PMBA8155 Operations Management

# Inventory Control

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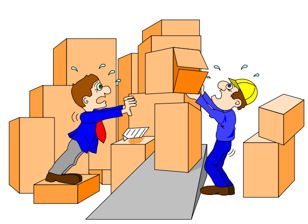
## Learning Objectives

- Understand
  - advantages and disadvantages of carrying inventory
  - independent and dependent demand
  - various inventory related costs
  - fixed-order-quantity and fixed-time-period systems
  - ABC classification system, optional replenishment system, and bin systems
- Be Able to Apply Inventory Control Models
  - Single-period Model
  - Basic EOQ Model
  - EOQ with Uncertain Demand
  - Fixed Time Period Model



## What Is Inventory?

- Definition
  - Stock of items held to meet future needs
- Types of Inventory
  - Raw materials
  - Component parts
  - Work in process
  - Finished products
  - Supplies



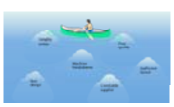
## Why Holding Inventory?

- To meet anticipated and unexpected demand
- To protect against stockouts
- To maintain independence of operations
- To smooth production requirements
- To protect against variation in delivery time from suppliers
- To take advantage of quantity discounts or economies of scale



## Why Not Holding Inventory?

- Hides Problems
  - Poor quality, inadequate maintenance, poor production scheduling, unreliable suppliers
- Costs Money and Ties Up Resources
  - Annual investment in inventory in U.S. is several trillion dollars

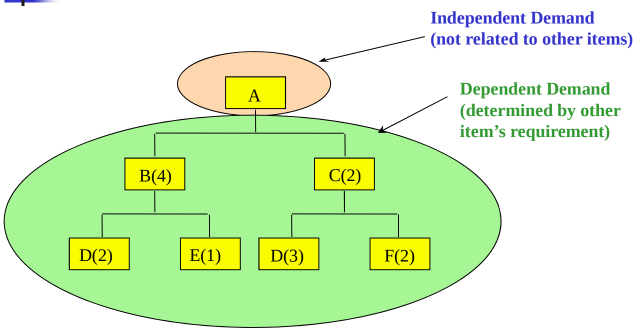




## Inventory Costs

- Carrying (or holding) costs
  - Storage cost (space, utilities, personnel)
  - Opportunity cost of capital
  - Insurance, taxes, possible loss of value
- Ordering or Setup (production change) costs
  - Order processing cost
  - Transportation and receiving cost
- Shortage costs
  - Backorder related cost (tracking, rush shipment)
  - Lost sales/customer or production time

# Independent vs. Dependent Demand






# Inventory Control Systems

- An inventory system is the set of policies that monitor and control the levels of inventory and determine what levels should be maintained, when stock should be replenished, and how large orders should be
- Two Fundamental Questions
  - When to order (timing)?
  - How much to order (quantity)?

# Inventory Control Systems

- Single-Period Inventory Systems
  - Inventory used only for one-period (Example: special t-shirts at a football game)
- Multi-Period Inventory Systems
  - Fixed-order-quantity system (Q)
    - constant amount ordered when inventory reaches a predetermined level
  - Fixed-time-period system (P)
    - order placed for variable amount after fixed passage of time

## Single Period Model

- Seeks to balance the costs of inventory overstock and under stock
- Marginal analysis

  Find the minimum order quantity (q) that satisfies

$$P \leq \frac{C_u}{C_o + C_u}$$

- $C_u$ = Cost per unit of demand underestimated
- $C_o$ = Cost per unit of demand overestimated
- P = Cumulative probability or P = Prob[demand<q]



Single Period Model

## Example 1

Our college basketball team is playing in a tournament game this weekend. Based on our past experience we sell on average 2,400 T-shirts with a standard deviation of 350. We make $10 on every shirt we sell at the game, but lose $5 on every shirt not sold. Assuming normal distribution for T-shirts sales, how many shirts should we make for the game?

***Solution:***

$C_u$ = $10 and $C_o$ = $5; $P \leq$ $10 / ($10 + $5) = .667;

Since z = .45 (use Appendix E (p. 745) or NORMSINV(.667) in Excel),

we need q = 2,400 + .45(350) = 2,558 shirts



Single Period Model

## Example 2—Yield Management Application

Atlanta Airlines has found that the number of people who purchased tickets and did not show up for a flight is normally distributed with mean of 15 and standard deviation of 8.6. The ill will and penalty costs associated with not being able to board a passenger are estimated to be $699. Assume that the average cost for a ticket is $249. How many seats should be overbooked?

***Solution:***

$C_u$ = $249 and $C_o$ = $699; $P \leq$ 249/(249+699)=0.2627

Using Appendix E, we can find z= -0.65.

So overbooking = 15-0.65(8.6) = 9 seats

Fixed Order Quantity System

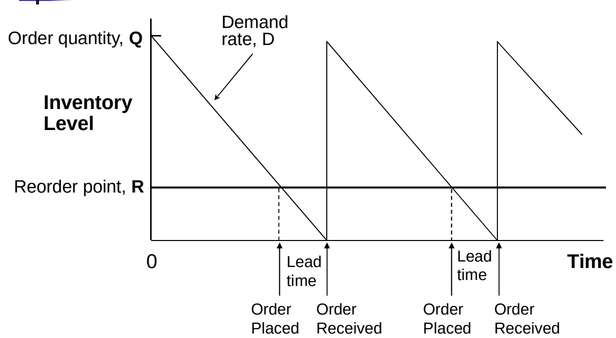
## Basic EOQ Model

- Assumptions
  - Demand for the product is known and constant
  - Lead time (time from ordering to receipt) is constant
  - Price per unit of product is constant
  - Ordering or setup costs are constant
  - All demands for the product will be satisfied (No shortages are allowed)
  - The order quantity is received all at once



Basic EOQ Model

## The Inventory Order Cycle






Basic EOQ Model

## Model Development - I

- Objective: To find the order quantity **Q** and reorder point **R** that minimize total cost

$$\text{Total Annual Cost} = \begin{matrix}\text{Annual}\\ \text{Purchase}\\ \text{Cost}\end{matrix} + \begin{matrix}\text{Annual}\\ \text{Ordering}\\ \text{Cost}\end{matrix} + \begin{matrix}\text{Annual}\\ \text{Holding}\\ \text{Cost}\end{matrix}$$

**Symbols used in EOQ model**

| | |
|---|---|
| TC - Total annual cost | R - Reorder point |
| D - Annual demand | L - Lead time |
| C – Purchase cost per unit | H - Annual holding and storage cost per unit of inventory |
| Q - Order quantity | |
| S - Cost of placing an order or setup cost | (H = i C, i=percentage rate) |

Basic EOQ Model

## Model Development - II

Total Annual Cost = Annual Purchase Cost + Annual Ordering Cost + Annual Holding Cost

Annual Purchase Cost = (# units)(cost/unit) = DC

Annual Ordering Cost = (# orders)(cost/order) = (D/Q)S

Annual Holding Cost = (avg. inventory)(unit holding cost) = (Q/2)H

$$TC = DC + \left(\frac{D}{Q}\right) \cdot S + \left(\frac{Q}{2}\right) \cdot H$$



Basic EOQ Model

## Model Development - III

- Using calculus, we can find the best (optimal) order quantity, $Q_{opt}$ (the economic order quantity)

$$Q_{opt} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(\text{Annual Demand})(\text{Order or Setup Cost})}{\text{Annual Holding Cost}}}$$

**When to order?**

The reorder point, $R = \bar{d}\,L$

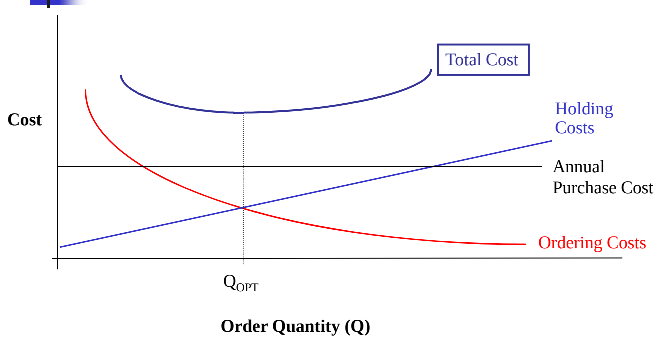
$\bar{d}$ = Average demand per time period

L = Lead time



Basic EOQ Model

## EOQ Model Cost Curves



Basic EOQ Model

## Some Facts

- The number of orders made per year is $n=D/Q_{opt}$
- The time between orders is $T = Q_{opt}/D$
- At the optimal solution $Q_{opt}$,

  Annual holding cost = Annual ordering cost
- The annual purchase cost does not affect $Q_{opt}$



Basic EOQ Model

## Example

Annual Demand = 1,000 units

Days per year considered in average daily demand = 365

Cost to place an order = $10

Holding cost per unit per year = $2.50

Lead time = 7 days

Cost per unit = $15

***Determine the economic order quantity and the reorder point.***



Basic EOQ Model

## Solution

$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1{,}000)(10)}{2.50}} = 89.443 \text{ or } 90 \text{ units}$$

$$\bar{d} = \frac{1{,}000 \text{ units / year}}{365 \text{ days / year}} = 2.74 \text{ units / day}$$

$$\text{Reorder point, } R = \bar{d}\ L = 2.74 \text{units / day (7days)} = 19.18 \text{ or } \mathbf{20\ units}$$

**Inventory Control Policy:** ***When the inventory level reaches 20, order 90 units.***

**How many orders will be made annually?** 1000/90=11.11

**What is the time between two orders?** 90/1000*365=32.85 days

**What is the average inventory level?** 90/2=45 units

Fixed Order Quantity System

## EOQ with Uncertain Demand

- When demand is uncertain, shortage (stockout) is possible
- Safety stock is often used to hedge against the risk of stockout
  - Safety stock is the additional inventory held above the expected demand (buffer on top of forecast demand)
- That is, we carry a little more when we order

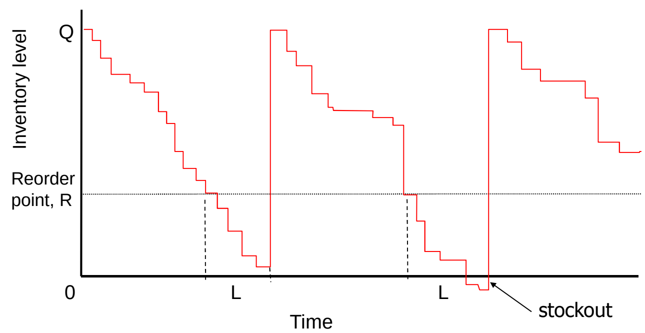
**R** = Avg. lead time demand + **safety stock**

= dL + **safety stock**



EOQ Model with Uncertain Demand

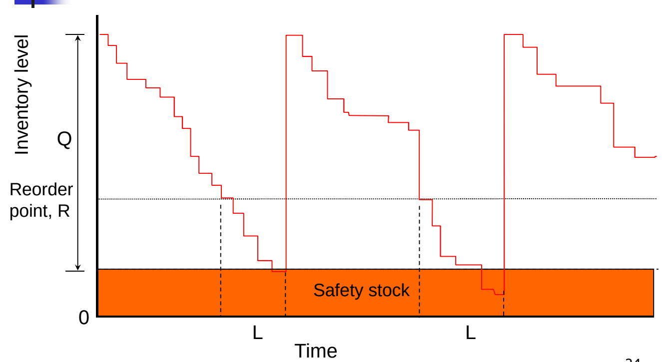
## Illustration--without Safety Stock






EOQ Model with Uncertain Demand

## Illustration--with Safety Stock



## Determine Safety Stock

- One way to determine the amount of safety stock is using desired service level
  - Service level: probability of no shortage
- Given a desired service level, the safety stock can be calculated as

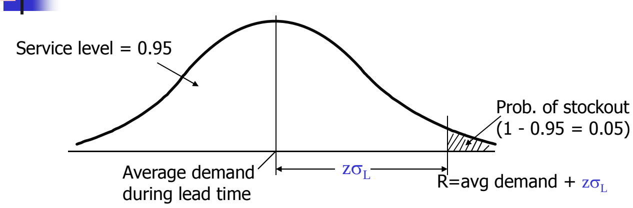
  Safety stock = $z\sigma_L$

  where $\sigma_L$ is the standard deviation of demand during the lead time and z is found from a standard Normal distribution table based on the service level



Determine Safety Stock

## How to Find z




- For example for a 95% service level, the chance of running out is 5% and we can find z ≈1.65
  - Using Appendix E (p. 745), z ≈ 1.65
  - Using Excel function, z = NORMSINV(0.95) ≈ 1.6449



Safety Stock and Reorder Point

## Example

If the lead time demand for an item is normally distributed with a mean of 25 and a standard deviation of 5. What are the safety stock and reorder point for satisfying a 99% (or 95%, 90%, or 50%) probability of not stocking out?

| Prob. of no stockout | z | Safety Stock | R |
|---|---|---|---|
| 0.99 | 2.35 | 11.75 | 36.75 |
| 0.95 | 1.65 | 8.25 | 33.25 |
| 0.90 | 1.3 | 6.5 | 31.5 |
| 0.50 | 0 | 0 | 25 |

If the safety stock is 5, what is the service level? 0.8413

EOQ Model with Uncertain Demand

## Computing $\sigma_L$

- Often times, we are only given $\sigma_d$ -- the standard deviation of *daily* demand. The relationship between $\sigma_L$ and $\sigma_d$ is

$$\sigma_L = \sqrt{\sigma_d^2 L} = \sigma_d \sqrt{L}$$

$\sigma_d$ = standard deviation of *daily* demand

Example: If L = 5 days, $\sigma_d$ = 2, what is $\sigma_L$?

$\sigma_L = 2\sqrt{5} = 4.472$



EOQ Model with Uncertain Demand

## How Many to Order?


- The best order quantity $Q_{opt}$ for uncertain demand is the same as that for known demand

$$Q_{opt} = \sqrt{\frac{2DS}{H}}$$

D = *average* annual demand



EOQ Model with Uncertain Demand

## Example

Avg. daily demand for a product is 60 and standard deviation is 7. The lead time is 6 days. The ordering cost is \$10 per order and holding cost is \$0.50 per item per year. Assume sales occur over 365 days of the year. What is the inventory control policy to satisfy a 95% probability of not stocking out during the lead time?

**Solution:** The best order quantity is

$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(60)(365)\ (10)}{0.50}} = 936$$

The reorder point is R = Lead time demand + Safety stock

= $(60)(6) + z\sigma_L$

$z = 1.65$ and $\sigma_L = 7\sqrt{6} = 17.146$

Therefore, R = 360 + (1.65)(17.146) = 388.29 or 389

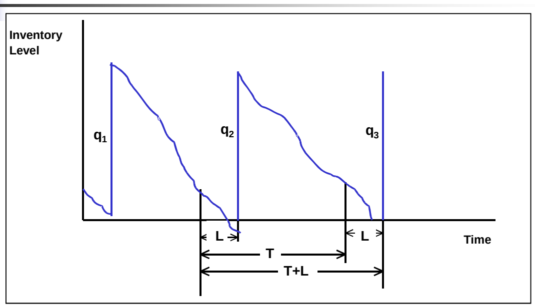
## Fixed-Time Period Model

- Check the inventory once every review period and then order a quantity that is large enough to cover demand until the next order will come in
  - time between orders is constant
  - order size may vary
  - safety stock is used to protect against uncertain demand



Fixed-Time Period Model

## Illustration




T: Time between two review points

L: Lead time



Inventory Control with

## Fixed-Time Period Model

- When to order?
  - At pre-specified times such as every Friday, last day of month etc.
- How much to order?

$$q = \bar{d}(T + L) + z\sigma_d\sqrt{T + L} - I$$

where:

q = quantity to be ordered

T = the number of time units (e.g., days) between reviews

L = lead time

$\bar{d}$ = forecast average demand per day

z = number of standard deviations for a specified service level

$\sigma_d$ = standard deviation of daily demand

I = current inventory level (includes items on order)

Fixed-Time Period System

## Example

The Mediterranean Restaurant stocks a red Chilean table wine it purchases from a wine merchant in a nearby city. The daily demand for the wine at the restaurant is normally distributed, with a mean of 18 bottles and a standard deviation of 4 bottles. The wine merchant sends a representative to check the restaurant's wine cellar every 30 days, and during a recent visit there were 25 bottles in stock. The lead time to receive an order is 2 days. The restaurant manager has requested an order size that will enable him to limit the probability of stockout to 2 percent. What is the order size?



Fixed-Time Period System

## Example (cont.)

| | |
|---|---|
| Average demand rate | d = 18 bottles |
| The fixed time between reviews | T = 30 days |
| Lead time | L = 2 days |
| Standard deviation of demand | $\sigma_d$ = 4 |
| Inventory level | I = 25 |
| Service Level = 1 – 2% = 98% | |

Step 1: Look the table in the appendix E and find the z value

Z = 2.05

Step 2: Calculate the order quantity

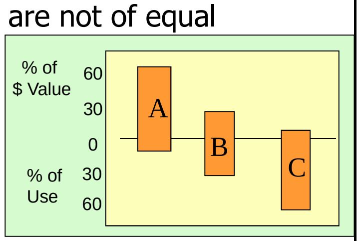
$$Q = 18 \times (30 + 2) + 2.05 \times 4 \times \sqrt{30 + 2} - 25 = 598$$

Safety stock = 46.39



## ABC Classification System


- Items kept in inventory are not of equal importance in terms of
  - dollars invested
  - profit potential
  - sales or usage volume
  - stockout penalties




Classify inventory items based on percentage of total dollar value, where "A" items are roughly top 15 %, "B" items as next 35 %, and the lower 50% are the "C" items