

# Introduction to production scheduling

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# Introduction to production scheduling

- Scheduling
- Production scheduling
- Gantt Chart
- Scheduling environment
- Constraints
- Scheduling objectives

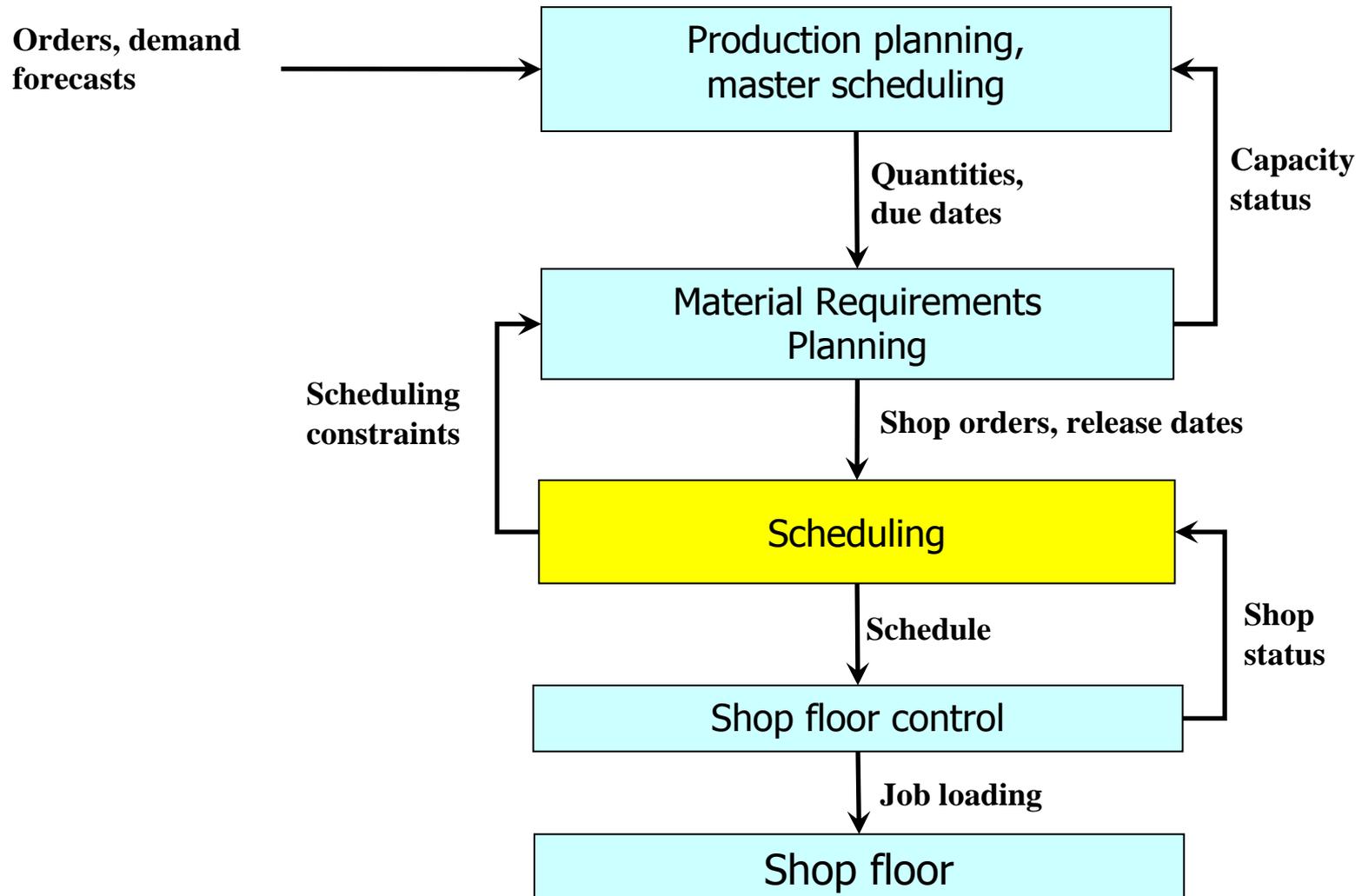
# Scheduling (I)

- Definition
  - Scheduling deals with the efficient allocation of tasks over resources
  - The general scheduling problem is, given a number of tasks and a number of resources, set the dates when each task should be accomplished on each resource
- We are interested in scheduling in the manufacturing context, although it has many applications in other fields

# Scheduling (II)

- Real-life examples
  - Timetabling
    - Scheduling of the PC room at Duisburg University: a number of different courses (tasks) have to be given using the PC room (resource)
  - Workforce scheduling
    - Assign shifts for nurses and doctors in a Hospital
  - Sports scheduling

# Production Scheduling (I)



# Production Scheduling (II)

- The MRP tell us the quantities of products to manufacture in every time bucket
  - However, MRP does not make any assumption about the resources (i.e. labour, machines) currently available in the factory
    - E.g. two different components have to be manufactured in the same section. How to schedule them?
- Therefore, we have a number of jobs ( $j$ ) to be manufactured over a number of machines ( $i$ )
  - The production scheduling problem deals with obtaining the date for each job to enter on each machine
  - Not necessarily physical machines, they may be stages (consisting on several machines or labour) in a manufacturing process

# Production Scheduling (III)

- Jobs have to be manufactured in each machine in a certain order (known as route) during a certain time period (known as processing time)
  - Processing time of job  $j$  in machine  $i$  is usually denoted by  $p_{i,j}$
- Both route and time period are given by the technological process

# Example1: Computer assembly (I)

- Tomcat Ltd. is a company that assembles computers. Three main steps can be distinguished in this process:
  - Motherboard & microprocessor are installed
  - Peripheral devices are plugged in the motherboard
    - The number and type of devices that have been order by each customer
  - The computer (all its components) are tested
    - This process depends on the number and type of components
- The route through the steps is given by the technological process and does not depend on the specific order of the customer

# Example1: Computer assembly (II)

- The plant in which Tomcat Ltd. assembles the computers is organised in three sections, according to the three main steps:
  - Section 1: Motherboard & microprocessor
  - Section 2: Peripheral devices
  - Section 3: Computer test
- On each section, one worker is performing the corresponding operation
  - Obviously, a new order cannot start until the worker completes the current order
  - Let us assume that an order cannot overtake another order, i.e. the job sequence is the same for all steps

# Example1: Computer assembly (III)

- Let us assume that we have three orders (computers to manufacture).
  - According to the nature of each order (components, type, etc.), we can have some estimate of the average times (minutes) for each step:

|                 | <b>Motherboard</b> | <b>Devices</b> | <b>Test</b> |
|-----------------|--------------------|----------------|-------------|
| <b>Order #1</b> | 2                  | 15             | 7           |
| <b>Order #2</b> | 3                  | 10             | 5           |
| <b>Order #3</b> | 2                  | 12             | 5           |

- The objective of the company is to keep the average time to assembly a computer as lowest as possible

# Example1: Computer assembly (IV)

- Which of the following sequences is the most convenient for the objective of the company:
  - #1, #2, #3?
  - #3, #2, #1?
  - Or the order of the jobs is not relevant for the final result?

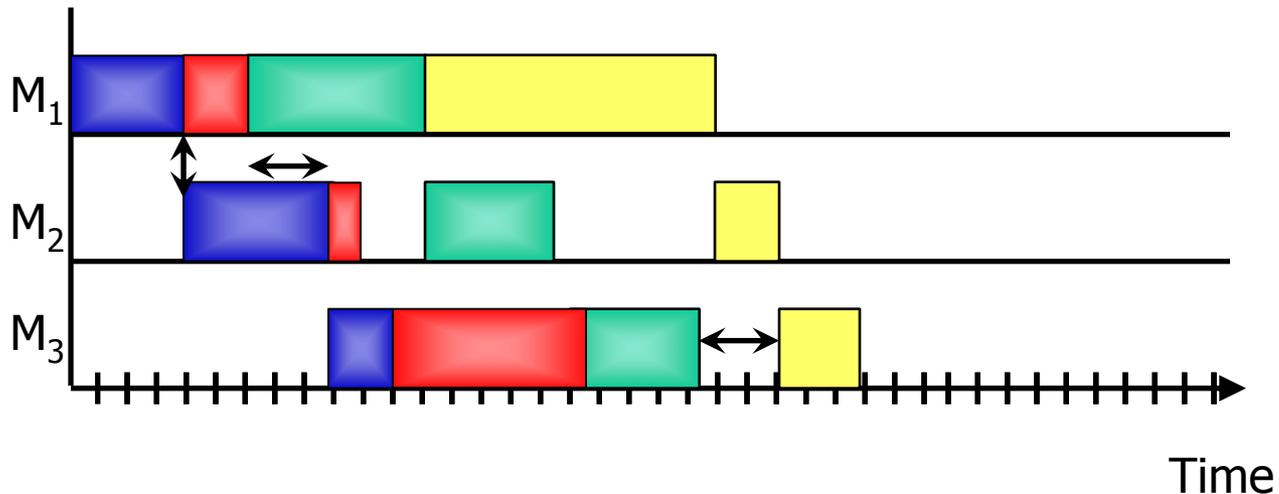
|          | Motherboard | Devices | Test |
|----------|-------------|---------|------|
| Order #1 | 2           | 15      | 7    |
| Order #2 | 3           | 10      | 5    |
| Order #3 | 2           | 12      | 5    |



# Gantt chart

- A representation of an specific solution of a scheduling problem in terms of the machines and jobs

- Eg. job1 , job2 , job3 , job4 
- Sequence [1,2,3,4]



# Example2: Representation of solutions (I)

- Using the data of the Example1, try to represent the sequences [1,2,3] and [3,2,1] by a Gantt chart
- job1 , job2 , job3 

|          | Motherboard | Devices | Test |
|----------|-------------|---------|------|
| Order #1 | 2           | 15      | 7    |
| Order #2 | 3           | 10      | 5    |
| Order #3 | 2           | 12      | 5    |



# Solution of Example2

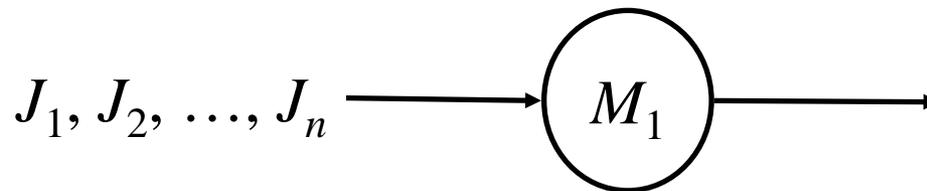
# Scheduling environment (I)

- The environment or framework of a scheduling problem refers to the way the jobs must visit the machines:
  - Single machine
  - Parallel machines
  - Flow shops
  - Flexible flow shops
  - Job shops
  - Flexible job shops
  - Open shop
- The environment largely determines the difficulty of the scheduling problem

# Scheduling environment (II)

- Single machine

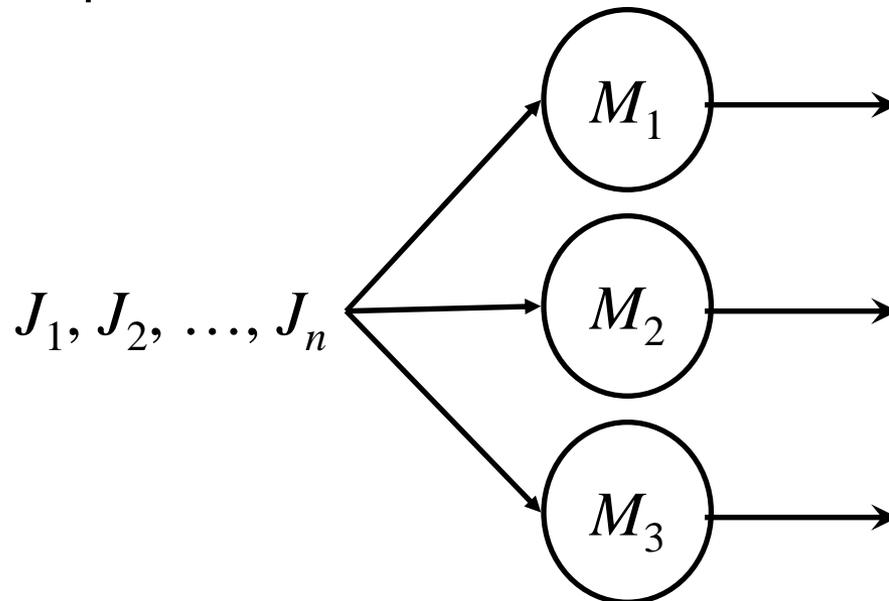
- The simplest of all machine environments
- One may reduce the different steps (sections) in the plant to a single machine
- Interesting case: bottleneck process, the important issue is scheduling jobs in the bottleneck



# Scheduling environment (III)

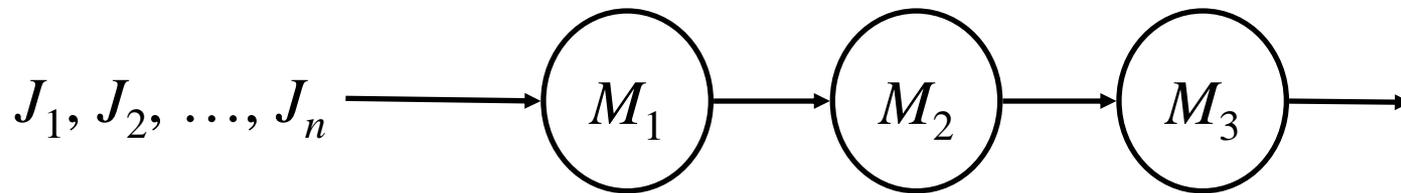
- Parallel machines

- All machines are identical
- A job can be processed on any machine
- Generalization of the single machine
- Special case of the flexible flow shop and flexible job shop



# Scheduling environment (IV)

- Flow shops
  - All jobs have the same routing

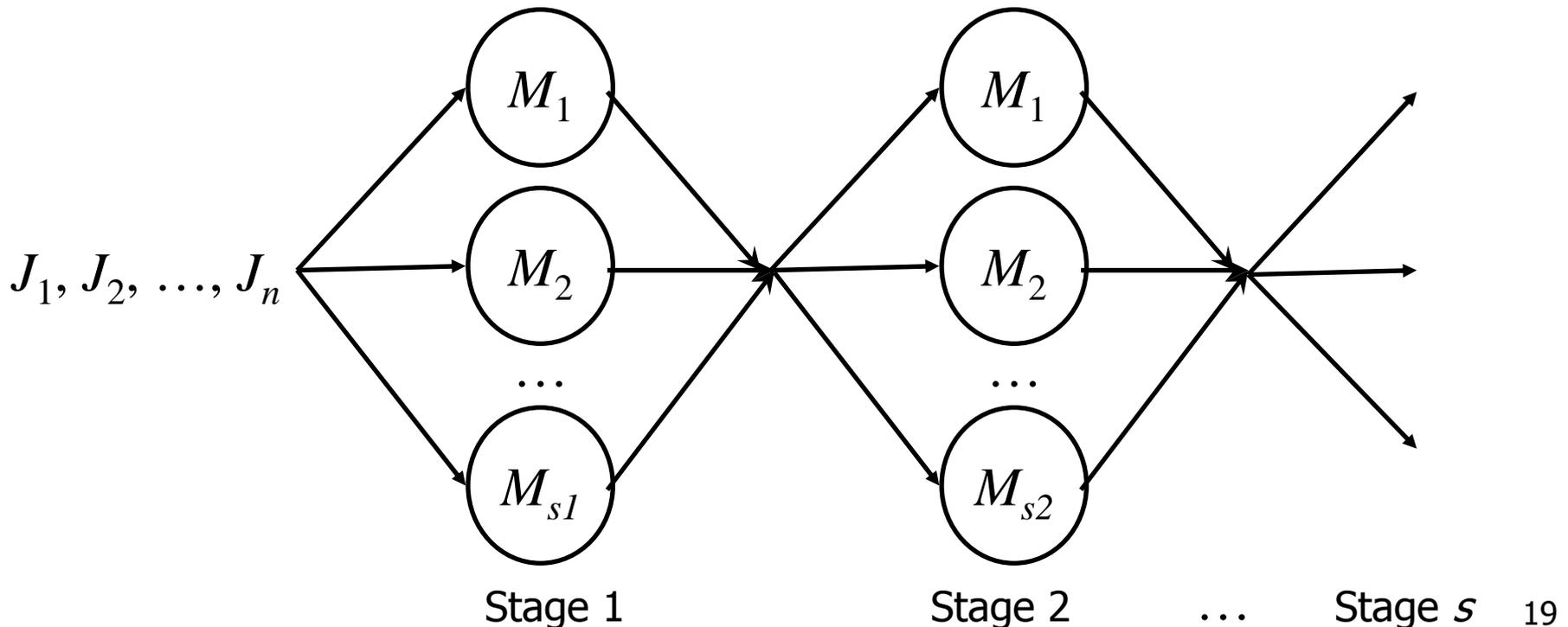


- Additionally, most of the times it is consider than the sequence is the same for all machines
  - Permutation flow shop, e.g. in Tomcat Ltd

# Scheduling environment (V)

- Flexible flow shop

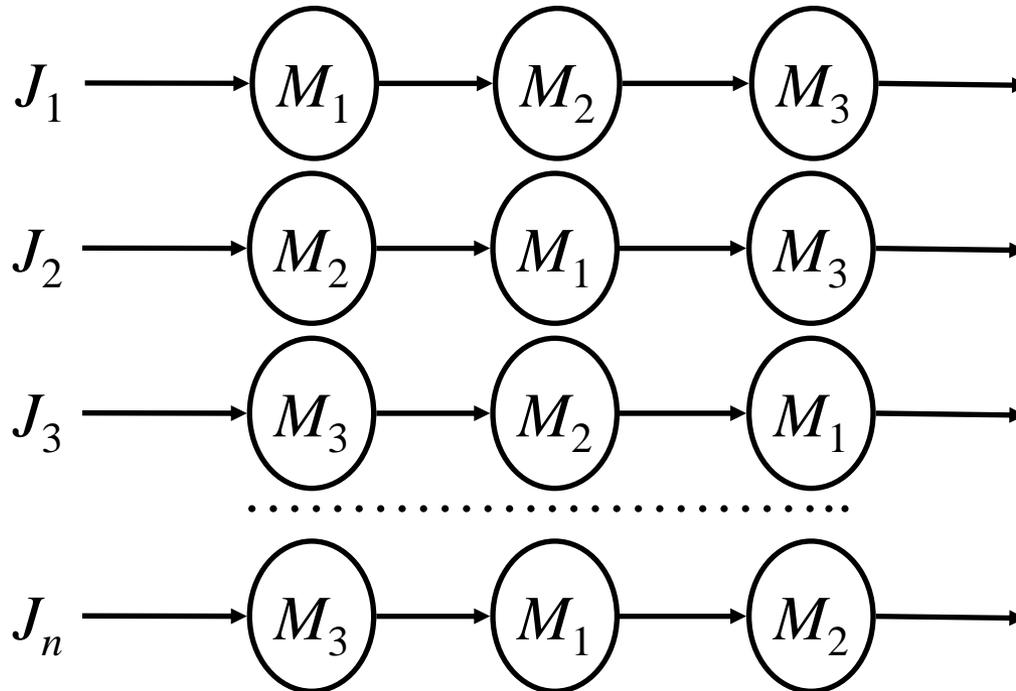
- $s$  stages with  $m_s$  machines in parallel
- A job can be processed on any machine at each stage
- Generalization of parallel machines and flow shop



# Scheduling environment (VI)

- Job shops

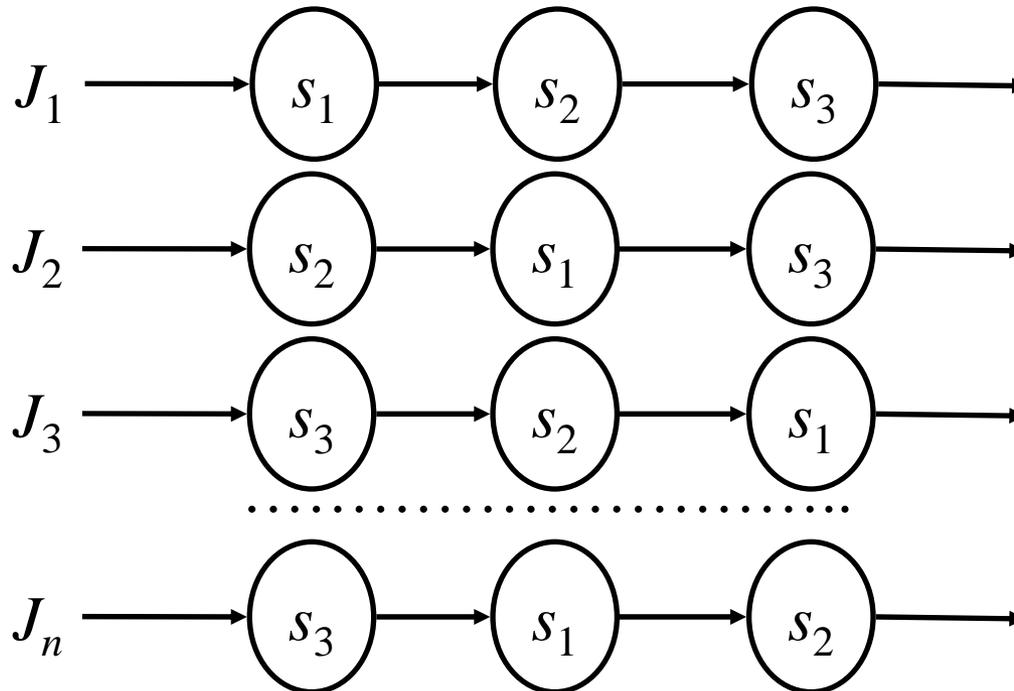
- Each job has, in general, a different route to be processed by the machines
- It is one of the most complex cases



# Scheduling environment (VII)

- Flexible Job shops

- Each job has, in general, a different route to be processed at all stages
- Each stage has  $m_s$  machines in parallel
- It is even more complex than the job shop



# Constraints

- There may be additional constraints for the scheduling problem:
  - Release dates
  - Setup-times
  - Pre-emption
  - Precedence constraints
  - Blocking
  - No-wait
  - ....

# Scheduling objectives (I)

- A scheduling objective is a measure to evaluate the quality of certain schedule
  - In real-life situations, there are many (sometimes conflicting) objectives
- There are based on the completion times:
  - $C_{ij}$ : Time in which job  $j$  is finished in machine  $I$
  - $C_j$ : Time in which the job  $j$  is finished in the last machine
- It is easy to see that, for the flow shop case, the completion time for a job in the position  $[k]$  is:

$$C_{[k],j} = \max(C_{[k-1],j}; C_{[k],j-1}) + p_{[k],j}$$

assuming  $C_{[k],0} = 0$ , and  $C_{[0],j} = 0$

# Scheduling objectives (II)

- Rather often, not all jobs (customers) are equally important
  - Therefore, one can assign a weight  $w_j$  to each job representing the relative importance of each job
- In general, one can distinguish two types of objectives:
  - Due date related objectives
  - Non-due date related objectives

# Scheduling objectives (III)

## Due date related objectives (I)

- For this kind of problems, we assume that each job  $j$  has, in general, a due date  $d_j$  and a release date  $r_j$ 
  - The due date represents the commitment of the company with a customer
  - The release date implies the non availability of raw materials from the beginning
- When each job has a due date, a basic objective is fulfilling this due date
  - Indicator of the service level
  - However, finishing the order as soon as possible (much before the due date) is not a good idea
    - Inventory costs

# Scheduling objectives (IV)

## Due date related objectives (II)

- Lateness of a job:  $L_j = C_j - d_j$ 
  - Maximum lateness:  $L_{max} = \max(L_j)$
  - Average (total) lateness:  $\underline{L} = \sum L_{ij} / n$
  - Weighted lateness:  $wL = \sum w_j L_j$
- Tardiness of a job:  $T_j = \max(0, L_j)$ 
  - Maximum tardiness  $T_{max} = \max(T_j)$
  - Average (total) tardiness:  $\underline{T} = \sum T_j / n$
  - Weighted tardiness:  $wT = \sum w_j T_j$
  - Number of tardy jobs:  $U = \sum U_j$ ;  $U_j = 0$  if  $T_j = 0$ ,  $U_j = 1$  if  $T_j \neq 0$
- Earliness of a job:  $E_j = \max(0, -L_j)$ 
  - Maximum earliness  $E_{max} = \max(E_j)$
  - Average (total) earliness:  $\underline{E} = \sum E_j / n$
  - Weighted earliness:  $wE = \sum w_j E_j$

# Scheduling objectives (V)

## Non due date objectives

### ■ Machine utilisation

- Makespan:  $C_{max} = \max(C_j)$
- Idle time
  - Time after finishing one job and before starting the next one
  - It can be shown that minimising makespan is equivalent to minimising idle time

### ■ Average lead time

- Average (total) completion time:  $\sum C_j$
- Average weighted completion times  $\sum w_j C_j$

# Break

