Introduction to production scheduling

Industrial Management Group
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Introduction to production scheduling

- Scheduling
- Production scheduling
- Gantt Chart
- Scheduling environment
- Constraints
- Scheduling objectives
Scheduling (I)

- Definition
  - Scheduling deals with the efficient allocation of tasks over resources
  - The general scheduling problem is, given a number of tasks and a number of resources, set the dates when each task should be accomplished on each resource

- We are interested in scheduling in the manufacturing context, although it has many applications in other fields
Scheduling (II)

- Real-life examples
  - Timetabling
    - Scheduling of the PC room at Duisburg University: a number of different courses (tasks) have to be given using the PC room (resource)
  - Workforce scheduling
    - Assign shifts for nurses and doctors in a Hospital
  - Sports scheduling
Production Scheduling (I)

Production planning, master scheduling

Quantities, due dates

Material Requirements Planning

Shop orders, release dates

Scheduling

Schedule

Shop floor control

Job loading

Shop floor

Orders, demand forecasts

Capacity status

Scheduling constraints
The MRP tell us the quantities of products to manufacture in every time bucket

- However, MRP does not make any assumption about the resources (i.e. labour, machines) currently available in the factory
  - E.g. two different components have to be manufactured in the same section. How to schedule them?

Therefore, we have a number of jobs \((j)\) to be manufactured over a number of machines \((i)\)

- The production scheduling problem deals with obtaining the date for each job to enter on each machine
- Not necessarily physical machines, they may be stages (consisting on several machines or labour) in a manufacturing process
Production Scheduling (III)

- Jobs have to be manufactured in each machine in a certain order (known as route) during a certain time period (known as processing time)
  - Processing time of job $j$ in machine $i$ is usually denoted by $p_{i,j}$

- Both route and time period are given by the technological process
Tomcat Ltd. is a company that assembles computers. Three main steps can be distinguished in this process:
- Motherboard & microprocessor are installed
- Peripheral devices are plugged in the motherboard
  - The number and type of devices that have been ordered by each customer
- The computer (all its components) are tested
  - This process depends on the number and type of components

The route through the steps is given by the technological process and does not depend on the specific order of the customer.
Example 1: Computer assembly (II)

- The plant in which Tomcat Ltd. assembles the computers is organised in three sections, according to the three main steps:
  - Section 1: Motherboard & microprocessor
  - Section 2: Peripheral devices
  - Section 3: Computer test

- On each section, one worker is performing the corresponding operation
  - Obviously, a new order cannot start until the worker completes the current order
  - Let us assume that an order cannot overtake another order, i.e. the job sequence is the same for all steps
Example 1: Computer assembly (III)

- Let us assume that we have three orders (computers to manufacture).
  - According to the nature of each order (components, type, etc.), we can have some estimate of the average times (minutes) for each step:

<table>
<thead>
<tr>
<th>Order</th>
<th>Motherboard</th>
<th>Devices</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>2</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>#2</td>
<td>3</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>#3</td>
<td>2</td>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>

- The objective of the company is to keep the average time to assembly a computer as lowest as possible.
Example 1: Computer assembly (IV)

- Which of the following sequences is the most convenient for the objective of the company:
  - #1, #2, #3?
  - #3, #2, #1?
  - Or the order of the jobs is not relevant for the final result?

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A representation of a specific solution of a scheduling problem in terms of the machines and jobs

- Eg. job1, job2, job3, job4
- Sequence [1,2,3,4]
Example 2: Representation of solutions (I)

- Using the data of Example 1, try to represent the sequences [1,2,3] and [3,2,1] by a Gantt chart.

<table>
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Solution of Example2
The environment or framework of a scheduling problem refers to the way the jobs must visit the machines:

- Single machine
- Parallel machines
- Flow shops
- Flexible flow shops
- Job shops
- Flexible job shops
- Open shop

The environment largely determines the difficulty of the scheduling problem
Scheduling environment (II)

- **Single machine**
  - The simplest of all machine environments
  - One may reduce the different steps (sections) in the plant to a single machine
  - Interesting case: bottleneck process, the important issue is scheduling jobs in the bottleneck

\[ J_1, J_2, \ldots, J_n \rightarrow M_1 \]
Scheduling environment (III)

- Parallel machines
  - All machines are identical
  - A job can be processed on any machine
  - Generalization of the single machine
  - Special case of the flexible flow shop and flexible job shop

\[ J_1, J_2, \ldots, J_n \]
Flow shops

- All jobs have the same routing

Additionally, most of the times it is consider than the sequence is the same for all machines

- Permutation flow shop, e.g. in Tomcat Ltd
Scheduling environment (V)

- Flexible flow shop
  - $s$ stages with $m_s$ machines in parallel
  - A job can be processed on any machine at each stage
  - Generalization of parallel machines and flow shop

$$J_1, J_2, \ldots, J_n$$

Stage 1  Stage 2  ...  Stage $s$
Scheduling environment (VI)

- **Job shops**
  - Each job has, in general, a different route to be processed by the machines
  - It is one of the most complex cases

```
J_1 -> M_1 -> M_2 -> M_3
J_2 -> M_2 -> M_1 -> M_3
J_3 -> M_3 -> M_2 -> M_1
J_n -> M_3 -> M_1 -> M_2
```
Scheduling environment (VII)

- **Flexible Job shops**
  - Each job has, in general, a different route to be processed at all stages
  - Each stage has $m_s$ machines in parallel
  - It is even more complex than the job shop

![Diagram of flexible job shops](image-url)
Constraints

- There may be additional constraints for the scheduling problem:
  - Release dates
  - Setup-times
  - Pre-emption
  - Precedence constraints
  - Blocking
  - No-wait
  - ....
Scheduling objectives (I)

- A scheduling objective is a measure to evaluate the quality of certain schedule
  - In real-life situations, there are many (sometimes conflicting) objectives

- There are based on the completion times:
  - $C_{i,j}$: Time in which job $j$ is finished in machine $I$
  - $C_j$: Time in which the job $j$ is finished in the last machine

- It is easy to see that, for the flow shop case, the completion time for a job in the position $[k]$ is:
  \[
  C_{[k],j} = \max(C_{[k-1],j} ; C_{[k],j-1}) + p_{[k],j}
  \]
  assuming $C_{[k],0} = 0$, and $C_{[0],j} = 0$
Scheduling objectives (II)

- Rather often, not all jobs (customers) are equally important
  - Therefore, one can assign a weight $w_j$ to each job representing the relative importance of each job
- In general, one can distinguish two types of objectives:
  - Due date related objectives
  - Non-due date related objectives
Due date related objectives (I)

- For this kind of problems, we assume that each job $j$ has, in general, a due date $d_j$ and a release date $r_j$
  - The due date represents the commitment of the company with a customer
  - The release date implies the non-availability of raw materials from the beginning
- When each job has a due date, a basic objective is fulfilling this due date
  - Indicator of the service level
  - However, finishing the order as soon as possible (much before the due date) is not a good idea
    - Inventory costs
Scheduling objectives (IV)

Due date related objectives (II)

- Lateness of a job: \( L_j = C_j - d_j \)
  - Maximum lateness: \( L_{\text{max}} = \max(L_j) \)
  - Average (total) lateness: \( L = \sum L_j / n \)
  - Weighted lateness: \( wL = \sum w_j L_j \)

- Tardiness of a job: \( T_j = \max(0, L_j) \)
  - Maximum tardiness \( T_{\text{max}} = \max(T_j) \)
  - Average (total) tardiness: \( T = \sum T_j / n \)
  - Weighted tardiness: \( wT = \sum w_j T_j \)
  - Number of tardy jobs: \( U = \sum U_j ; U_j = 0 \text{ if } T_j=0, U_j=1 \text{ if } T_j\neq0 \)

- Earliness of a job: \( E_j = \max(0, -L_j) \)
  - Maximum earliness \( E_{\text{max}} = \max(E_j) \)
  - Average (total) earliness: \( E = \sum E_j / n \)
  - Weighted earliness: \( wE = \sum w_j E_j \)
Scheduling objectives (V)

Non due date objectives

- Machine utilisation
  - Makespan: $C_{max} = \max(C_j)$
  - Idle time
    - Time after finishing one job and before starting the next one
    - It can be shown that minimising makespan is equivalent to minimising idle time

- Average lead time
  - Average (total) completion time: $\Sigma C_j$
  - Average weighted completion times $\Sigma w_j C_j$
Break