make a total of four pies with 36 apples; **and**
- Explains or shows work (3 x 12 = 36, 36 ÷ 8 = 4 R4).
- Does not state that Maria must buy one more bag.
- Has no explanation.

OR

- Does not show or explain that Maria can make a total of four pies.
- Does not explain or show work (3 x 12 = 36, 36 ÷ 8 = 4 R4).
- Shows or explains that Maria will need to buy one more bag of apples to make a total of six pies; **and**
- Gives an appropriate explanation or shows work (must use 12).

OR

- Shows or explains that Maria can make a total of four pies with 36 apples; **but**
- Explains or shows work with one minor computation error; **and**
- Shows or explains the correct number of bags to buy based on the computation error made.

OR

- Shows or explains that Maria can make a total of four pies, **and**
- Indicates that Maria will need to buy one more bag of apples.

0 points
- The response shows insufficient understanding of the problem's essential mathematical concepts. The response is incomplete, inaccurate, and contains major errors.

NJ ASK MATH Grade 4
Abby's House
Scoring Rubric

3 points
The student:
- Correctly lists the places Abby passed on her walk, including the place where she ended.
- Correctly writes directions for Bryan to follow from school to his house.
- Correctly writes directions for Bryan to follow from his house to Abby's house.

Sample 3–point response:
- hospital, post office, school
- walk one block north and three blocks west (other routes are possible)
- walk five blocks south and one block east (other routes are possible)

2 points
The student:
- lists the hospital, post office, and school.
- has a minor error in the directions for the second bullet.

OR

- incorrectly lists one or more of the places passed.
- answers the second bullet correctly and completely.

1 point
The student:
- lists the hospital, post office, and school.
- does not answer the second bullet correctly.

OR

- shows limited understanding of the problem and may attempt to follow and give directions; however, the response contains major errors or omissions.

0 points
- The response shows insufficient understanding of the problem's essential mathematical concepts.

APPENDIX F:

DIRECTORY OF TEST SPECIFICATIONS FOR GRADE 4

Standards and Strands

4.1. Number and Numerical Operations
A. Number Sense
B. Numerical Operations
C. Estimation

4.2. Geometry and Measurement
A. Geometric Properties
B. Transforming Shapes
C. Coordinate Geometry
D. Units of Measurement
E. Measuring Geometric Objects

4.3. Patterns and Algebra
A. Patterns and Relationships
B. Functions
C. Modeling
D. Procedures

4.4. Data Analysis, Probability, and Discrete Mathematics
A. Data Analysis (Statistics)
B. Probability
C. Discrete Mathematics--Systematic Listing and Counting
D. Discrete Mathematics--Vertex-Edge Graphs and Algorithms

4.5 Mathematical Processes
A. Problem Solving
B. Communication
C. Connections
D. Reasoning
E. Representation
F. Technology

MATHEMATICS TEST SPECIFICATIONS

> **STANDARD 4.1   (NUMBER AND NUMERICAL OPERATIONS)   ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.**

Descriptive Statement: Numbers and arithmetic operations are what most of the general public think about when they think of mathematics; and, even though other areas like geometry, algebra, and data analysis have become increasingly important in recent years, numbers and operations remain at the heart of mathematical teaching and learning. Facility with numbers, the ability to choose the appropriate types of numbers and the appropriate operations for a given situation, and the ability to perform those operations as well as to estimate their results, are all skills that are essential for modern day life.

Number Sense. Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.

Numerical Operations. Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

Estimation. Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

Number and operation skills continue to be a critical piece of the school mathematics curriculum and, indeed, a very important part of mathematics. But, there is perhaps a greater need for us to rethink our approach here than to do so for any other curriculum component. An enlightened mathematics program for today's children will empower them to use all of today's tools rather than require them to meet yesterday's expectations.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 4,** students will:

A. Number Sense (4.1.4.A.1-7)
1. Use real-life experiences, physical materials, and technology to construct meanings for numbers (**unless otherwise noted, all indicators for grade 4 pertain to these sets of numbers as well**).
   - Whole numbers through millions
   - Commonly used fractions (denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 16) as part of a whole, as a subset of a set, and as a location on a number line
   - Decimals through hundredths
2. Demonstrate an understanding of place value concepts.
3. Demonstrate a sense of the relative magnitudes of numbers.
4. Understand the various uses of numbers.
   - Counting, measuring, labeling (e.g., numbers on baseball uniforms), locating (e.g., Room 235 is on the second floor)
5. Use concrete and pictorial models to relate whole numbers, commonly used fractions, and decimals to each other, and to represent equivalent forms of the same number.
6. Compare and order numbers.
7. Explore settings that give rise to negative numbers.
   - Temperatures below $0°$, debts
   - Extension of the number line

B. Numerical Operations (4.1.4.B.1-10)
1. Develop the meanings of the four basic arithmetic operations by modeling and discussing a large variety of problems.
   - Addition and subtraction: joining, separating, comparing
   - Multiplication: repeated addition, area/array
   - Division: repeated subtraction, sharing
2. Develop proficiency with basic multiplication and division number facts using a variety of fact strategies (such as "skip counting" and "repeated subtraction") and then commit them to memory.
3. Construct, use, and explain procedures for performing whole number calculations and with:
   - Pencil-and-paper
   - Mental math
   - Calculator
4. Use efficient and accurate pencil-and-paper procedures for computation with whole numbers.
   - Addition of 3-digit numbers
   - Subtraction of 3-digit numbers
   - Multiplication of 2-digit numbers
   - Division of 3-digit numbers by 1-digit numbers
5. Construct and use procedures for performing decimal addition and subtraction.
6. Count and perform simple computations with money.
   - Standard dollars and cents notation
7. Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.
8. Check the reasonableness of results of computations.
9. Use concrete models to explore addition and subtraction with fractions.

10. Understand and use the inverse relationships between addition and subtraction and between multiplication and division.

C. Estimation (4.1.4.C.1-4)
   1. **Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set.**
   2. Construct and use a variety of estimation strategies (e.g., rounding and mental math) for estimating both quantities and the results of computations.
   3. Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.
   4. Use estimation to determine whether the result of a computation (either by calculator or by hand) is reasonable.

STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.

Descriptive Statement: Spatial sense is an intuitive feel for shape and space. Geometry and measurement both involve describing the shapes we see all around us in art, nature, and the things we make. Spatial sense, geometric modeling, and measurement can help us to describe and interpret our physical environment and to solve problems.

Geometric Properties. This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.

Transforming Shapes. Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.

Coordinate Geometry. Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

Units of Measurement. Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

Measuring Geometric Objects. This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important that they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.

Students of all ages should realize that geometry and measurement is all around them. Through study of these areas and their applications, they should come to better understand and appreciate the role of mathematics in their lives.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 4,** students will:

A. Geometric Properties (4.2.4.A.1-5)
1. Identify and describe spatial relationships of two or more objects in space.
   - Direction, orientation, and perspectives (e.g., which object is on your left when you are standing here?)
   - Relative shapes and sizes
   - Shadows (projections) of everyday objects
2. Use properties of standard three-dimensional and two-dimensional shapes to identify, classify, and describe them.
   - Vertex, edge, face, side, angle
   - 3D figures – cube, rectangular prism, sphere, cone, cylinder, and pyramid
   - 2D figures – square, rectangle, circle, triangle, quadrilateral, pentagon, hexagon, octagon
   - Inclusive relationships – squares are rectangles, cubes are rectangular prisms
3. Identify and describe relationships among two-dimensional shapes.
   - Congruence
   - Lines of symmetry
4. Understand and apply concepts involving lines, angles, and circles.
   - Point, line, line segment, endpoint
   - Parallel, perpendicular
   - Angles – acute, right, obtuse
   - Circles – diameter, radius, center
5. Recognize, describe, extend, and create space-filling patterns.

B. Transforming Shapes (4.2.4.B.1-3)
1. Use simple shapes to cover an area (tessellations).
2. Describe and use geometric transformations (slide, flip, turn).
3. Investigate the occurrence of geometry in nature and art.

C. Coordinate Geometry (4.2.4.C.1-2)
1. Locate and name points in the first quadrant on a coordinate grid.
2. Use coordinates to give or follow directions from one point to another on a map or grid.

D. Units of Measurement (4.2.4.D.1-5)
1. Understand that everyday objects have a variety of attributes, each of which can be measured in many ways.
2. Select and use appropriate standard units of measure and measurement tools to solve real-life problems
   - Length – fractions of an inch (1/8, 1/4, 1/2), mile, decimeter, kilometer
   - Area – square inch, square centimeter
   - Volume – cubic inch, cubic centimeter
   - Weight – ounce
   - Capacity – fluid ounce, cup, gallon, milliliter
3. Develop and use personal referents to approximate standard units of measure (e.g., a common paper clip is about an inch long).
4. Incorporate estimation in measurement activities (e.g., estimate before measuring).
5. Solve problems involving elapsed time.

E. Measuring Geometric Objects (4.2.4.E.1-3)
1.   Determine the area of simple two-dimensional shapes on a square grid.
2.   Distinguish between perimeter and area and use each appropriately in problem-solving situations.
3.   Measure and compare the volume of three–dimensional objects using materials such as rice or cubes.

STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.

Descriptive Statement: Algebra is a symbolic language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

Patterns. Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.

Functions and Relationships. The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

Modeling. Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

Procedures. Techniques for manipulating algebraic expressions – procedures – remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

Algebra is a gatekeeper for the future study of mathematics, science, the social sciences, business, and a host of other areas. In the past, algebra has served as a filter, screening people out of these opportunities. For New Jersey to be part of the global society, it is important that algebra play a major role in a mathematics program that opens the gates for all students.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 4,** students will:

A. Patterns (4.3.4.A.1)
  1. Recognize, describe, extend, and create patterns.
   - Descriptions using words, number sentences/expressions, graphs, tables, variables (e.g., shape, blank, or letter)
   - Sequences that stop or that continue infinitely
   - Whole number patterns that grow or shrink as a result of repeatedly adding, subtracting, multiplying by, or dividing by a fixed number (e.g., 5, 8, 11, . . . or 800, 400, 200, . . .)
   - Sequences can often be extended in more than one way (e.g., the next term after 1, 2, 4, . . . could be 8, or 7, or …)

B. Functions and Relationships (4.3.4.B.1)
  1. Use concrete and pictorial models to explore the basic concept of a function.
   - Input/output tables, T-charts
   - Combining two function machines
   - Reversing a function machine

C. Modeling (4.3.4.C.1-2)
  1. Recognize and describe change in quantities.
   - Graphs representing change over time (e.g., temperature, height)
   - How change in one physical quantity can produce a corresponding change in another (e.g., pitch of a sound depends on the rate of vibration)
  2. Construct and solve simple open sentences involving any one operation (e.g., $3 \times 6 =$ __, $n = 15 \div 3$, $3 \times$ __ $= 0$, $16 - c = 7$).

D. Procedures (4.3.4.D.1-2)
  1. Understand, name, and apply the properties of operations and numbers.
   - Commutative (e.g., $3 \times 7 = 7 \times 3$)
   - Identity element for multiplication is 1 (e.g., $1 \times 8 = 8$)
   - Associative (e.g., $2 \times 4 \times 25$ can be found by first multiplying either 2×4 or 4×25)
   - Division by zero is undefined
   - Any number multiplied by zero is zero.
  2. Understand and use the concepts of equals, less than, and greater than in simple number sentences.
   - Symbols $(=, <, >)$

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS) ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.

Descriptive Statement: Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

Data Analysis (or Statistics). In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

Probability. Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

Discrete Mathematics—Systematic Listing and Counting. Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

Discrete Mathematics—Vertex-Edge Graphs and Algorithms. Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

These topics provide students with insight into how mathematics is used by decision-makers in our society, and with important tools for modeling a variety of real-world situations. Students will better understand and interpret the vast amounts of quantitative data that they are exposed to daily, and they will be able to judge the validity of data-supported arguments.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 4,** students will:

A. Data Analysis (4.4.4.A.1-2)
1. Collect, generate, organize, and display data in response to questions, claims, or curiosity.
   - Data collected from the school environment
2. Read, interpret, construct, analyze, generate questions about, and draw inferences from displays of data.
   - Pictograph, bar graph, line plot, line graph, table
   - Average (mean), most frequent (mode), middle term (median)

B. Probability (4.4.4.B.1-3)
1. Use everyday events and chance devices, such as dice, coins, and unevenly divided spinners, to explore concepts of probability.
   - Likely, unlikely, certain, impossible, improbable, fair, unfair
   - More likely, less likely, equally likely
   - Probability of tossing "heads" does not depend on outcomes of previous tosses
2. Determine probabilities of simple events based on equally likely outcomes and express them as fractions.
3. Predict probabilities in a variety of situations (e.g., given the number of items of each color in a bag, what is the probability that an item picked will have a particular color).
   - What students think will happen (intuitive)
   - Collect data and use that data to predict the probability (experimental)
   - Analyze all possible outcomes to find the probability (theoretical)

C. Discrete Mathematics—Systematic Listing and Counting (4.4.4.C.1-2)
1. Represent and classify data according to attributes, such as shape or color, and relationships.
   - Venn diagrams
   - Numerical and alphabetical order
2. Represent all possibilities for a simple counting situation in an organized way and draw conclusions from this representation.
   - Organized lists, charts, tree diagrams
   - Dividing into categories (e.g., to find the total number of rectangles in a grid, find the number of rectangles of each size and add the results)

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms (4.4.4.D.1-4)
1. Follow, devise, and describe practical sets of directions (e.g., to add two 2-digit numbers).
2. Play two-person games and devise strategies for winning the games (e.g., "make 5" where players alternately add 1 or 2 and the person who reaches 5, or another designated number, is the winner).
3. Explore vertex-edge graphs and tree diagrams.
   - Vertex, edge, neighboring/adjacent, number of neighbors
   - Path, circuit (i.e., path that ends at its starting point)
4. Find the smallest number of colors needed to color a map or a graph.

MATHEMATICAL PROCESSES

> **STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.**

Descriptive Statement: The mathematical processes described here highlight ways of acquiring and using the content knowledge and skills delineated in the first four mathematics standards. These mathematical processes will be embedded within specific items contained on the assessment.

Problem Solving. Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

Communication. Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.

Connections. Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

Reasoning. Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

Representations. Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

Technology. Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of

mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

Strands (A, B, C, D, E, F) and associated Cumulative Progress Indicators

At each grade level, with respect to content appropriate for that grade level, students will:

A. Problem Solving (4.5.A.1-5)
    1.   Learn mathematics through problem solving, inquiry, and discovery.
    2.   Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
- Open-ended problems
- Non-routine problems
- Problems with multiple solutions
- Problems that can be solved in several ways
    3.   Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.
    4.   Pose problems of various types and levels of difficulty.
    5.   Monitor their progress and reflect on the process of their problem solving activity.

B. Communication (4.5.B.1-4)
    1.   Use communication to organize and clarify their mathematical thinking.
- Reading and writing
- Discussion, listening, and questioning
    2.   Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
    3.   Analyze and evaluate the mathematical thinking and strategies of others.
    4.   Use the language of mathematics to express mathematical ideas precisely.

C. Connections (4.5.C.1-6)
    1.   Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
    2.   Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
    3.   Recognize that mathematics is used in a variety of contexts outside of mathematics.
    4.   Apply mathematics in practical situations and in other disciplines.
    5.   Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
    6.   Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning (4.5.D.1-6)
1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
   - Counterexamples as a means of disproving conjectures
   - Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations (4.5.E.1-3)
1. Create and use representations to organize, record, and communicate mathematical ideas.
   - Concrete representations (e.g., base-ten blocks or algebra tiles)
   - Pictorial representations (e.g., diagrams, charts, or tables)
   - Symbolic representations (e.g., a formula)
   - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology (4.5.F.1-6)
1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness standard 8.4-D).
3. Use graphing calculators and computer software to investigate properties of function and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the science (cf. science standards).

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