OBJECTIVES

1 | Compute the monthly payment and interest costs for a mortgage.

2 | Prepare a partial loan amortization schedule.

3 | Compute payments and interest for other kinds of fixed installment loans.

4 | Find the interest, the balance due, and the minimum monthly payment for credit card loans.

8.5 Installment Loans, Amortization, and Credit Cards

Do you buy products with a credit card? Although your card lets you use a product while paying for it, the costs associated with such cards, including their high interest rates, fees, and penalties, stack the odds in favor of your getting hurt by them. In 2008, the average credit-card debt per U.S. household was $11,211. If you use a credit card, you are engaging in **installment buying**, in which you repay a loan for the cost of a product on a monthly basis. A loan that you pay off with weekly or monthly payments, or payments in some other time period, is called an **installment loan**. The advantage of an installment loan is that the consumer gets to use a product immediately. In this section, we will see that the disadvantage is that it can add a substantial amount to the cost of a purchase. When it comes to installment buying, consumer beware!

1 Compute the monthly payment and interest costs for a mortgage.

Mortgages

A **mortgage** is a long-term installment loan (perhaps up to 30, 40, or even 50 years) for the purpose of buying a home, and for which the property is pledged as security for payment. If payments are not made on the loan, the lender may take possession of the property. The **down payment** is the portion of the sale price of the home that the buyer initially pays to the seller. The minimum required down payment is computed as a percentage of the sale price. For example, suppose you decide to buy a $220,000 home that requires you to pay the seller 10% of the sale price. You must pay 10% of $220,000, which is $0.10 \times \$220,000$, or $22,000, to the seller. Thus, $22,000 is the down payment. The **amount of the mortgage** is the difference between the sale price and the down payment. For your $220,000 home, the amount of the mortgage is $220,000 − $22,000, or $198,000.

Monthly payments for a mortgage depend on the amount of the mortgage, or the principal, the interest rate, and the time of the mortgage. Mortgages can have a fixed interest rate or a variable interest rate. **Fixed-rate mortgages** have the same monthly payment during the entire time of the loan. A loan like this that has a schedule for paying a fixed amount each period is called a **fixed installment loan**. **Variable-rate mortgages**, also known as **adjustable-rate mortgages** (ARMs), have payment amounts that change from time to time depending on changes in the interest rate. ARMS are less predictable than fixed-rate mortgages. They start out at lower rates than fixed-rate mortgages. Caps limit how high rates can go over the term of the loan.

Computation Involved with Buying a Home

Although monthly payments for a mortgage depend on the amount of the mortgage, the time of the loan, and the interest rate, the rate is not the only cost of a mortgage. Most lending institutions require the buyer to pay one or more **points** at the time of closing—that is, the time at which the mortgage begins. A point is a one-time charge that equals 1% of the loan amount. For example, two points means that the buyer must pay 2% of the loan amount at closing. Often, a buyer can pay fewer points in exchange for a higher interest rate or more points for a lower rate. A document, called the **Truth-in-Lending Disclosure Statement**, shows the buyer the APR for the mortgage. The APR takes into account the interest rate and points.

A monthly mortgage payment is used to repay the principal plus interest. In addition, lending institutions can require monthly deposits into an **escrow account**, an account used by the lender to pay real estate taxes and insurance. These deposits increase the amount of the monthly payment.

We can use formulas for compound interest and the value of an annuity to determine the amount of mortgage payments for fixed-rate mortgages. Suppose that you borrow P dollars at interest rate r over t years.

The lender expects A dollars at the end of t years.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

You save the A dollars in an annuity by paying PMT dollars n times per year.

$$A = \frac{PMT\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}$$

To find your regular payment amount, PMT, we set the amount the lender expects to receive equal to the amount you will save in the annuity:

$$P\left(1 + \frac{r}{n}\right)^{nt} = \frac{PMT\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}.$$

Solving this equation for PMT, we obtain a formula for the loan payment for any installment loan, including payments on fixed-rate mortgages.

LOAN PAYMENT FORMULA FOR FIXED INSTALLMENT LOANS

The regular payment amount, PMT, required to repay a loan of P dollars paid n times per year over t years at an annual rate r is given by

$$PMT = \frac{P\left(\dfrac{r}{n}\right)}{\left[1 - \left(1 + \dfrac{r}{n}\right)^{-nt}\right]}.$$

EXAMPLE 1   **Computing the Monthly Payment and Interest Costs for a Mortgage**

The price of a home is $195,000. The bank requires a 10% down payment and two points at the time of closing. The cost of the home is financed with a 30-year fixed-rate mortgage at 7.5%.

   a. Find the required down payment.

   b. Find the amount of the mortgage.

   c. How much must be paid for the two points at closing?

   d. Find the monthly payment (excluding escrowed taxes and insurance).

   e. Find the total interest paid over 30 years.

Solution

   a. The required down payment is 10% of $195,000 or
$$0.10 \times \$195,000 = \$19,500.$$

   b. The amount of the mortgage is the difference between the price of the home and the down payment.

$$\begin{aligned} \text{Amount of the mortgage} &= \text{sale price} - \text{down payment} \\ &= \$195,000 - \$19,500 \\ &= \$175,500 \end{aligned}$$

   c. To find the cost of two points on a mortgage of $175,500, find 2% of $175,500.
$$0.02 \times \$175,500 = \$3510$$

The down payment ($19,500) is paid to the seller and the cost of two points ($3510) is paid to the lending institution.

   d. We are interested in finding the monthly payment for a $175,500 mortgage at 7.5% for 30 years. We use the loan payment formula for installment loans.

P, the mortgage amount, is $175,500.
Fixed rate, r, is 7.5%.
12 payments per year
The mortgage time, t, is 30 years.

$$PMT = \frac{P\left(\dfrac{r}{n}\right)}{\left[1 - \left(1 + \dfrac{r}{n}\right)^{-nt}\right]} = \frac{175,500\left(\dfrac{0.075}{12}\right)}{\left[1 - \left(1 + \dfrac{0.075}{12}\right)^{-12(30)}\right]}$$

$$= \frac{1096.875}{1 - (1.00625)^{-360}} \approx 1227$$

The monthly mortgage payment for principal and interest is approximately $1227.00. (Keep in mind that this payment does not include escrowed taxes and insurance.)

e. The total cost of interest over 30 years is equal to the difference between the total of all monthly payments and the amount of the mortgage. The total of all monthly payments is equal to the amount of the monthly payment multiplied by the number of payments. We found the amount of each monthly payment in (d): $1227. The number of payments is equal to the number of months in a year, 12, multiplied by the number of years in the mortgage, 30: $12 \times 30 = 360$. Thus, the total of all monthly payments = 1227×360.

Now we can calculate the interest over 30 years.

$$\boxed{\text{Total interest paid}} = \boxed{\begin{array}{c}\text{total of all} \\ \text{monthly payments}\end{array}} \quad \boxed{\text{minus}} \quad \boxed{\begin{array}{c}\text{amount of} \\ \text{the mortgage.}\end{array}}$$

$$= \ \$1227 \times 360 - \$175,500$$

$$= \ \$441,720 - \$175,500 = \$266,220$$

The total interest paid over 30 years is approximately $266,220.

> **CHECK POINT 1** In Example 1, the $175,500 mortgage was financed with a 30-year fixed rate at 7.5%. The total interest paid over 30 years was approximately $266,220.
>
> **a.** Use the loan payment formula for installment loans to find the monthly payment if the time of the mortgage is reduced to 15 years. Round to the nearest dollar.
>
> **b.** Find the total interest paid over 15 years.
>
> **c.** How much interest is saved by reducing the mortgage from 30 to 15 years?

2 Prepare a partial loan amortization schedule.

Loan Amortization Schedules

When a loan is paid off through a series of regular payments, it is said to be **amortized**, which literally means "killed off." In working Check Point 1(c), were you surprised that nearly $150,000 was saved when the mortgage was amortized over 15 years rather than over 30 years? What adds to the interest cost is the long period over which the loan is financed. **Although each payment is the same, with each successive payment the interest portion decreases and the portion applied toward paying off the principal increases.** The interest is computed using the simple interest formula $I = Prt$. The principal, P, is equal to the balance of the loan, which changes each month. The rate, r, is the annual interest rate of the mortgage loan. Because a payment is made each month, the time, t, is

$$\frac{1 \text{ month}}{12 \text{ months}} = \frac{1 \cancel{\text{ month}}}{12 \cancel{\text{ months}}}$$

or $\frac{1}{12}$ of a year.

A document showing how the payment each month is split between interest and principal is called a **loan amortization schedule**. Typically, this document includes the number of the most recent payment and those of any previous monthly payments, the interest for each payment, the amount of each payment applied to the principal, and the balance of the loan.

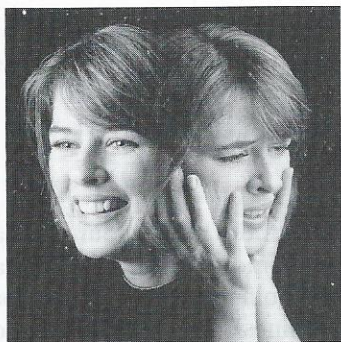**EXAMPLE 2** **Preparing a Loan Amortization Schedule**

Prepare a loan amortization schedule for the first two months of the mortgage loan shown in the table at the top of the next page.

LOAN AMORTIZATION SCHEDULE

Annual % rate: 9.5% Amount of Mortgage: $130,000 Number of Monthly Payments: 180		Monthly payment: $1357.50 Term: Years 15, Months 0	
Payment Number	**Interest Payment**	**Principal Payment**	**Balance of Loan**
1			
2			

Solution   We begin with payment number 1.

$$\text{Interest for the month} = Prt = \$130,000 \times 0.095 \times \frac{1}{12} \approx \$1029.17$$

$$\begin{aligned} \text{Principal payment} &= \text{Monthly payment} - \text{Interest payment} \\ &= \$1357.50 - \$1029.17 = \$328.33 \end{aligned}$$

$$\begin{aligned} \text{Balance of loan} &= \text{Principal balance} - \text{Principal payment} \\ &= \$130,000 - \$328.33 = \$129,671.67 \end{aligned}$$

Now, starting with a loan balance of $129,671.67, we repeat these computations for the second month.

$$\text{Interest for the month} = Prt = \$129,671.67 \times 0.095 \times \frac{1}{12} \approx \$1026.57$$

$$\begin{aligned} \text{Principal payment} &= \text{Monthly payment} - \text{Interest payment} \\ &= \$1357.50 - \$1026.57 = \$330.93 \end{aligned}$$

$$\begin{aligned} \text{Balance of loan} &= \text{Principal balance} - \text{Principal payment} \\ &= \$129,671.67 - \$330.93 = \$129,340.74 \end{aligned}$$

The results of these computations are included in **Table 8.4**, a partial loan amortization schedule. By using the simple interest formula month-to-month on the loan's balance, a complete loan amortization schedule for all 180 payments can be calculated.

TABLE 8.4 Loan Amortization Schedule

Annual % rate: 9.5% Amount of Mortgage: $130,000 Number of Monthly Payments: 180		Monthly payment: $1357.50 Term: Years 15, Months 0	
Payment Number	**Interest Payment**	**Principal Payment**	**Balance of Loan**
1	$1029.17	$328.33	$129,671.67
2	1026.57	330.93	129,340.74
3	1023.96	333.54	129,007.22
4	1021.32	336.18	128,671.04
30	944.82	412.68	118,931.35
31	941.55	415.95	118,515.52
125	484.62	872.88	60,340.84
126	477.71	879.79	59,461.05
179	21.26	1336.24	1347.74
180	9.76	1347.74	

Many lenders supply a loan amortization schedule like the one in Example 2 at the time of closing. Such a schedule shows how the buyer pays slightly less in interest and more in principal for each payment over the entire life of the loan.

CHECK POINT 2 Prepare a loan amortization schedule for the first two months of the mortgage loan shown in the following table:

Annual % rate: 7.0% Amount of Mortgage: $200,000 Number of Monthly Payments: 240		Monthly payment: $1550.00 Term: Years 20, Months 0	
Payment Number	Interest Payment	Principal Payment	Balance of Loan
1			
2			

3 Compute payments and interest for other kinds of fixed installment loans.

Monthly Payments and Interest Costs for Other Kinds of Fixed Installment Loans

The loan payment formula can be used to determine how much your regular payments will be on fixed installment loans other than mortgages, including car loans and student loans. The portions of each payment going toward the principal and toward the interest will vary as the loan balance declines. Near the beginning of the loan term, the portion going toward the interest will be relatively high and the portion going toward the principal will be relatively low. As the loan term continues, the principal portion will gradually increase and the interest portion will gradually decrease.

EXAMPLE 3 **Comparing Car Loans**

You decide to borrow $20,000 for a new car. You can select one of the following loans, each requiring regular monthly payments:

Installment Loan A: 3-year loan at 7%.
Installment Loan B: 5-year loan at 9%.

a. Find the monthly payments and the total interest for Loan A.
b. Find the monthly payments and the total interest for Loan B.
c. Compare the monthly payments and total interest for the two loans.

Solution For each loan, we use the loan payment formula to compute the monthly payments.

a. We first determine monthly payments and total interest for Loan A.

P, the loan amount, is $20,000.

Rate, r, is 7%.

12 payments per year

The loan is for 3 years.

$$PMT = \frac{P\left(\frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]} = \frac{20{,}000\left(\frac{0.07}{12}\right)}{\left[1 - \left(1 + \frac{0.07}{12}\right)^{-12(3)}\right]} \approx 618$$

The monthly payments are approximately $618.

Now we calculate the interest over 3 years, or 36 months.

$$\text{Total interest over 3 years} = \text{Total of all monthly payments} \quad \text{minus} \quad \text{amount of the loan.}$$

$$= \$618 \times 36 - \$20{,}000$$
$$= \$2248$$

The total interest paid over 3 years is approximately $2248.

b. Next, we determine monthly payments and total interest for Loan B.

P, the loan amount, is $20,000.   Rate, *r*, is 9%.

$$PMT = \frac{P\left(\dfrac{r}{n}\right)}{\left[1 - \left(1 + \dfrac{r}{n}\right)^{-nt}\right]} = \frac{20{,}000\left(\dfrac{0.09}{12}\right)}{\left[1 - \left(1 + \dfrac{0.09}{12}\right)^{-12(5)}\right]} \approx 415$$

12 payments per year

The loan is for 5 years.

The monthly payments are approximately $415.
Now we calculate the interest over 5 years, or 60 months.

$$\text{Total interest over 5 years} = \text{Total of all monthly payments} \quad \text{minus} \quad \text{amount of the loan.}$$

$$= \$415 \times 60 - \$20{,}000$$
$$= \$4900$$

The total interest paid over 5 years is approximately $4900.

c. **Table 8.6** compares the monthly payments and total interest for the two loans.

TABLE 8.6 Comparing Car Loans

$20,000 loan	Monthly Payment	Total Interest
3-year loan at 7%	$618	$2248
5-year loan at 9%	$415	$4900

Monthly payments are less with the longer-term loan.

Interest is more with the longer-term loan.

CHECK POINT 3   You decide to borrow $15,000 for a new car. You can select one of the following loans, each requiring regular monthly payments:

Installment Loan A: 4-year loan at 8%

Installment Loan B: 6-year loan at 10%

a. Find the monthly payments and the total interest for Loan A.

b. Find the monthly payments and the total interest for Loan B.

c. Compare the monthly payments and total interest for the two loans.

Open-End Installment Loans

Using a credit card is an example of an open-end installment loan, commonly called **revolving credit**. Open-end loans differ from fixed installment loans such as the car loans in Example 3 in that there is no schedule for paying a fixed amount each period. Credit card loans require users to make only a minimum monthly payment that depends on the unpaid balance and the interest rate. Credit cards have high interest rates compared to other kinds of loans. The interest on credit cards is computed using the simple interest formula $I = Prt$. However, r represents the *monthly* interest rate and t is time in months rather than in years. A typical interest rate is 1.57% monthly. This is equivalent to a yearly rate of $12 \times 1.57\%$, or 18.84%. With such a high annual percentage rate, credit card balances should be paid off as quickly as possible.

Most credit card customers are billed every month. A typical billing period is May 1 through May 31, but it can also run from, say, May 5 through June 4. Customers receive a statement, called an **itemized billing**, that includes the unpaid balance on the first day of the billing period, the total balance owed on the last day of the billing period, a list of purchases and cash advances made during the billing period, any finance charges or other fees incurred, the date of the last day of the billing period, the payment due date, and the minimum payment required.

Customers who make a purchase during the billing period and pay the entire amount of the purchase by the payment due date are not charged interest. By contrast, customers who make cash advances using their credit cards must pay interest from the day the money is advanced until the day it is repaid.

4 | Find the interest, the balance due, and the minimum monthly payment for credit card loans.

Interest on Credit Cards: The Average Daily Balance Method

Methods for calculating interest, or finance charges, on credit cards may vary and the interest can differ on credit cards that show the same annual percentage rate, or APR. The method used for calculating interest on most credit cards is called the *average daily balance method*.

THE AVERAGE DAILY BALANCE METHOD

Interest is calculated using $I = Prt$, where r is the monthly rate and t is one month. The principal, P, is the average daily balance. The **average daily balance** is the sum of the unpaid balances for each day in the billing period divided by the number of days in the billing period.

Average daily balance

$$= \frac{\text{Sum of the unpaid balances for each day in the billing period}}{\text{Number of days in the billing period}}$$

In Example 4, we illustrate how to determine the average daily balance. At the conclusion of the example, we summarize the steps used in the computation.

EXAMPLE 4 Balance Due on a Credit Card

A particular VISA card calculates interest using the average daily balance method. The monthly interest rate is 1.3% of the average daily balance. The following transactions occurred during the May 1–May 31 billing period.

THE MINIMUM-PAYMENT TRAP

Credit-card debt is made worse by paying only the required minimum, a mistake made by 11% of credit-card debtors. Pay the minimum and most of it goes to interest charges.

Average Credit-Card Debt for United States College Students

	2038	2362	2912	4138
	Freshmen	Sophomores	Juniors	Seniors
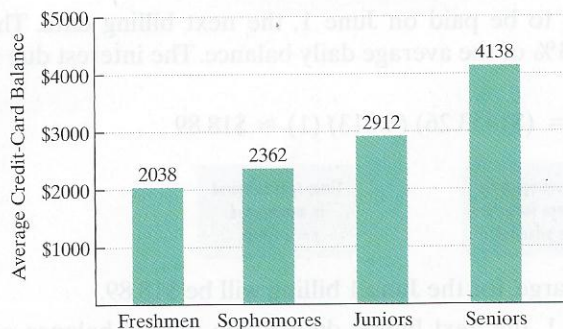
Source: Nellie Mae

Transaction Description	Transaction Amount
Previous balance, $1350.00	
May 1 Billing date	
May 8 Payment	$250.00 Credit
May 10 Charge: Airline Tickets	$375.00
May 20 Charge: Books	$ 57.50
May 28 Charge: Restaurant	$ 65.30
May 31 End of billing period	
Payment Due Date: June 9	

a. Find the average daily balance for the billing period. Round to the nearest cent.

b. Find the interest to be paid on June 1, the next billing date. Round to the nearest cent.

c. Find the balance due on June 1.

d. This credit card requires a $10 minimum monthly payment if the balance due at the end of the billing period is less than $360. Otherwise, the minimum monthly payment is $\frac{1}{36}$ of the balance due at the end of the billing period, rounded up to the nearest whole dollar. What is the minimum monthly payment due by June 9?

Solution

a. We begin by finding the average daily balance for the billing period. First make a table that shows the beginning date of the billing period, each transaction date, and the unpaid balance for each date.

Date	Unpaid Balance	
May 1	$1350.00	*previous balance*
May 8	$1350.00 − $250.00 = $1100.00	*$250.00 payment*
May 10	$1100.00 + $375.00 = $1475.00	*$375.00 charge*
May 20	$1475.00 + $57.50 = $1532.50	*$57.50 charge*
May 28	$1532.50 + $65.30 = $1597.80	*$65.30 charge*

We now extend our table by adding two columns. One column shows the number of days at each unpaid balance. The final column shows each unpaid balance multiplied by the number of days that the balance is outstanding.

Date	Unpaid Balance	Number of Days at Each Unpaid Balance	$\left(\begin{array}{c}\text{Unpaid}\\\text{Balance}\end{array}\right) \cdot \left(\begin{array}{c}\text{Number}\\\text{of Days}\end{array}\right)$
May 1	$1350.00	7	($1350.00)(7) = $9450.00
May 8	$1100.00	2	($1100.00)(2) = $2200.00
May 10	$1475.00	10	($1475.00)(10) = $14,750.00
May 20	$1532.50	8	($1532.50)(8) = $12,260.00
May 28	$1597.80	4	($1597.80)(4) = $6391.20
		Total: 31	Total: $45,051.20

There are 4 days at this unpaid balance, May 28, 29, 30, and 31 before the beginning of the next billing period, June 1.

This is the number of days in the billing period.

This is the sum of the unpaid balances for each day in the billing period.

Notice that we found the sum of the products in the final column of the table. This dollar amount, $45,051.20, gives the sum of the unpaid balances for each day in the billing period.

Now we divide the sum of the unpaid balances for each day in the billing period, $45,051.20, by the number of days in the billing period, 31. This gives the average daily balance.

Average daily balance

$$= \frac{\text{Sum of the unpaid balances for each day in the billing period}}{\text{Number of days in the billing period}}$$

$$= \frac{\$45,051.20}{31} \approx \$1453.26$$

The average daily balance is approximately $1453.26.

b. Now we find the interest to be paid on June 1, the next billing date. The monthly interest rate is 1.3% of the average daily balance. The interest due is computed using $I = Prt$.

$$I = Prt = (\$1453.26)(0.013)(1) \approx \$18.89$$

> The average daily balance serves as the principal.

> Time, t, is measured in months, and $t = 1$ month.

The interest, or finance charge, for the June 1 billing will be $18.89.

c. The balance due on June 1, the next billing date, is the unpaid balance on May 31 plus the interest.

$$\text{Balance due} = \$1597.80 + \$18.89 = \$1616.69$$

> Unpaid balance on May 31, obtained from the table at the bottom of the previous page

> Interest, or finance charge, obtained from part (b)

The balance due on June 1 is $1616.69.

d. Because the balance due, $1616.69, exceeds $360, the customer must pay a minimum of $\frac{1}{36}$ of the balance due.

$$\text{Minimum monthly payment} = \frac{\text{balance due}}{36} = \frac{\$1616.69}{36} \approx \$45$$

Rounded up to the nearest whole dollar, the minimum monthly payment due by June 9 is $45.

BLITZER BONUS

THE CHANGING-TERMS TRAP

Consumer Reports (October 2008) cited a credit card with an enticing 9.9% annual interest rate. But the fine print revealed a $29 account-setup fee, a $95 program fee, a $48 annual fee, and a $7 monthly servicing fee. Nearly 40% of the $40 billion in profits that U.S. card issuers earned in 2008 came from fees. Furthermore, issuers can hike interest rates and fees at any time, for any reason. In 2009, these abuses had Congress listening, with proposed reforms that would bring sweeping changes to the lightly regulated, $160 billion credit-card industry.

The following box summarizes the steps used in Example 4 to determine the average daily balance. Calculating the average daily balance can be quite tedious when there are numerous transactions during a billing period.

DETERMINING THE AVERAGE DAILY BALANCE

Step 1 Make a table that shows the beginning date of the billing period, each transaction date, and the unpaid balance for each date.

Step 2 Add a column to the table that shows the number of days at each unpaid balance.

Step 3 Add a final column to the table that shows each unpaid balance multiplied by the number of days that the balance is outstanding.

Step 4 Find the sum of the products in the final column of the table. This dollar amount is the sum of the unpaid balances for each day in the billing period.

Step 5 Compute the average daily balance.

Average daily balance

$$= \frac{\text{Sum of the unpaid balances for each day in the billing period}}{\text{Number of days in the billing period}}$$

CHECK POINT 4

A credit card calculates interest using the average daily balance method. The monthly interest rate is 1.6% of the average daily balance. The following transactions occurred during the May 1–May 31 billing period.

Transaction Description	Transaction Amount
Previous balance, $8240.00	
May 1 Billing date	
May 7 Payment	$ 350.00 Credit
May 15 Charge: Computer	$1405.00
May 17 Charge: Restaurant	$ 45.20
May 30 Charge: Clothing	$ 180.72
May 31 End of billing period	
Payment Due Date: June 9	

Answer parts (a) through (d) in Example 4 using this information.

Exercise Set 8.5

Practice and Application Exercises

In Exercises 1–18, use

$$PMT = \frac{P\left(\dfrac{r}{n}\right)}{\left[1 - \left(1 + \dfrac{r}{n}\right)^{-nt}\right]}$$

to determine the regular payment amount, rounded to the nearest dollar.

Exercises 1–8 involve home mortgages.

1. The price of a home is $220,000. The bank requires a 20% down payment and three points at the time of closing. The cost of the home is financed with a 30-year fixed-rate mortgage at 7%.

   a. Find the required down payment.

   b. Find the amount of the mortgage.

   c. How much must be paid for the three points at closing?

   d. Find the monthly payment (excluding escrowed taxes and insurance).

   e. Find the total cost of interest over 30 years.

2. The price of a condominium is $180,000. The bank requires a 5% down payment and one point at the time of closing. The cost of the condominium is financed with a 30-year fixed-rate mortgage at 8%.

   a. Find the required down payment.

   b. Find the amount of the mortgage.

   c. How much must be paid for the one point at closing?

   d. Find the monthly payment (excluding escrowed taxes and insurance).

   e. Find the total cost of interest over 30 years.

3. The price of a small cabin is $100,000. The bank requires a 5% down payment. The buyer is offered two mortgage options: 20-year fixed at 8% or 30-year fixed at 8%. Calculate the amount of interest paid for each option. How much does the buyer save in interest with the 20-year option?

4. The price of a home is $160,000. The bank requires a 15% down payment. The buyer is offered two mortgage options: 15-year fixed at 8% or 30-year fixed at 8%. Calculate the amount of interest paid for each option. How much does the buyer save in interest with the 15-year option?

5. In terms of paying less in interest, which is more economical for a $150,000 mortgage: a 30-year fixed-rate at 8% or a 20-year fixed-rate at 7.5%? How much is saved in interest?

6. In terms of paying less in interest, which is more economical for a $90,000 mortgage: a 30-year fixed-rate at 8% or a 15-year fixed-rate at 7.5%? How much is saved in interest?

In Exercises 7–8, which mortgage loan has the greater total cost (closing costs + the amount paid for points + total cost of interest)? By how much?

7. A $120,000 mortgage with two loan options:

   Mortgage A: 30-year fixed at 7% with closing costs of $2000 and one point

   Mortgage B: 30-year fixed at 6.5% with closing costs of $1500 and four points

8. A $250,000 mortgage with two loan options:

   Mortgage A: 30-year fixed at 7.25% with closing costs of $2000 and one point

   Mortgage B: 30-year fixed at 6.25% with closing costs of $350 and four points

Exercises 9–18 involve installment loans other than mortgages.

9. Your credit card has a balance of $4200 and an annual interest rate of 18%. You decide to pay off the balance over two years. If there are no further purchases charged to the card,

a. How much must you pay each month?

b. How much total interest will you pay?

10. Your credit card has a balance of $3600 and an annual interest rate of 16.5%. You decide to pay off the balance over two years. If there are no further purchases charged to the card,

a. How much must you pay each month?

b. How much total interest will you pay?

11. To pay off the $4200 credit-card balance in Exercise 9, you can get a bank loan at 10.5% with a term of three years.

a. How much will you pay each month? How does this compare with your credit-card payment in Exercise 9?

b. How much total interest will you pay? How does this compare with your total credit-card interest in Exercise 9?

12. To pay off the $3600 credit-card balance in Exercise 10, you can get a bank loan at 9.5% with a term of three years.

a. How much will you pay each month? How does this compare with your credit-card payment in Exercise 10?

b. How much total interest will you pay? How does this compare with your total credit-card interest in Exercise 10?

13. Rework Exercise 9 if you decide to pay off the balance over one year rather than two. How much more must you pay each month and how much less will you pay in total interest?

14. Rework Exercise 10 if you decide to pay off the balance over one year rather than two. How much more must you pay each month and how much less will you pay in total interest?

In Exercises 15–18, round to the nearest cent.

15. You borrow $10,000 for four years at 8% toward the purchase of a car.

a. Find the monthly payments and the total interest for the loan.

b. Prepare a loan amortization schedule for the first three months of the car loan. Round entries to the nearest cent.

Payment Number	Interest	Principal	Loan Balance
1			
2			
3			

16. You borrow $30,000 for four years at 8% toward the purchase of a car.

a. Find the monthly payments and the total interest for the loan.

b. Prepare a loan amortization schedule for the first three months of the car loan. Use the table in Exercise 15(b) and round entries to the nearest cent.

17. A student graduates from college with a loan of $40,000. The interest rate is 8.5% and the loan term is 20 years.

a. Find the monthly payments and the total interest for the loan.

b. Prepare a loan amortization schedule for the first three months of the student loan. Use the table in Exercise 15(b) and round entries to the nearest cent.

c. If the interest rate remains at 8.5% and the loan term is reduced to ten years, how much more must the student pay each month and how much less will be paid in total interest?

18. A student graduates from college with a loan of $50,000. The interest rate is 7.5% and the loan term is 20 years.

a. Find the monthly payments and the total interest for the loan.

b. Prepare a loan amortization schedule for the first three months of the student loan. Use the table in Exercise 15(b) and round entries to the nearest cent.

c. If the interest rate remains at 7.5% and the loan term is reduced to ten years, how much more must the student pay each month and how much less will be paid in total interest?

Exercises 19–20 involve credit cards that calculate interest using the average daily balance method. The monthly interest rate is 1.5% of the average daily balance. Each exercise shows transactions that occurred during the March 1–March 31 billing period. In each exercise,

a. Find the average daily balance for the billing period. Round to the nearest cent.

b. Find the interest to be paid on April 1, the next billing date. Round to the nearest cent.

c. Find the balance due on April 1.

d. This credit card requires a $10 minimum monthly payment if the balance due at the end of the billing period is less than $360. Otherwise, the minimum monthly payment is $\frac{1}{36}$ of the balance due at the end of the billing period, rounded up to the nearest whole dollar. What is the minimum monthly payment due by April 9?

19.

Transaction Description	Transaction Amount
Previous balance, $6240.00	
March 1 Billing date	
March 5 Payment	$300.00 credit
March 7 Charge: Restaurant	$ 40.00
March 12 Charge: Groceries	$ 90.00
March 21 Charge: Car Repairs	$230.00
March 31 End of billing period	
Payment Due Date: April 9	

20.

Transaction Description	Transaction Amount
Previous balance, $7150.00	
March 1 Billing date	
March 4 Payment	$ 400.00 credit
March 6 Charge: Furniture	$1200.00
March 15 Charge: Gas	$ 40.00
March 30 Charge: Groceries	$ 50.00
March 31 End of billing period	
Payment Due Date: April 9	

Exercises 21–22 involve credit cards that calculate interest using the average daily balance method. The monthly interest rate is 1.2% of the average daily balance. Each exercise shows transactions that occurred during the June 1-June 30 billing period. In each exercise,

a. Find the average daily balance for the billing period. Round to the nearest cent.

b. Find the interest to be paid on July 1, the next billing date. Round to the nearest cent.

c. Find the balance due on July 1.

d. This credit card requires a $30 minimum monthly payment if the balance due at the end of the billing period is less than $400. Otherwise, the minimum monthly payment is $\frac{1}{25}$ of the balance due at the end of the billing period, rounded up to the nearest whole dollar. What is the minimum monthly payment due by July 9?

21.

Transaction Description	Transaction Amount
Previous balance, $2653.48	
June 1 Billing date	
June 6 Payment	$1000.00 credit
June 8 Charge: Gas	$ 36.25
June 9 Charge: Groceries	$ 138.43
June 17 Charge: Gas	$ 42.36
Charge: Groceries	$ 127.19
June 27 Charge: Clothing	$ 214.83
June 30 End of billing period	
Payment Due Date: July 9	

22.

Transaction Description	Transaction Amount
Previous balance, $4037.93	
June 1 Billing date	
June 5 Payment	$350.00 credit
June 10 Charge: Gas	$ 31.17
June 15 Charge: Prescriptions	$ 42.50
June 22 Charge: Gas	$ 43.86
Charge: Groceries	$112.91
June 29 Charge: Clothing	$ 96.73
June 30 End of billing period	
Payment Due Date: July 9	

Writing in Mathematics

23. What is a mortgage?

24. What is a down payment?

25. How is the amount of a mortgage determined?

26. Describe why a buyer would select a 30-year fixed-rate mortgage instead of a 15-year fixed-rate mortgage if interest rates are $\frac{1}{4}$% to $\frac{1}{2}$% lower on a 15-year mortgage.

27. Describe one advantage and one disadvantage of an adjustable-rate mortgage over a fixed-rate mortgage.

28. What is a loan amortization schedule?

29. Describe what happens to the portions of payments going to principal and interest over the life of an installment loan.

30. Describe one advantage and one disadvantage of home ownership over renting.

31. Describe the difference between a fixed installment loan and an open-end installment loan.

32. For a credit card billing period, describe how the average daily balance is determined. Why is this computation somewhat tedious when done by hand?

Critical Thinking Exercises

Make Sense?   *In Exercises 33–36, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

33. There must be an error in the loan amortization schedule for my mortgage because the annual interest rate is only 3.5%, yet the schedule shows that I'm paying more on interest than on the principal for many of my payments.

34. Assuming that a 3-year car loan has a lower interest rate than a 5-year car loan, people should always select the 3-year loan.

35. I like to keep all my money, so I pay only the minimum required payment on my credit card.

36. I used the formula

$$PMT = \frac{P\left(\dfrac{r}{n}\right)}{\left[1 - \left(1 + \dfrac{r}{n}\right)^{-nt}\right]}$$

to determine the payment due on my credit card.

37. Use the discussion at the bottom of page 482 to prove the loan payment formula shown in the box on page 483. Work with the equation in which the amount the lender expects to receive is equal to the amount saved in the annuity. Multiply both sides of this equation by $\frac{r}{n}$ and then solve for *PMT* by dividing both sides by the appropriate expression. Finally, divide the numerator and the denominator of the resulting formula for *PMT* by $\left(1 + \frac{r}{n}\right)^{nt}$ to obtain the form of the loan payment formula shown in the box.

38. The unpaid balance of an installment loan is equal to the present value of the remaining payments. The unpaid balance, *P*, is given by

$$P = PMT \frac{\left[1 - \left(1 + \dfrac{r}{n}\right)^{-nt}\right]}{\left(\dfrac{r}{n}\right)},$$

where *PMT* is the regular payment amount, *r* is the annual interest rate, *n* is the number of payments per year, and *t* is the number of years remaining in the mortgage.

a. Use the loan payment formula to derive the unpaid balance formula.