Digital Papercutting

Yanxi Liu and James Hays
Carnegie Mellon University

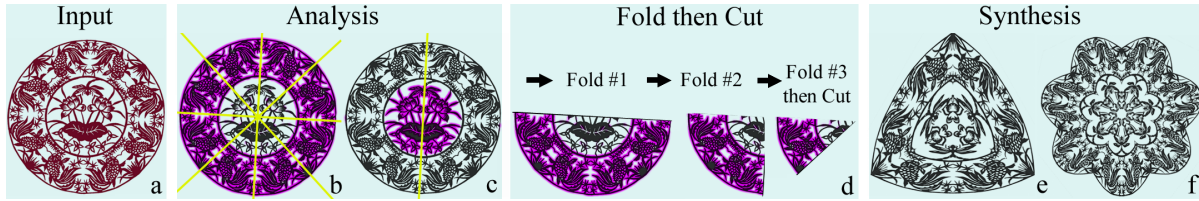Ying-Qing Xu and Heung-Yeung Shum
Microsoft Research Asia

Figure 1: Given a papercut-pattern (a), our algorithm performs analysis, fold-then-cut plan generation and synthesis. The algorithm identifies the dihedral symmetry group D_{2x4} in the outer-ring of the pattern (b), the inner bilateral symmetry (c) as well as the asymmetry region (the tail of the dragonfly). The fold-then-cut plan is generated by symmetry group indexing of each sub-region of the papercut pattern (d). Finally, new papercut patterns with the same motif as the input but different symmetry groups (D_{2x3}, D_{2x7} respectively in e and f) are synthesized.

SYNOPSIS We develop a symmetry-based computer analysis and synthesis platform that, given a digitized image of a real-world papercut-pattern, is able to (1) identify dihedral and frieze symmetry as well as asymmetric sub-patterns of the input; (2) generate a fold-then-cut plan that can be used to recreate the input papercut-pattern; and (3) synthesize new papercut patterns by varying the motif automatically extracted from the input, or by varying the symmetry group describing the structural organization of the input papercut pattern. We formalize a papercut-pattern as a composition of multiple sub-patterns that are either reflection symmetric (with dihedral or frieze symmetry groups) or reflection asymmetric.

INTRODUCTION Symmetry or deviation from symmetry is the central theme of many different art forms, ranging from music, painting, sculpture to various decorative and folk arts. One of the most popular and accessible such art forms is that of papercutting, where symmetry is used both in its artistic design and its generation process. Intrigued by this effective, seamless combination, we are interested in using computational means to reveal the relationship between the form and generative process for a digitized papercut-pattern. Regardless of how complicated and diverse a papercutting design might be, each papercut pattern is connected and often presents global or local symmetry. We are interested in those papercut-patterns containing non-trivial global and/or local *reflection symmetry* such that fold-then-cut operations are *necessary* steps in their generative process. The computational challenges in symmetry-based analysis of real-world patterns include discriminating different types of symmetry structures in global and local regions; and distinguishing between "near-symmetries" as deviations from implied symmetry and truly asymmetric designs. [Demaine and Lubiw 2000] show that any polygonal shapes can be generated through multiple foldings followed by a single straight-cut. The simultaneous foldings required, however, are often rather challenging physically. Formal and algorithmic treatment of general papercut patterns with a non-polyhedral nature remains scarce.

ANALYSIS, SYNTHESIS and FOLD-then-CUT We define a papercut-pattern P as $P = \cup S_i$, where S_i are edge point sets such that $S_i \cap S_j = \emptyset, i \neq j$. Each S_i belongs to one of the three types: (1) patterns with dihedral symmetry groups $D_{2n}, n = 1, 2, ...$; (2) patterns with frieze groups that have reflections; and (3) patterns that have no reflection symmetries. Only pattern types (1) and (2) above can lead to a fold-then-cut generative procedure in production, while type (3) can only be generated by direct cutting without folding. Thus P can either be *globally* or *locally* symmetrical. Geometrically, each reflection symmetry in a papercut-pattern provides a valid fold which reduces the workload of cutting in half. Therefore continued folding reduces workload exponentially to $1/2, 1/4, 1/8, ... 1/2^N$ of the original work, where N is the number of folds. Such folds can continue until the remaining pattern contains no more reflection symmetry, such a region is called a *fundamental region* or *motif* of the corresponding symmetry group. Topologically, each fold also achieves a reduction in genus numbers in the remaining unfolded region, leaving less holes to cut. In its simplest topological form, a fundamental region can be isomorphic to a disk. Our symmetry-based papercut-pattern analysis and synthesis procedure captures these insights: (1) (analysis) find all the reflection symmetries in a given image of a papercut-pattern at different scales, and capture those symmetry structures that form a relevant symmetry group G as well as that portion of the pattern supporting G; (2) (synthesis) use the automatically found fundamental region to generate papercut-patterns of different forms with the same symmetry group, or similar forms of different symmetry groups. Let the user choose his/her favorite pattern. (3) (generation) the symmetry group-based analysis provides a proper index into the different fold-then-cut plans for different subregions. This last step reflects once again the practical value of a symmetry group-based approach on "compression": the foldings of the fold-then-cut plan of a specific pattern only depend on the symmetry axes of the pattern, not the exact contours of the pattern. In another word, two papercut-patterns that have the same symmetry group can be folded in exactly the same way before cutting.

SYMMETRY DETECTION ALGORITHM Given a papercut-pattern P, for each potential reflection symmetry axis $l(r, \theta)$ on P, our algorithm verifies whether l is a potential reflection symmetry axis of P by: (1) reflect P about line l to obtain $P_{ref(l)}$. (2) compute a symmetry score $S_l = \Sigma cos(\phi)$ where ϕ is the angle between the orientations of corresponding edges of $P(x, y)$ and $P_{ref(l)}(x, y)$ at pixel (x, y). We search peaks in $M(r, \theta) = S_{l(r, \theta)}$ to locate those axes l with the highest scores. Furthermore, we can capture collections of $l(r, \theta)$ whose structure indicates a desired symmetry group. Near-symmetry is treated locally (tolerate differences in a small neighborhood) and globally (symmetry group based clustering) The algorithm is implemented in MATLAB. For each pattern P, 1 to 5 minutes are needed to complete the symmetry detection task (depending on the resolution and the complexity of the papercut).

References

DEMAINE, E.D., D. M., AND LUBIW, A. 2000. Folding and cutting paper. In *JCDCG'98: Revised Papers from the Japanese Conference on Discrete and Computational Geometry, Springer-Verlag*, 104–118.