

Revenue-Sharing vs. Wholesale-Price Contracts in Assembly Systems with Random Demand[†]

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Abstract: Assembly and kitting operations, as well as jointly sold products, are rather basic yet intriguing decentralized supply chains, where achieving coordination through appropriate incentives is very important, especially when demand is uncertain. We investigate two very distinct types of arrangements between an assembly firm/retailer and its suppliers. One scheme is a vendor managed inventory with revenue sharing, and the other wholesale-price driven contract. In the VMI case, each supplier faces strategic uncertainty as to the amounts of components, which need to be mated with its own, other suppliers will deliver. We explore the resulting components' delivery quantities equilibrium in this simple decentralized supply chain and their implication for participants' and system's expected profits. We derive the revenue shares the assembly firm should select in order to maximize its own profits. We then explore a revenue-plus-surplus-subsidy incentive scheme, where in addition to a share of revenue, the assembly firm also provides a subsidy to component suppliers for their unsold components. We show that by using this two-parameter contract the assembly firm can easily achieve channel coordination and increase the profits of all parties involved. We then explore a wholesale-price-driven scheme, both as a single lever and in combination with buybacks. The channel performance of a wholesale-price-only scheme is shown to degrade with the number of suppliers, which is not the case with a revenue-shares-only contract.

1. Introduction

The sales and profits of suppliers often depend on delivery quantities and timing of other suppliers of highly complementary components or products, as well as on realized demand. In assembly systems (e.g., personal computers) complete sets of components, supplied by various manufacturers, are needed to put units together. In distribution centers, packaging material is

needed in proportion to the amount of hardware packaged, and is typically purchased from a different supplier. At the retail level some products that are almost always purchased jointly (e.g., solder and flux in plumbing stores, marscapone cream and savoirdi biscuit, used to make Tiramisú, in food stores) are produced and delivered to stores by different firms. Depending on the financial arrangements or incentive system, a supplier operating in such an environment may then base its own production/delivery decisions on its anticipation of how much suppliers of complementary components/items will deliver, as well as its forecast of demand.

Such decentralized supply chain gives rise to interesting strategic inter-supplier considerations. Moreover, anticipating the suppliers' behavior, the assembly firm or retailer will choose an incentive scheme which will maximize its own expected profits. While in recent years operations management researchers have been stressing the importance of coordination mechanisms in decentralized supply chains (e.g., Cachon and Lariviere 1999, Cachon 1999, Lariviere 1999, Corbett and Tang 1999, Moynadeh and Bassok 1998, Tsay et al 1999 and references therein), a decentralized *assembly* system of the type described above, has not yet been explored. As stated, for example, by Cachon and Lariviere 1999: "In the GM example above, we are concerned only with ashtrays and not body panels or drive trains". One exception is a recent study by Gurnani and Gerchak 1998, who addressed a scenario with random component production yields but known demand. It is the purpose of this paper to initiate an investigation of coordination in decentralized assembly/joint-purchase systems with random common-belief demand.

Some behavioral economists have investigated "minimum effort" or "weakest link" games, where the payoff of each player equals the efforts of the player that chooses the least

amount of effort (Cachon and Camerer 1996 and references therein). In these games, however, everyone, *ceteris paribus*, prefers a higher minimum effort, while in our setting each player's individually-optimal effort level is its preferred system effort level. Also, in these “minimum effort” settings the only uncertainty is strategic, while in ours there is also an environmental uncertainty (demand volume).

Economists, marketing researchers and recently operations management researchers, have identified and explored the consequences of several main types of incentive mechanisms which one might consider in order to coordinate a decentralized supply chain. These are profit sharing (Atkinson, 1979, Jeuland and Shugan 1983), consignment (Kandel 1996), buy-backs (Pasternack 1985, Cachon 1999, Lariviere 1999) and quantity-flexibility (Tsay and Lovejoy 1998). Vendor Managed Inventory (VMI) is also commonly used (e.g., Clark and Hammond 1997, Cachon and Fisher 1997, Cachon 1997, Narayan and Raman 1997).

We explore and compare two types of settings. One is a VMI system with revenue sharing led from downstream, and the other a wholesale-price-based system led from upstream. The VMI system is one where suppliers choose how much to deliver, and are paid only for units (of assembled/combined product) sold. Thus here it is the retailer who sets the parameters (revenue shares), and the suppliers then decide what to do. Cachon and Lariviere (1999) analyze contracts of this type, in the case of a single supplier. VMI systems of this type are common in retail settings (e.g., with dairy products), and seem to be used even in cases where complementary products are delivered by different suppliers. The above mentioned ingredients of Tiramisú are a case in point. We note that there also exist revenue-sharing contracts where the shares are selected by the supplier, while the retailer then selects the quantity (Pasternack 1999,

Cachon and Lariviere 2000). Ours is a revenue-sharing scheme led by the downstream player (the retailer), accompanied by a VMI quantity choice. At first thought such an arrangement does not seem effective, and we initially explored it primarily since it has been observed in practice. The conclusions of how it will perform with informed and rational suppliers are, however, quite surprising.

The basic system we consider has component suppliers who, when choosing how much to produce, trade-off their production costs against the revenue, which is uncertain since: i) the number of kits assembled, and hence everyone's revenue, is constrained by the supplier who delivered the least; ii) demand is random. Each supplier knows the production costs and revenue share of others, and they all have the same probabilistic beliefs about demand (referred to as "full information" by Cachon and Lariviere 1999; we prefer "common beliefs"). It turns out that the decentralized Nash equilibrium will equal the smallest component lotsize, determined by the solution of an appropriate newsvendor problem.

We consider the selection of the best revenue shares from the retailer's/assembly firm's point of view. We show that they will be such that all suppliers' independent lot sizes would be equal. We further show that such an optimal quantity exists, uniquely, for almost all type of demand distributions.

In general, the above incentive scheme based on revenue share alone can not coordinate the decentralized assembly system. That is, the final quantity delivered and assembled, which is determined by the revenue shares set up by the assembly firm to maximize its own profit, is in general not equal to the quantity which would optimize the entire/centralized system. To achieve coordination, we then propose a revenue-plus-surplus-subsidy scheme where, in addition to a

share of revenue, a supplier is partially paid by the assembly firm for its delivered components that are not sold. Surplus subsidies transfer some of the demand risk from the suppliers to the assembler. In that sense, they bear economic similarity to buybacks (to be discussed shortly) which transfer risk from retailer to manufacturer in wholesale-price-based contracts (Pasternack 1985, Lariviere 1999). We show that, in such environment, there exists a continuum of supplier-specific two-parameter contracts for the assembly firm to choose from to coordinate the decentralized system. We further demonstrate that each such coordinating contract simply corresponds to a different amount of profit (out of the *maximum* total channel profit) allocated to the supplier. Thus, in addition to achieving channel coordination, the assembly firm can also easily make sure that some or all parties involved improve their benefits (in terms of total profits), and no one is worse off (compared with any situation where coordination was not achieved). Additional aspects and implications of this scheme are analyzed in Gerchak and Wang (1999).

Since VMI with revenue shares does not *a priori* seem as the most natural policy for assembly systems or complementary products, we also analyze a system with a more “conventional” wholesale-price-based contract. That system, a generalization of Lariviere and Porteus (1999) “selling to a newsvendor” model, works as follows. First, the n suppliers, simultaneously, choose their individual component wholesale prices. The assembler then chooses the quantity to order from all suppliers. We prove the existence, and in the case of identical suppliers also uniqueness, of a Nash equilibrium. For a given total production cost, we show that the production quantity and channel profit are decreasing in the number of suppliers, which is not the case for the revenue-sharing contract.

For a particular demand distribution, we provide an example for which the VMI with revenue shares system *dominates* the wholesale-price-based system even for a single supplier, and increasingly so for more suppliers.

As is well known from the single-supplier literature, a wholesale price alone cannot coordinate the channel (e.g., Lariviere 1999). The natural additional financial lever to supplement it for achieving coordination are *buybacks* (returns). Unsold units are returned to the supplier for a pre-arranged price (Pasternack 1985, Lariviere 1999). We thus endow our n suppliers wholesale-price-driven model with these additional supplier-specific levers, analyze the resulting model, and show that the channel can be coordinated.

The rest of the paper is organized as follows. We introduce the basic model and its centralized solution in Section 2. Section 3 discusses a decentralized system with VMI revenue-sharing contract. At first, only revenue shares are used, and exemplified. We then introduce the revenue-plus-surplus-subsidy incentive scheme and discuss its channel coordination property. Section 4 analyzes a wholesale-price-based contract. Initially, only wholesale prices are used. Then buybacks are added as a second lever. An example is given. Some concluding remarks comparing the effectiveness of the two schemes and their informational requirements are provided in Section 5.

2. The Centralized System

A final product faces a random demand D , with common-belief CDF F and PDF f . Unit revenue from selling the product is scaled to equal one. The product consists of n

components, or sets thereof (without loss of generality one unit of each) produced by independent suppliers. The components' unit production costs are c_i , $i=1, \dots, n$, and the assembly firm incurs a unit cost of c_0 mating the components together (in a retail setting it is likely that $c_0 = 0$). The decision variables are the suppliers' components production/delivery quantities Q_i , $i=1, \dots, n$, and the firm's assembly quantity Q_0 . All decisions have to be made before the demand is realized. For simplicity, assume there are no holding costs or salvage value for unsold products or components.

If the system were centralized, the firm would want to maximize the expected system-wide revenue. Clearly, one should set $Q_1 = \dots = Q_n = Q_0 \equiv Q_c$, since any unmated or unassembled components will be wasted. The choice of optimal Q_c would then be a simple newsvendor problem. The expected profit is:

$$\begin{aligned}\pi(Q_c) &= E\{-(\sum_{i=0}^n c_i)Q_c + \min(Q_c, D)\} \\ &= -(\sum_{i=0}^n c_i)Q_c + \int_0^{Q_c} xf(x)dx + Q_c \bar{F}(Q_c),\end{aligned}\tag{1}$$

where $\bar{F} = 1 - F$. Assuming that $\sum_{i=0}^n c_i < 1$ (i.e., that costs do not exceed revenue), then since the expected profit function in (1) is concave, the optimal production quantity Q_c^* for the centralized system satisfies the first-order condition

$$\bar{F}(Q_c^*) = \sum_{i=0}^n c_i.\tag{2}$$

Substituting $Q_c = Q_c^*$ into (1), we can show that the optimal system-wide expected profit is given by

$$\pi(Q_c^*) = \int_0^{Q_c^*} xf(x)dx, \quad (3)$$

Both the production quantity Q_c^* and profit $\pi(Q_c^*)$ are decreasing in the total unit production costs $\sum_{i=0}^n c_i$, as expected.

We now consider decentralized systems.

3. Decentralized System with Revenue-Sharing Contracts

Assume now that the component lot sizes Q_i , $i = 1, \dots, n$, and the assemble quantity Q_0 are chosen by individual suppliers and the assembly firm respectively. All parties (the suppliers and the firm) are assumed to have the same beliefs concerning the demand distribution (i.e., “common knowledge” is assumed), and they also know each other’s production costs.

3.1 Basic Revenue-Sharing Contract

A basic revenue-share contract specifies that, *for each unit of final product sold*, the assembly firm pays Supplier i (S_i) α_i , $i = 1, \dots, n$, $0 < \alpha_i < 1$, out of the \$1 total revenue. Thus, the firm keeps $\alpha_0 = 1 - \sum_{i=1}^n \alpha_i$ for itself. This revenue sharing scheme is known to the

suppliers. Clearly, a necessary condition for each party to stay in business, is

$$\alpha_i > c_i, \quad i = 0, 1, \dots, n. \quad (4)$$

For a such a given revenue-sharing scheme, if a party's final revenue did not depend on others' deliveries or the firm's assembly decision, it would solve its own newsvendor problem, i.e., would produce

$$\bar{F}(Q_i^*) = c_i / \alpha_i, \quad i = 0, 1, \dots, n. \quad (5)$$

Suppose WLOG that $c_1 / \alpha_1 = \max_i \{c_i / \alpha_i\}$. Then, $Q_1^* = \min_i Q_i^*$. We then refer to S_1 as the “critical supplier”.

Returning to a situation with inter-dependent revenues (and decisions), we now argue that, at equilibrium, all suppliers will deliver, and the firm will assemble, no more than Q_1^* . The reason: Q_1^* is the optimal amount for S_1 , who thus clearly does not want to deliver more. The other suppliers (the firm) would have liked to deliver (assemble) more than Q_1^* if they had a chance to be paid for these extra units; but since profit is a function of the number of complete and assembled kits, there is no benefit for the other suppliers to deliver more than Q_1^* , and it is thus infeasible for the firm to assemble more than Q_1^* . Any amount in $[0, Q_1^*]$ is here a Nash equilibrium and Q_1^* is the one among them which maximizes all parties' profits (i.e., a Pareto-optimal point). To summarize

Proposition 1

If $c_1/\alpha_1 = \max_i \{c_i/\alpha_i\}$, all points in $[0, Q_1^]$ are Nash equilibria, and $Q_d^* = Q_1^*$ is the Pareto-optimal among them.* ||

Note that this result does not really depend on whether choices are simultaneous or sequential. It also does not depend on the type of incentive system; it is a direct consequence of $Q_1^* = \min_i Q_i^*$.

We observe that since $\bar{F}(Q_d^*) = c_1/\alpha_1$, then while $\bar{F}(Q_c^*) = \sum_{i=0}^n c_i$, the decentralized solution Q_d^* is, in general, not equal to the system-optimal one, Q_c^* . In fact, since by assumption $c_1/\alpha_1 = \max_i \{c_i/\alpha_i\}$, then $c_1\alpha_i \geq \alpha_1 c_i$, $i = 0, 1, \dots, n$, so $c_1 \sum_{i=0}^n \alpha_i \geq \alpha_1 \sum_{i=0}^n c_i$. Thus, $c_1/\alpha_1 \geq \sum_{i=0}^n c_i$, since $\sum_{i=0}^n \alpha_i = 1$, and, hence, $Q_d^* \leq Q_c^*$. We can show that the two quantities will be equal *if and only if* $\alpha_i = c_i / \sum_{i=0}^n c_i$ for $i = 0, 1, \dots, n$. So, we further have the following proposition:

Proposition 2

1) *The decentralized production quantity can not be larger than the centralized quantity.*

That is, $Q_d^ \leq Q_c^*$.*

2) *The decentralized production quantity will be the same as the centralized quantity if and only if the revenue allocation is such that the revenue share of each party equals to its cost share. That is, $Q_d^* = Q_c^*$ iff $\alpha_i = c_i / \sum_{i=0}^n c_i$ for $i = 0, 1, \dots, n$.* ||

3.1.1 Revenue Shares Maximizing Assembler's Expected Profit

How is the assembly firm or retailer going to set up the incentive scheme (i.e., $\alpha_1, \dots, \alpha_n$, and so $\alpha_0 = 1 - \sum_{i=1}^n \alpha_i$), if it has the power to do so? It is plausible to assume that the firm tries to maximize its own expected profit. So, we have a Stackelberg type game: First, the firm sets up the revenue shares, and then the suppliers decide the number of units Q_d^* to deliver. Denote the firm's expected profit by $\pi_0(\alpha_1, \dots, \alpha_n)$. The firm faces the following optimization problem

$$\max_{\alpha_1, \dots, \alpha_n} \pi_0(\alpha_1, \dots, \alpha_n) = E[-c_0 Q_d^* + (1 - \sum_{i=1}^n \alpha_i) \min(Q_d^*, X)]. \quad (7)$$

The following property partially characterizes the firm's optimal policy:

Proposition 3

The firm will always set $\alpha_1, \dots, \alpha_n$ such that

$$c_1 / \alpha_1 = \dots = c_n / \alpha_n \geq c_0 / \alpha_0. \quad \parallel \quad (8)$$

Proof We first show that the revenue shares allocated to the n suppliers ought to be such that $c_1 / \alpha_1 = \dots = c_n / \alpha_n$. Otherwise, assume WLOG that $c_1 / \alpha_1 = \max\{c_i / \alpha_i : i = 1, \dots, n\}$ and $c_1 / \alpha_1 > c_i / \alpha_i$ for some $i \neq 1$. Then, by reducing α_i to a value such that $c_1 / \alpha_1 = c_i / \alpha_i$, the assembler will increase its own share c_0 of the unit revenue *without* reducing the delivery quantity of complete sets of components, and, hence, the firm will increase its expected profit $\pi_0(\alpha_1, \dots, \alpha_n)$ in (7).

Now, for the second part in (8), assume otherwise, i.e., $c_1/\alpha_1 = \dots = c_n/\alpha_n < c_0/\alpha_0$.

Then, we know from (5) that each component supplier is willing to delivery *more than* that the firm is willing to assemble. Thus, by reducing the revenue share allocated to each supplier *at least* to a value such that $c_1/\alpha_1 = \dots = c_n/\alpha_n = c_0/\alpha_0$, the firm can again only improve its own expected profit. ||

Thus, if the assembly firm behaves optimally, *all* suppliers will be “critical”, and the decentralized production quantity Q_d^* is determined by Q_1^* . Now, the n -dimensional problem in (7) reduces to a one-dimensional one, since it follows from (8) that $\sum_{i=1}^n \alpha_i = \alpha_1 \sum_{i=1}^n c_i / c_1$ and also $\alpha_1 \leq c_1 / \sum_{i=0}^n c_i$. That is,

$$\max_{c_1 \leq \alpha_1 \leq c_1 / \sum_{i=0}^n c_i} \pi_0(\alpha_1) = E\{-c_0 Q_1^* + [1 - (\sum_{i=1}^n c_i / c_1) \alpha_1] \min(Q_1^*, D)\}. \quad (9)$$

Since at suppliers' optimum $\alpha_1 = c_1 / \bar{F}(Q_1^*)$, a monotone increasing function, one can perform the optimization over Q_1^* rather than over α_1 (see Lariviere and Porteus 1999 for a similar approach). We also know that $Q_1^* = Q_c^*$ when $\alpha_1 = c_1 / \sum_{i=0}^n c_i$. Thus, suppressing the super/subscripts on Q , problem (9) becomes

$$\begin{aligned} \max_{0 \leq Q \leq Q_c^*} \pi_0(Q) &= E\{-c_0 Q + (1 - \frac{\sum_{i=1}^n c_i}{\bar{F}(Q)}) \min(Q, X)\} \\ &= -c_0 Q + [1 - \frac{\sum_{i=1}^n c_i}{\bar{F}(Q)}] \int_0^Q \bar{F}(x) dx. \end{aligned} \quad (10)$$

Proposition 3 states that the revenue shares allocated among the n suppliers by the assembly firm will always be proportional to their production costs. As a result, if the total production costs, namely $\sum_{i=1}^n c_i$, is constant, the number of suppliers n (and their relative production costs) will not affect the assembly firm's order-size decision. This is also evident in (10), where the total costs $\sum_{i=1}^n c_i$ appears as a single parameter in the assembly firm's profit function. So, we have,

Corollary 1

For a given total components production cost $\sum_{i=1}^n c_i$, the decentralized production quantity Q_d^ and, hence, total channel profit $\pi(Q_d^*)$ are not affected by the number of suppliers and the allocation of the total cost among them.* ||

Since the number of suppliers will not affect the decentralized decision, problem (10) is equivalent to the problem studied by Cachon and Lariviere (1999) where the downstream firm provides incentives to induce a single supplier to build up production capacity. It should be noted, however, that the observation, in our decentralized assembly system, that *if* the assembly firm acts optimally the problem becomes equivalent to one with a single supplier is a *result* rather than something which is obvious from the outset. Second, in the following, we derive a concavity condition, which is weaker than that obtained by Cachon and Lariviere.

Now, the first order condition of optimality for problem (10) is

$$\frac{d\pi_0(Q)}{dQ} = -c_0 + \bar{F}(Q) - (\sum_{i=1}^n c_i) \left\{ 1 + \frac{f(Q)}{[\bar{F}(Q)]^2} \int_0^Q \bar{F}(x) dx \right\} = 0. \quad (11)$$

Note that at $Q=0$, $d\pi_0(Q)/dQ = 1 - \sum_{i=0}^n c_i > 0$,

and at $Q = Q_c^*$,

$$d\pi_0(Q)/dQ = -(\sum_{i=1}^n c_i) \{f(Q_c^*)/[\bar{F}(Q_c^*)]^2\} \int_0^{Q_c^*} \bar{F}(x)dx < 0.$$

Thus, the following proposition follows immediately:

Proposition 4

$$\text{If } \frac{f(Q)}{[\bar{F}(Q)]^2} \int_0^Q \bar{F}(x)dx \quad (12)$$

is increasing, then $\pi_0(Q)$ is concave and has a unique interior maximum which can be found by solving $d\pi_0(Q)/dQ = 0$. ||

The assumption above is very weak. It is implied by the IFR property (increasing f/\bar{F}), and will be satisfied by essentially any practical unimodal demand distribution. Now, the assumption is equivalent to the first derivative of (12) being non-negative. That is,

$$2 \frac{f(Q)}{\bar{F}(Q)} + \frac{f'(Q)}{f(Q)} + \frac{\bar{F}(Q)}{\int_0^Q \bar{F}(x)dx} \geq 0,$$

which is weaker than the following condition used by Cachon and Lariviere (1999; Theorem 3):

$$2 \frac{f(Q)}{\bar{F}(Q)} + \frac{f'(Q)}{f(Q)} \geq 0.$$

We note also that the condition in Proposition 4 here differs from Lariviere and Porteus's (1999) "increasing generalized failure rate" (increasing Qf/\bar{F}); neither one of these two conditions implies the other.

When $\{f(Q)/[\bar{F}(Q)]^2\} \int_0^Q \bar{F}(x)dx$ is increasing, the following properties can be established through (11):

Corollary 2

The decentralized production quantity Q_d^* and, hence, total channel profit $\pi(Q_d^*)$ are

- 1) decreasing in $\sum_{i=1}^n c_i$ and c_0 ;
- 2) increasing in the ratio of $c_0 / (c_0 + \sum_{i=1}^n c_i)$ for any given total costs $c_0 + \sum_{i=1}^n c_i$. \parallel

With α_i , $i=1, \dots, n$, being set by the assembly firm such that $c_1 / \alpha_1 = \dots = c_n / \alpha_n \geq c_0 / \alpha_0$, the decentralized production quantity Q_d^* , obtained by solving (11), is the Newsvendor-optimal delivery quantity for each of the suppliers, i.e., $\bar{F}(Q_d^*) = c_1 / \alpha_1 = \dots = c_n / \alpha_n$. Their corresponding expected profits will be

$$\pi_i = \alpha_i \int_0^{Q_d^*} x f(x) dx = \frac{c_i}{\bar{F}(Q_d^*)} \int_0^{Q_d^*} x f(x) dx, \text{ for } i=1, \dots, n. \quad (13)$$

Note that each supplier's expected profit is also proportional to its marginal production cost, i.e., $c_1 / \pi_1 = \dots = c_n / \pi_n$.

Example 1:

Assume demand for the final product is exponentially distributed with a mean μ . So, we have $f(x) = \frac{1}{\mu} e^{-(1/\mu)x}$ and $F(x) = 1 - e^{-(1/\mu)x}$, $x > 0$. This IFR distribution clearly satisfies (12). We compare the production quantity and profits of a centralized system with those of a decentralized one.

For centralized decision, we obtain the optimal production quantity from (2) and system-wide profit from (3) as

$$Q_c^* = -\mu \log(\sum_{i=0}^n c_i), \quad (\text{E.1})$$

and
$$\pi(Q_c^*) = \mu[(\sum_{i=0}^n c_i) \log(\sum_{i=0}^n c_i) - (\sum_{i=0}^n c_i) + 1], \quad (\text{E.2})$$

respectively.

For the decentralized system, solving for Q in (11) yields the production quantity of

$$Q_d^* = -\mu \log K, \quad (\text{E.3})$$

where,
$$K \equiv \frac{c_0 + \sqrt{c_0^2 + 4 \sum_{i=1}^n c_i}}{2}.$$

Substituting Q_d^* into (1), we obtain the system-wide profit as

$$\pi(Q_d^*) = \mu[(\sum_{i=0}^n c_i) \log K - K + 1]. \quad (\text{E.4})$$

The deviations of the decentralized production quantity Q_d^* and profit $\pi(Q_d^*)$ from the centralized Q_c^* and $\pi(Q_c^*)$ will depend on the total production cost $\sum_{i=0}^n c_i$ as well as its allocation between the assembly cost c_0 and components' cost $\sum_{i=1}^n c_i$. For the limiting case with $c_0 = 0$ (i.e., a free assembly stage), it is interesting to note that, for this demand distribution, the decentralized production quantity is exactly one half of the centralized quantity,

i.e., $Q_d^* = 0.5Q_c^*$, independent of mean demand and components' production costs. When $\sum_{i=1}^n c_i = 0$, we have $Q_d^* = Q_c^*$, as expected.

Assuming that the total production costs per unit equal half of the unit revenue (i.e., $c_0 + \sum_{i=1}^n c_i = \0.5), **Figure 1** illustrates how the deviations (as percentage) of the decentralized production quantity and system profit from their centralized counterparts change with the allocation of cost between the assembly firm and the suppliers (i.e., the ratio of $c_0 / (c_0 + \sum_{i=1}^n c_i)$). (Note that both $(Q_c^* - Q_d^*) / Q_c^*$ and $[\pi(Q_c^*) - \pi(Q_d^*)] / \pi(Q_c^*)$ do *not* depend on the mean demand level μ .) First, we see that, when the assembly firm bears a relative low portion of the total channel costs, the decentralized decision can result in a channel profit which is over 20% lower than that of a centralized decision. In general, the decentralized system performance improves as the assembly firm bears a greater fraction of the total costs. The managerial implication is that if the party capturing most of the revenue also bears most of the channel cost, “double marginalization” will not cause significant deterioration in channel performance for decentralized supply chains.

In the decentralized system, the channel profit $\pi(Q_d^*)$ is shared by the assembly firm and the suppliers. Substituting $Q = Q_d^*$ in (E.3) into (10), we obtain the assembly firm's profit as

$$\pi_0 = \mu[c_0 \log K - (\sum_{i=1}^n c_i) / K + \sum_{i=1}^n c_i - K + 1], \quad (\text{E.6})$$

which is decreasing in $\sum_{i=1}^n c_i$. From (13), each supplier's profit is given by

$$\pi_i = \mu c_i [\log K + 1 / K - 1], \quad i = 1, 2, \dots, n. \quad (\text{E.7})$$

Setting the scalar $\mu = 1$, **Figure 2** illustrates that, as the assembly firm's portion of cost increases, the total channel profit as well as the profit of the firm increase while the (total) profit of suppliers decreases. This is again rather intuitive.

3.2 Surplus Subsidy and Channel Coordination

Suppose now that, in addition to a share α_i of revenue from sales of final product, the assembly firm will pay supplier i s_i per unit for its delivered components that are *not* sold – a surplus subsidy. To avoid trivial cases, we assume that $\alpha_i > c_i > s_i$, for $i = 1, \dots, n$, and $\sum_{i=1}^n \alpha_i \leq 1 - c_0$. Note that a delivered component may end up unsold either due to low demand or due to shortage of mating components, or both. The surplus subsidy does not distinguish between causes. However, as we shall see, rational suppliers will actually deliver equal amounts, so unmated components will not actually occur. Thus, such subsidy, in effect, transfers some of the risk due to uncertain demand from the suppliers to the firm. Economically, that is similar to manufacturers' reducing retailers' risk by committing to buy-backs (returns) (Pasternack 1985, Lariviere 1999) within a different type of contract discussed in later sections.

If supplier i 's revenue did not depend on other suppliers' deliveries, it would now face the following newsvendor profit function:

$$\begin{aligned} \pi_i(Q_i) &= E\{-c_i Q_i + \alpha_i \min(Q_i, D) + s_i [Q_i - D]^+\} \\ &= -c_i Q_i + \alpha_i \left[\int_0^{Q_i} x f(x) dx + Q_i \bar{F}(Q_i) \right] + s_i \int_0^{Q_i} (Q_i - x) f(x) dx. \end{aligned} \quad (14)$$

This yields its most desired delivery quantity Q_i^* , which satisfies

$$\bar{F}(Q_i^*) = \frac{c_i - s_i}{\alpha_i - s_i}, \quad i = 1, \dots, n. \quad (15)$$

As expected, Q_i^* is increasing in s_i .

But, since the over-delivery subsidy alone will not help to fully recover the production costs of components (i.e., $c_i > s_i$), none of the suppliers will be willing to deliver more than anyone else. Furthermore, it is optimal for the assembly firm to set the incentives scheme (α_i, s_i) for each i such that all suppliers willingly deliver the same quantity: by reducing either the revenue share α_i and/or subsidy s_i for those who are willing to deliver more than that the “critical” supplier’s newsvendor quantity, the assembly firm can only benefit. That is,

Proposition 5

The assembly firm will set (α_i, s_i) , $i = 1, \dots, n$, such that

$$\frac{c_1 - s_1}{\alpha_1 - s_1} = \frac{c_2 - s_2}{\alpha_2 - s_2} = \dots = \frac{c_n - s_n}{\alpha_n - s_n}. \quad \parallel \quad (16)$$

When the delivery quantity of *each* supplier in the decentralized system equals the centralized decision Q_c^* , we say that the supply chain is coordinated. Comparing (15) with (2), it’s then obvious that

Proposition 6

To coordinate the system, the assembly firm only needs to set (α_i, s_i) for each i , $i = 1, \dots, n$, such that

$$\frac{c_i - s_i}{\alpha_i - s_i} = \sum_{j=0}^n c_j, \quad \text{or} \quad s_i = \frac{c_i - \alpha_i \sum_{j=0}^n c_j}{1 - \sum_{j=0}^n c_j}. \quad \parallel \quad (17)$$

Equation (17) determines s_i as a function of the other variable α_i . That is, for any *given* revenue share α_i , as long as the corresponding surplus subsidy s_i is determined by (17), the resulting contract (α_i, s_i) will coordinate the decentralized system. Thus, for each supplier i , there actually exists a continuum of contracts that can coordinate the supply chain. Second, we note that for purpose of coordination, the contract (α_i, s_i) of one supplier does not have to depend on those of other suppliers. Thus, the assembly firm can negotiate the contracts independently with different suppliers. These two properties make this two-parameter contract structure especially attractive from a practical point of view.

When the supply chain is coordinated through the incentive scheme in (17), supplier i 's expected profit can be calculated by substituting $Q_i = Q_c^*$ and $s_i = (c_i - \alpha_i \sum_{i=1}^n c_i) / (1 - \sum_{i=1}^n c_i)$ into (14). After some algebra, we have

$$\pi_i = (\alpha_i - c_i) \frac{1}{1 - \sum_{i=1}^n c_i} \int_0^{Q_c^*} x f(x) dx. \quad (18)$$

Thus, supplier i 's profit π_i is determined *solely* by its revenue share α_i .

With coordination achieved, the supply chain reaches its highest possible total profit. Now, the best strategy for the assembly firm is simply to try to allocate as low revenue shares α_i and, hence, as little profits, as possible to each of the suppliers; the rest of the *maximum* channel profit goes to himself.

Obviously, this revenue-plus-surplus-subsidy contract dominates the revenue-only contract and any other contract types that cannot achieve channel coordination. For example, due to the ‘profit surplus’ generated through coordination, the assembly firm can allocate to each supplier at least the same profit as when the channel is not coordinated and still leave himself with more.

4. Wholesale Price Contract

With a wholesale price contract, the n suppliers first *simultaneously* choose their individual component wholesale prices w_i , $i = 1, 2, \dots, n$, charged to the assembler; then, the assembler chooses a quantity Q ordered from *all* suppliers. Thus, mis-matching of components will never happen in this environment. This setting is a multi-supplier generalization of Lariviere and Porteus (1999).

When the wholesale prices w_i , $i = 1, 2, \dots, n$, are offered by the suppliers, the assembler faces the simple Newsvendor problem,

$$\max_Q \pi_0(Q) = E\{-(\sum_{i=1}^n w_i + c_0)Q + \min(D, Q)\}, \quad (19)$$

and its optimal order quantity will be

$$Q = \bar{F}^{-1}\left(\sum_{i=1}^n w_i + c_0\right). \quad (20)$$

In the simultaneous sub-game of choosing component wholesale prices, all suppliers know the production quantity decision made by the assembler. Obviously, we require $w_i > c_i$, $i = 1, 2, \dots, n$, and $\sum_{i=1}^n w_i + c_0 < 1$ to ensure that every one remains in business. Then, comparing (20) with (2), we have that the decentralized production quantity in this setting will again never be more than the centralized quantity Q_c^* .

Now, for given wholesale prices of all other suppliers w_i , $i \neq j$, supplier j would choose its price w_j to maximize its own profit. That is, by (20)

$$\max_{w_j} \pi_j(w_j) = (w_j - c_j)Q = (w_j - c_j)\bar{F}^{-1}\left(\sum_{i=1}^n w_i + c_0\right), \quad j = 1, 2, \dots, n, \quad (21)$$

Since there is a one-to-one correspondence between w_j and Q for given w_i , $i \neq j$, i.e., $w_j = \bar{F}(Q) - (\sum_{i \neq j} w_i + c_0)$, choosing a value for w_j is equivalent to choosing a corresponding value for Q . Thus, the optimization over w_j in (21) can equivalently be written as the following optimization over Q :

$$\max_Q \pi_j(Q) = [\bar{F}(Q) - (\sum_{i \neq j} w_i + c_0) - c_j]Q, \quad j = 1, 2, \dots, n. \quad (22)$$

This transformation helps us to characterize the concavity of π_j , since we can show from (22) that

$$\frac{d\pi_j}{dQ} = \bar{F}(Q) \left(1 - \frac{f(Q)}{\bar{F}(Q)} Q \right) - (\sum_{i \neq j} w_i + c_0) - c_j. \quad (23)$$

Then, we have the following Lemma, where the second part follows from the one-to-one and monotone correspondence between w_j and Q :

Lemma 1

If $[f(Q)/\bar{F}(Q)]Q$ is increasing, then π_j is concave in Q as defined in (22) and, hence, concave in w_j as defined in (21).

Note that the above concavity condition is exactly the same as that proposed by Lariviere and Porteus (1999) for a *single-supplier-manufacturer* system.

Now, w_j , $j = 1, 2, \dots, n$, is constrained to be in $[c_j, 1 - c_0]$, which is nonempty, compact and convex. The payoff function π_j in (21) is continuous in w_j , assuming that the demand distribution function F is continuous. Then, Lemma 1 guarantees the existence of a Nash equilibrium (Theorem 1.2 of Fudenberg and Tirole 1991). That is,

Proposition 7

If $[f(Q)/\bar{F}(Q)]Q$ is increasing, there exists a pure-strategy Nash equilibrium for the sub-game of choosing the wholesale prices by the component suppliers.

Since the production quantity Q chosen by the assembler is uniquely determined by the sum of the wholesale prices as in (1), Proposition 1 implies that there exists an equilibrium production quantity for the decentralized assembly system.

4.1 Identical Suppliers

Assume that the n suppliers are identical in terms of their production costs, i.e., $c_i = c$ for all i . From (21), we can check that

$$\frac{\partial^2 \pi_j}{\partial w_j^2} - \frac{\partial^2 \pi_j}{\partial w_j \partial w_k} = \frac{1}{f(\sum_{i=1}^n w_i + c_0)} > 0, \quad j \neq k.$$

This eliminates the existence of non-symmetric equilibria (Theorem 4.1 of Anupindi et al. 1999). Thus, in equilibrium we will always have $w_i = w$ for all i . Now, from (20) we have $w = [\bar{F}(Q) - c_0]/n$. Substituting $w_i = w = [\bar{F}(Q) - c_0]/n$ for all i together with $c_j = c$ into (23) and letting it equal zero, the equilibrium production quantity Q can be found by solving

$$\bar{F}(Q) \left(1 - n \frac{f(Q)}{\bar{F}(Q)} Q \right) = nc + c_0. \quad (24)$$

Now, if $[f(Q)/\bar{F}(Q)]Q$ is increasing, the left-hand side of (24) is decreasing in Q . Furthermore, when $Q = 0$, the left-hand equal 1 (the unit product revenue), which is bigger than $nc + c_0$ (the total production cost), and as $Q \rightarrow \infty$, the left-hand becomes zero. Thus, the solution to (24) will be unique and finite. That is,

Proposition 8

With identical suppliers, if $[f(Q)/\bar{F}(Q)]Q$ is increasing, there exists a unique decentralized production quantity.

The following properties regarding the equilibrium production quantity can be obtained from (24):

Corollary 3

The decentralized production quantity Q_d^ and, hence, the total channel profit $\pi(Q_d^*)$*

- 1) are decreasing in the component cost c and assembly cost c_0 for any given number of suppliers;*
- 2) are decreasing in the number of suppliers n for any given total production cost $nc + c_0$;*
- 3) do not change with the allocation of production costs between the assembly firm and the suppliers, keeping the total costs $nc + c_0$ fixed.*

We note that while part 1) here is rather intuitive, part 2) and 3) may not be so. Also, part 2) and 3) contrast sharply with properties of the revenue-sharing systems, where the number of suppliers *does not* affect the production quantity (Corollary 1) while the allocation of costs between the assembly firm and suppliers *does* (Corollary 2). These structural differences suggest that the choice of incentive schemes (revenue-share vs. wholesale price) can be critical to system performances.

Example 2:

As in Example 1, we assume exponential demand distribution. Then, equation (24) reduces to

$$e^{-(1/\mu)Q}[1 - (n/\mu)Q] = nc + c_0, \quad (\text{E.8})$$

which we can easily solve numerically to find the decentralized production quantity Q .

Substituting such Q into (1), we can then evaluate the system-wide profit by

$$\pi(Q) = \mu(1 - e^{-(1/\mu)Q}) - (nc + c_0)Q. \quad (\text{E.9})$$

Setting the scalar $\mu = 1$ and the total production costs $nc + c_0 = 0.5$, **Figure 3** shows that the production quantity (hence, system-wide performance) decreases dramatically with the number of suppliers in the decentralized system with wholesale price contract. Illustrated in

Figure 3 are also what the production quantities would be if the same system were either coordinated through the revenue-share contract (with $c_0/(nc + c_0) = 0$ or $c_0/(nc + c_0) = 0.7$ respectively) or central controlled. First of all, for all these latter cases, system performance does not change with the number of suppliers in the system. More surprisingly, the system with revenue-share contract always performs better than with wholesale price contract. That is, the worst case (i.e., with $c_0/(nc + c_0) = 0$) of the revenue-share systems dominates the best case (i.e., with $n = 1$) of the wholesale price systems. Our extensive numerical testing seems to confirm that this conclusion is always true with the exponential demand distribution. Other distribution functions are yet to be explored. The managerial implications of this conclusion can be significant.

4.2 Inventory Buy-Back Policy and Channel Coordination

Pasternack (1985) showed that when a *single* supplier wholesales to a retailer/manufacturer, a properly designed inventory buy-back/return policy can coordinate the supply chain. In the following we show how such a policy can be adopted to coordinate our *multiple* suppliers decentralized assembly system.

Assume that while initially charging the assembly firm a wholesale price w_i for all units ordered, supplier i , $i = 1, 2, \dots, n$, also agrees to pay back the assembly firm s_i per unit for any quantity (ordered by the firm) *above* the realized demand. To avoid trivial cases, the following relationships must hold: $w_i > c_i$ and $w_i > s_i$ for $i = 1, 2, \dots, n$, and $\sum_{i=1}^n w_i + c_0 < 1$.

Now the firm faces the Newsvendor problem

$$\begin{aligned}\pi_0(Q) &= E\{-(\sum_{i=1}^n w_i + c_0)Q + \min(Q, D) + \sum_{i=1}^n s_i [Q - D]^+\} \\ &= -(\sum_{i=1}^n w_i + c_0)Q + \int_0^Q xf(x)dx + Q\bar{F}(Q) + \sum_{i=1}^n s_i \int_0^Q (Q-x)f(x)dx. \quad (25)\end{aligned}$$

Its optimal order quantity satisfies

$$\bar{F}(Q) = \frac{\sum_{i=1}^n w_i + c_0 - \sum_{i=1}^n s_i}{1 - \sum_{i=1}^n s_i}. \quad (26)$$

Comparing (26) with (2), we have

Proposition 9.

The assembly firm will choose the system-wide optimal production quantity Q_c^ , if and only if*

$$\frac{\sum_{i=1}^n w_i + c_0 - \sum_{i=1}^n s_i}{1 - \sum_{i=1}^n s_i} = \sum_{i=0}^n c_i. \quad (27)$$

How can the channel be coordinated? Assume that for *any* value of wholesale price w_j charged to the firm, *each* supplier j , $j=1,2,\dots,n$, further agrees to pay the firm a *corresponding* buy-back price of

$$s_j = \frac{1}{1 - \sum_{i=0}^n c_i} w_j - \frac{\frac{1}{n} \sum_{i=1}^n c_i}{1 - \sum_{i=0}^n c_i}. \quad (28)$$

Then, one can easily verify that condition (27) is always satisfied and, thus, the channel is coordinated!

Now, with the channel being coordinated (i.e., the production quantity being Q_c^* and, hence, the total channel profit pie being at its maximum size), supplier j 's profit is given by

$$\pi_j(w_j) = (w_j - c_j)Q_c^* - s_j \int_0^{Q_c^*} (Q_c^* - x)f(x)dx, \quad j = 1, 2, \dots, n.$$

In conjunction with (28), we can show that $\pi_j(w_j)$ is simply a linear function of its wholesale price w_j . Thus, in reality each supplier simply tries to bargain with the assembly firm for as high a wholesale price as possible. Obviously, there exists a continuum of such contracts for each supplier. Furthermore, one supplier's contract does not have to depend on that of any other's.

5. Concluding Remarks

Ours is the first study of coordination in decentralized assembly/joint-purchase systems with random demand. We believe that this setting is “natural” for exploring coordination and incentive issues. The basic observation that with either single-lever contract the decentralized inventory levels are less than the centralized ones (Proposition 2 and Corollary 3) should be viewed in the context of products' complementarity vs. substitution. Ours is a perfect complementarity setting. In a substitution environment, on the other hand, (e.g., Mahajan and van Ryzin 1999), the typical conclusion is that the decentralized inventory levels will be higher than system-optimal.

Viewed in a retail context, our complementary products were assumed to be jointly purchased in every case. There are products, however, like coffee and filters, which, although perfectly complementary, are not always purchased together or at fixed ratios (when you have guests you consume coffee faster than filters relative to when you make coffee just for yourself).

The implications of this weaker form of purchase-coupling need to be explored.

Delegating the inventory management of highly complementary products to individual, non-communicating, suppliers, while used in practice, sounds “wrong” at first. Yet the observation that at equilibrium all suppliers will deliver the same quantity “removes the danger” of mismatches. Furthermore, the performance of such a system is independent of the number of suppliers, and an additional lever, surplus subsidy, will coordinate the channel. The performance of a wholesale-price-based system, on the other hand, does degrade with the number of suppliers, and appears (in our example) to be inferior to the VMI one even for a small number of suppliers. Buybacks will coordinate such system, however.

That said, the practicality and relative attractiveness of the schemes would also heavily depend on informational assumptions/requirement and monitoring and enforcement issues. We shall only discuss the informational requirements. Again, at first the VMI system “sounds” as the one which will require more cost information; after all, in deciding how much to deliver a supplier in a VMI system needs to contemplate the other suppliers’ choices, and these are naturally based on their costs. But a closer look reveals that since the firm will set the revenue shares such that $c_1/\alpha_1 = \dots = c_n/\alpha_n$ (Proposition 3), a supplier can infer other suppliers’ cost/share ratios and determine his Nash strategy (Proposition 1) without explicit knowledge of others’, or assembler’s, costs. On the other hand, within the wholesale-price-based contract, although a supplier’s optimization problem [eq. (21)] does not explicitly involve other suppliers’ costs, that being a game *all* suppliers’ optimality conditions (reaction curves) will have to be solved simultaneously; thus each supplier will need to use information about others suppliers’, as well as assembler’s, costs. However, once the wholesale prices were chosen and announced, the

firm will not need to know the suppliers' costs to select its order quantity; on the other hand, the revenue-share selecting firm will need to know the suppliers' costs. Thus the VMI system requires the firm to have more information about the suppliers' costs than does the wholesale-price-system, but the latter requires the suppliers to be better informed.

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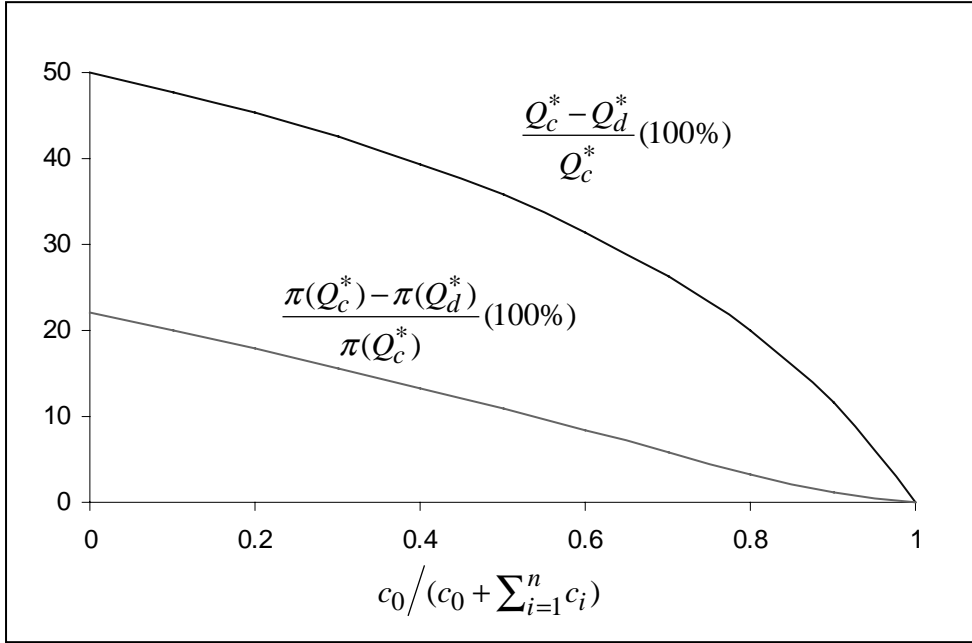


Figure 1. Revenue Sharing: Deviation of the Decentralized system from the Centralized System, with $c_0 + \sum_{i=1}^n c_i = 0.5$

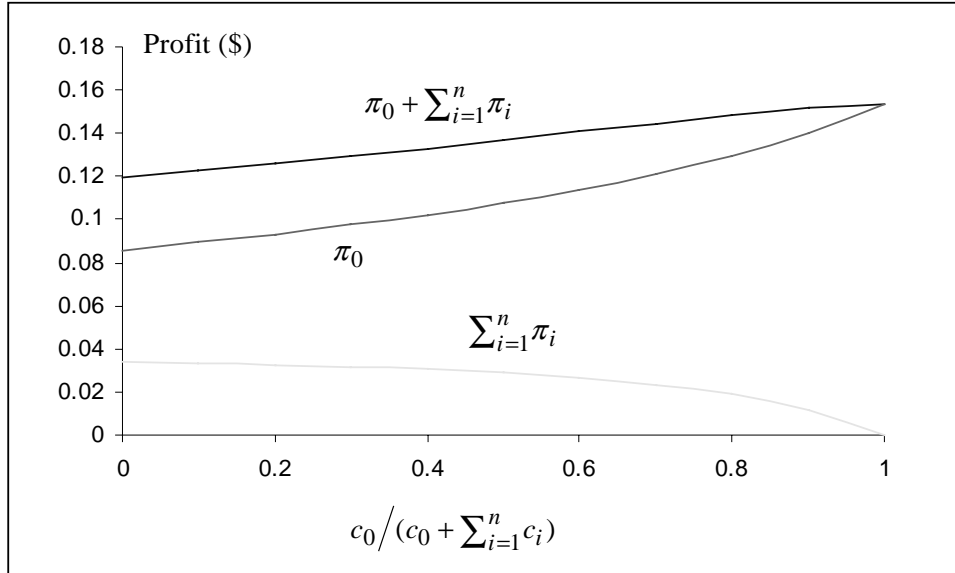


Figure 2. Revenue Sharing: Profits in the Decentralized System with $c_0 + \sum_{i=1}^n c_i = 0.5$ and $\mu = 1$

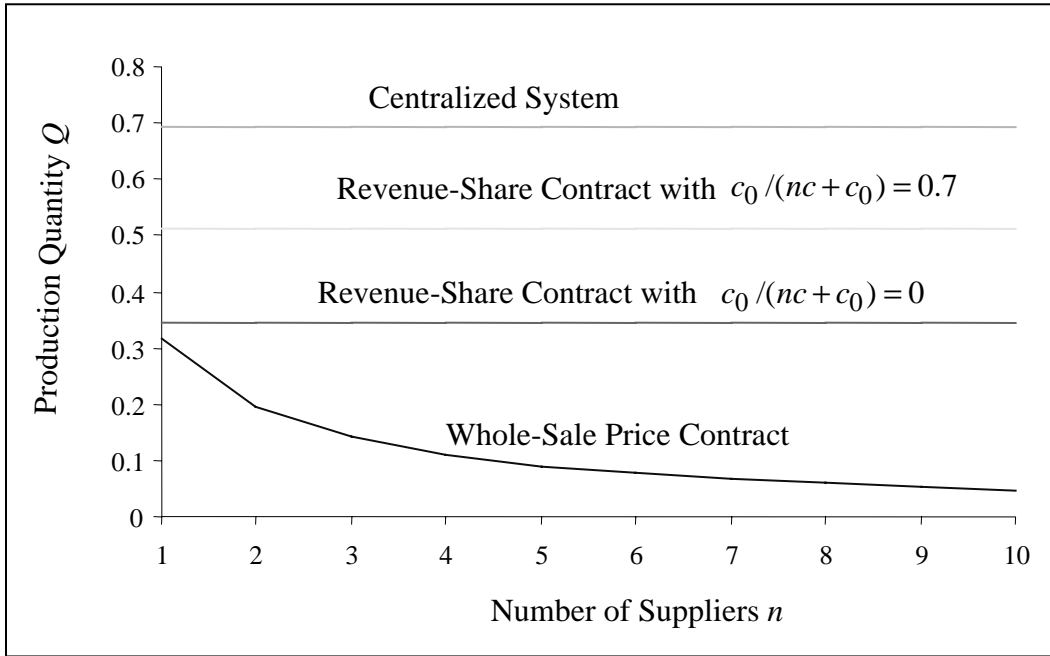


Figure 3. Performance Comparison of Systems with $nc + c_0 = 0.5$ and $\mu = 1$