

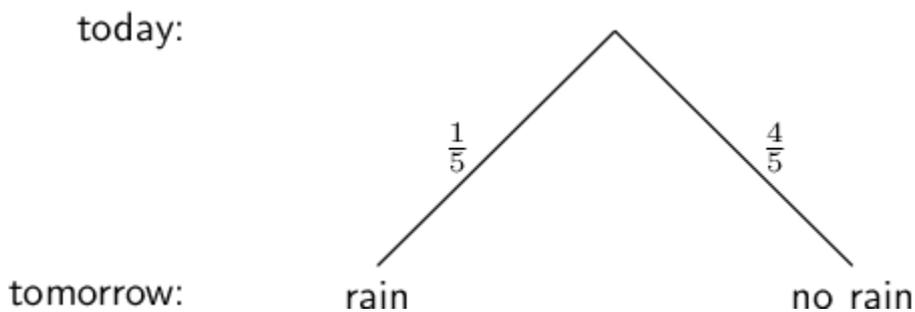
TREE DIAGRAMS:

Example: If it rains on a given day, the probability that it rains the next day is $\frac{1}{3}$. If it does not rain on a given day, the probability that it rains the next day is $\frac{1}{6}$. The probability that it will rain tomorrow is $\frac{1}{5}$. What is the probability that it will rain the day after tomorrow? Draw a tree diagram of all the possibilities to determine the answer.

Answer:

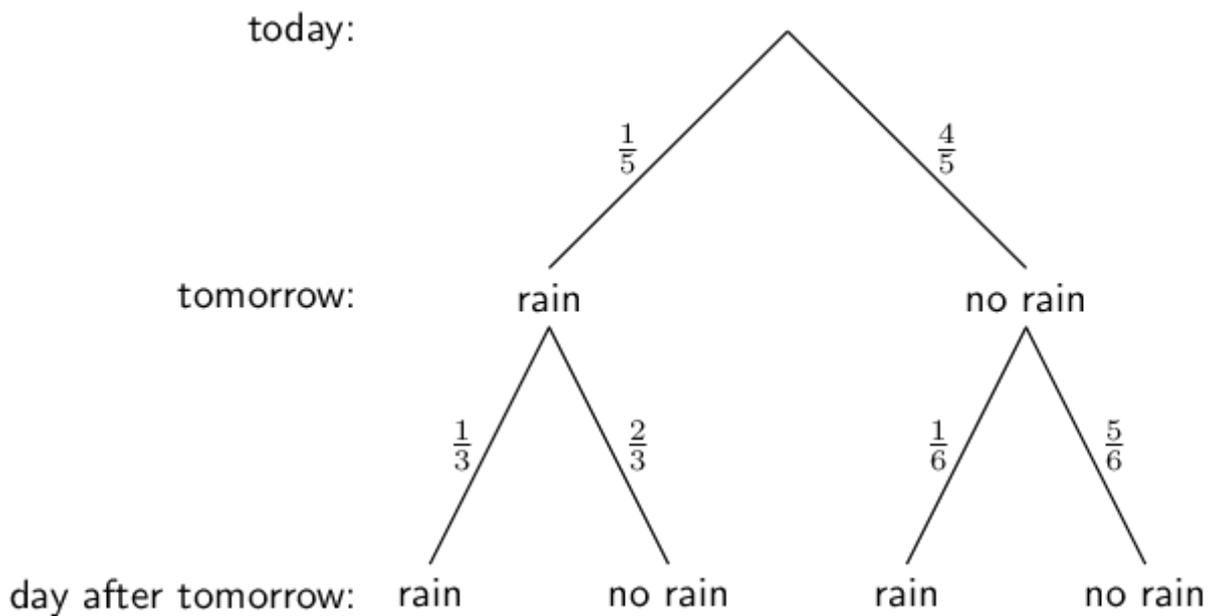
Draw the first level of the tree diagram

Before we can determine what happens on the day after tomorrow, we first have to determine what might happen tomorrow. We are told that there is a $\frac{1}{5}$ probability that it will rain tomorrow. Here is how to represent this information using a tree diagram:



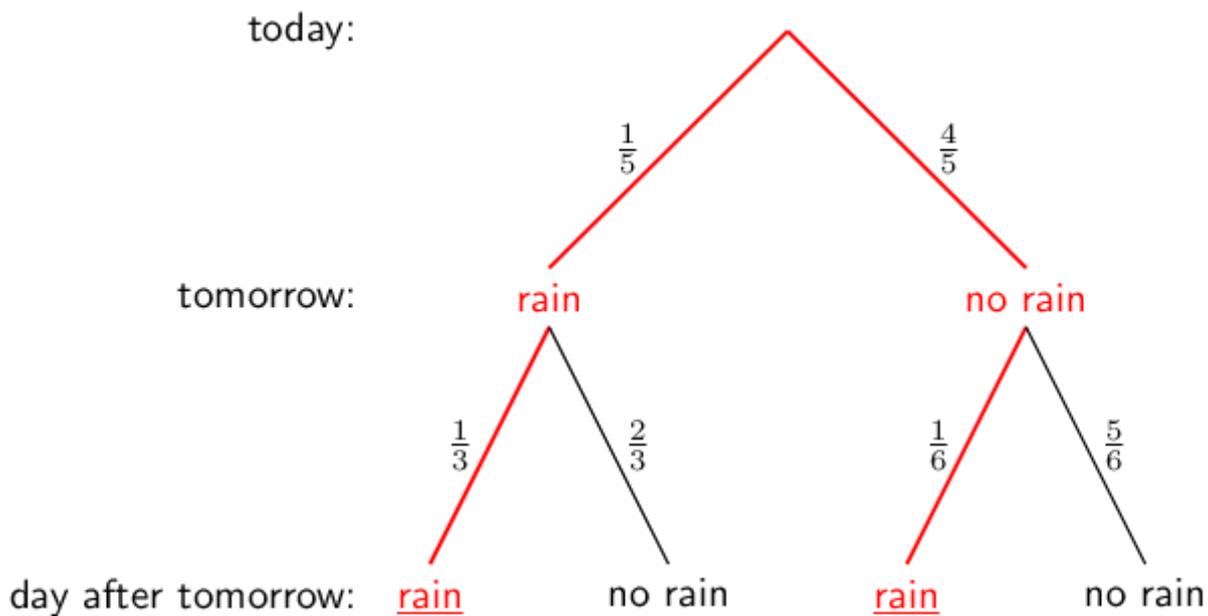
Draw the second level of the tree diagram

We are also told that if it *does* rain on one day, there is a $\frac{1}{3}$ probability that it will also rain on the following day. On the other hand, if it *does not* rain on one day, there is only a $\frac{1}{6}$ probability that it will also rain on the following day. Using this information we complete the tree diagram:



Compute the probability

We are asked what the probability is that it will rain the day after tomorrow. On the tree diagram above we can see that there are 2 situations where it rains on the day after tomorrow. They are marked in red below.



To get the probability for the first situation (that it rains tomorrow and the day after tomorrow) we have to multiply the probabilities along the first red line.

$$= = P(\text{rain tomorrow and rain day after tomorrow}) \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$$

To get the probability for the second situation (that it does not rain tomorrow, but it does rain the day after tomorrow) we have to multiply the probabilities along the second red line.

$$= = P(\text{not rain tomorrow and rain day after tomorrow}) \frac{4}{5} \times \frac{1}{6} = \frac{2}{15}$$

Therefore the total probability that it will rain the day after tomorrow is the sum of the probabilities along the two red paths, namely

$$\frac{1}{15} + \frac{2}{15} = \frac{1}{5}$$

Example 2: Drawing a tree diagram

Question:

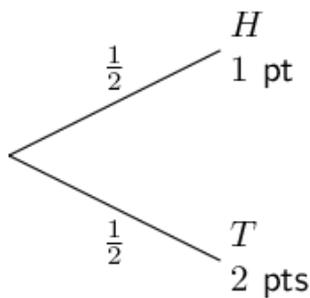
You play the following game. You flip a coin. If it comes up tails, you get 2 points and your turn ends. If it comes up heads, you get only 1 point, but you can flip the coin again. If you flip the coin multiple times in one turn, you add up the points. You can flip the coin at most 3 times in one turn. What is the probability that you will get exactly 3 points in one turn? Draw a tree diagram to visualise the different possibilities.

Answer

Write down the events and their symbols

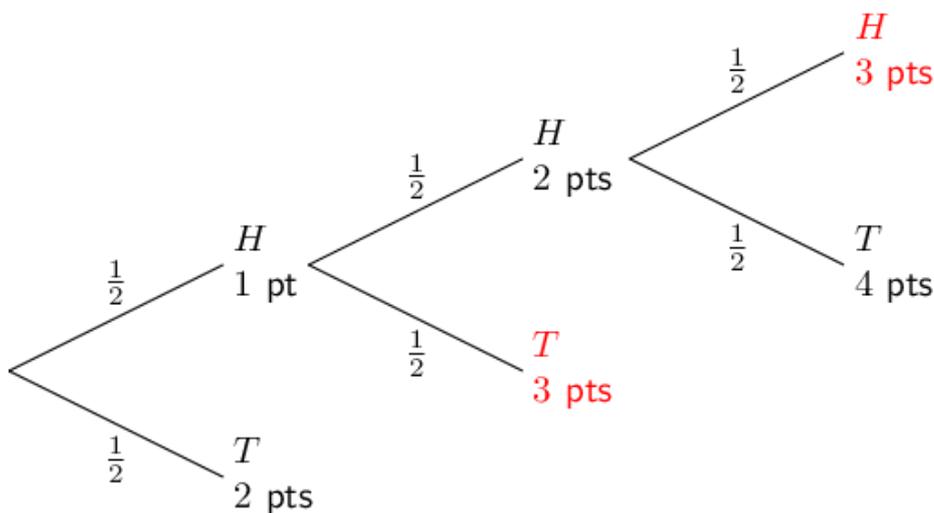
Each coin toss has one of two possible outcomes, namely heads (H) and tails (T). Each outcome has a probability of $\frac{1}{2}$. We are asked to count the number of points, so we will also indicate how many points we have for each outcome.

Draw the first level of the tree diagram



This tree diagram shows the possible outcomes after 1 flip of the coin. Remember that we can have up to 3 flips, so the diagram is not complete yet. If the coin comes up heads, we flip the coin again. If the coin comes up tails, we stop.

Draw the second and third level of the tree diagram



In this tree diagram you can see that we add up the points we get with each coin flip. After three coin flips, the game is over.

Find the relevant outcomes and compute the probability

We are interested in getting exactly 3 points during the game. To find these outcomes we look only at the tips of the tree. We end with exactly 3 points when the coin flips are

- (H; T) with probability $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$;
- (H; H; H) with probability $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Notice that we compute the probability of an outcome by multiplying all the probabilities along the path from the start of the tree to the tip where the outcome is. We add the above two probabilities to obtain the final probability of getting exactly 3 points as $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$.

